Signal Decorrelation Induced by Linear Distortion in a Transfer System

Décorrélation provoquée par la distorsion linéaire du système de transfert

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Abstract

Theoretical estimates of the changes in signal correlation function caused by nonlinear phase and non-constant amplitude characteristics of a transfer system are derived. Several simple approximations and bounds are presented. A review of the experimental verification is also given.

Key words

Signal Decorrelation, Linear Distortion.

Résumé

Nous obtenons des estimations théoriques sur les changements de la fonction de corrélation d'un signal causée par la non-linéarité de phase et les variations d'amplitude de la fonction du transfert d'un système. Nous donnons plusieurs bornes et approximations simples. Nous présentons une vérification expérimentale.

Mots clés

Décorrélation d'un signal, distorsion linéaire.

1. Introduction

Nonlinear phase or non-constant amplitude frequency characteristics of a linear transfer system cause linear distortion of a transferred signal. One of the consequences of the distortion is a change in signal correlation properties. Such correlation loss is particularly important if two or more correlated signals are transferred through a multi-channel transfer system. Examples of such systems are multi-sensor beam-forming receiving or transmitting systems, multi-channel magnetic tape recorders, etc.

In this paper theoretical estimates for the loss of signal correlation due to the nonlinearity of transfer system phase and the non-constancy of amplitude frequency characteristics are derived. A simple system consisting of two independent channels is assumed. The same signal is applied to both inputs to model the correlated component of input signals.

2. Estimate of the correlation loss

Referring to Figure 1, $H_a(f)$ and $H_b(f)$ are complex frequency characteristics of the two channels, $G(f)$ and $R(\tau)$ are power spectrum density and auto-correlation function, respectively, of a stationary signal applied to both inputs, and $R_{ba}(\tau)$ is the cross-correlation function of two output signals. In order to estimate the influence of the system characteristics, $R_{ba}(\tau)$ has to be compared to

$R(\tau)$. In the ideal case, when $\arg H_a(f) = \arg H_b(f)$ and $|H_a(f)| \equiv |H_b(f)| \equiv A_0^{1/2} = \text{const.}$, the following is valid : $R_{ba}(\tau) \equiv A_0 R(\tau)$. If the two channels differ in the slopes of phase characteristics only, $R_{ba}(\tau)$ would be a shifted version of $A_0 R(\tau)$. In all other cases distortion i.e. the correlation loss arises, and $R_{ba}(\tau)$ does not fit $A_0 R(\tau)$ even after a proper time shift is applied.

Denoting by $S(f) \equiv G(|f|)/2$ the two-sided power spectrum density of the input signal [1], two-sided cross-spectrum power density of the output signals follows: $H_b^*(f) H_a(f) S(f)$ [2]; here the asterisk denotes a complex-conjugate value. The two channels of the system in figure 1 act jointly, so that it seems appropriate to describe their characteristics by the constants A_0 and t_0, and the functions $A_n(f)$ and $\phi_n(f)$, in the following way:

$$(1)\quad H_b^*(f) H_a(f) = A_0[1 + A_n(f)]\, e^{-i[2\pi t_0 f + \phi_n(f)]};$$

A_0 is the mean value of the product of the channels amplitude frequency characteristics, $A_n(f)$ is the relative value of the non-constant component of this product, t_0 is the difference of the mean delays to which the signal is subjected when passing through the system, and $\phi_n(f)$ is the nonlinear component of the difference of the channels phase characteristics.

$R_{ba}(\tau)$ is the inverse Fourier transform of a two-sided cross-spectrum power density. Thus, taking into account equation (1), one has

$$(2)\quad R_{ba}(\tau) = A_0 \int_{-\infty}^{\infty} S(f)[1 + A_n(f)]\, e^{i 2\pi(\tau - t_0) f - i\phi_n(f)}\, df .$$

In the ideal case, when the characteristics are constant viz. linear, $A_n(f) \equiv 0$, $\phi_n(f) \equiv 0$, equation (2) yields

$$R_{ba}(\tau) = A_0 \int_{-\infty}^{\infty} S(f)\, e^{i 2\pi(\tau - t_0) f}\, df = A_0 R(\tau - t_0),$$

which is the shifted and scaled replica of the input correlation function, $R(\tau)$. Thus, in order to describe the distortion of $R(\tau)$ induced by both $A_n(f)$ and $\phi_n(f)$, the function

$$(3)\quad \Delta R(\tau) = R_{ba}(\tau + t_0)/A_0 - R(\tau)$$

has to be used.

If the parity of the functions is taken into account, $A_n(-f) = A_n(f)$, $\phi_n(-f) = -\phi_n(f)$, $S(-f) = S(f)$, and if a one-sided measurable function $G(f)$ is introduced instead of $S(f)$, the equations (2) and (3) yield

$$(4)\quad \Delta R(\tau) = \int_0^{\infty} G(f)\{[1 + A_n(f)] \cos(2\pi\tau f - \phi_n(f)) - \cos 2\pi\tau f\}\, df .$$

This is the general estimate of the correlation loss caused by the non-ideal form of the system frequency characteristics. The signal is described by $G(f)$, and the system by $A_n(f)$ and $\phi_n(f)$.

The definitions of $A_n(f)$ and $\phi_n(f)$, i.e. A_0 and t_0, as given above, are not appropriate for the present analysis. In what follows alternative definitions involving the signal are introduced. A_0 and $A_n(f)$ will be fixed by the requirement

$$(5)\quad \int_0^{\infty} G(f) A_n(f) = 0$$

which precises the meaning of the mean value, A_0: it is defined by means of the signal power spectrum density as a weight. As to the components of the difference,

$$(6)\quad \phi(f) = 2\pi t_0 f + \phi_n(f),$$

of the channels phase characteristics, their separation will be defined by the requirement that $R_{ba}(\tau)$ attains its maximum value in $\tau = t_0$. Let us find such a value of t_0, assuming the nonlinear component $\phi_n(f)$ to be small and $\phi_n(f)$ negligible.

Using equation (6) in (2), and replacing $\phi(f)$ by $2\pi\tau_f(f) f$, where $\tau_f(f)$ is the phase delay characteristic corresponding to the difference $\phi(f)$, one obtains

$$(7)\quad R_{ba}(\tau) = A_0 \int_{-\infty}^{\infty} S(f)\, e^{i 2\pi[\tau - \tau_f(f)] f}\, df$$

for the case $A_n(f) \equiv 0$. In the chosen approximation, when the changes of $\tau_f(f)$ are small, the maximum of $R_{ba}(\tau)$ comes in $\tau = t_0$, t_0 being equal to some value of $\tau_f(f)$. Thus, when $\tau \to t_0$, the exponent $2\pi[\tau - \tau_f(f)] f$ becomes small, so that exp(.) can be approximated by the first three terms of its power series. Taking into account the parity of these terms and replacing $S(f)$ by $G(|f|)/2$, equation (7) yields

$$R_{ba}(\tau) \simeq A_0 \int_0^{\infty} G(f)\{1 - 2\pi^2[\tau - \tau_f(f)]^2 f^2\}\, df, \quad (\tau \simeq t_0).$$

Now, searching for the position of the maximum, $dR_{ba}(\tau)/d\tau = 0 \Rightarrow \tau = t_0$, the estimate

$$(8)\quad t_0 = \int_0^{\infty} f^2 \tau_f(f) G(f)\, df \Big/ \int_0^{\infty} f^2 G(f)\, df ,$$

viz.

$$(9)\quad t_0 = \int_0^{\infty} f\phi(f) G(f)\, df \Big/ \int_0^{\infty} 2\pi f^2 G(f)\, df ,$$

follows. Being related to the derivative, it differs quite from the estimate

$$(10)\quad \int_0^{\infty} \tau_f(f) G(f)\, df \Big/ \int_0^{\infty} G(f)\, df ,$$

which would result if a requirement analogous to (5) were imposed on the phase delay characteristics.

The definitions given by equations (5) and (6)/(8)/(9) fix the functions $A_n(f)$ and $\phi_n(f)$, so that the resulting model of the change of the signal correlation function due to linear distortion in a transfer system, as presented by equation (4), becomes effective.

3. Approximations and bounds

Developing the functions in equation (4) in power series and omitting higher order terms, one obtains the approximation valid in the case of small non-constant viz. nonlinear components of the frequency characteristics:

$$(11) \quad \Delta R(\tau) \simeq \int_0^\infty G(f)\, A_n(f) \cos 2\pi\tau f\, df + \int_0^\infty G(f)\, \phi_n(f) \sin 2\pi\tau f\, df - \int_0^\infty G(f)[\phi_n^2(f)/2] \cos 2\pi\tau f\, df$$

$$(|A_n(f)| \ll 1 \quad \text{and} \quad |\phi_n(f)| \ll 2\pi, \quad \text{for} \quad f \in F);$$

F donotes the frequency band covered by the signal, i.e. the band in which $F(f)$ is not negligible.

Based on the estimate of the change of the correlation function maximum,

$$(12) \quad \Delta R(0) \simeq -\int_0^\infty G(f)[\phi_n^2(f)/2]\, df$$

$$(|A_n(f)| \ll 1 \quad \text{and} \quad |\phi_n(f)| \ll 2\pi \quad \text{for} \quad f \in F),$$

which follows from equations (11) and (5), one can compare the influence of $A_n(f)$ and $\phi_n(f)$. Since both $G(f)$ and $\phi_n^2(f)$ are non-negative, the effect induced by the nonlinearity of phase characteristics accumulates when F is broadened. At the same time, in the chosen approximation, the effect due to $A_n(f)$ disappeares. Thus, in most cases the phase characteristic nonlinearity will be the dominant cause of the signal correlation loss.

Simple estimates of the modulus of the integral by the integral of the modulus, simple upper bounds like x^2 for $\sin^2 x$, etc., accompanied by the identity $R(0) = \int_0^\infty G(f)\, df$, yield, owing to the fact that $G(f)$ is non-negative, the following :

$$|\Delta R(0)| \leq \int_0^\infty G(f)[|A_n(f)| + \phi_n^2(f)/2]\, df ,$$

$$(13) \quad \left| \frac{\Delta R(\tau)}{R(0)} \right| \leq 2 + \max_{f \in F} \{|A_n(f)|\} ,$$

$$(14) \quad \left| \frac{\Delta R(0)}{R(0)} \right| \leq \max_{f \in F} \{|A_n(f)| + \phi_n^2(f)/2\} .$$

If the signal spectrum can be considered almost constant, $G(f) \simeq$ const., $f \in F$, equation (12) yields an interesting simple estimate:

$$(15) \quad \frac{\Delta R(0)}{R(0)} \simeq -\frac{1}{2} \langle \phi_n^2(f) \rangle$$

$$(|A_n(f)| \ll 1 \quad \text{and} \quad |\phi_n(f)| \ll 2\pi \quad \text{for} \quad f \in F);$$

here $\langle . \rangle$ denotes averaging over the frequencies $f \in F$.

The case of (basically) constant amplitude characteristics is rather important; equation (4) yields for it:

$$\Delta R(0) \leq 0 \qquad (A_n(f) = 0 \quad \text{for} \quad f \in F) .$$

With equation (13) this implies $-2 \leq \Delta R(0)/R(0) \leq 0$, meaning that, as already shown by equation (15), only a decrease of the maximum $R(0)$ arises due to the phase nonlinearity:

$$-1 \leq \frac{R(0) + \Delta R(0)}{R(0)} \leq 1, \quad (A_n(f) = 0 \quad \text{for} \quad f \in F) .$$

Further, from this and equation (14) it follows

$$-\frac{\Delta R(0)}{R(0)} \leq \max_{f \in F} \{\phi_n^2(f)/2\}, \ (A_n(f) = 0 \quad \text{for} \quad f \in F) .$$

4. Experimental verification

For the special case $A_n(f) \equiv 0$ one obtains from equation (4)

$$(16) \quad \Delta R(\tau) = \int_0^\infty G(f)[-2 \sin^2(\phi_n(f)/2) \cos 2\pi f + \sin \phi_n(f) \sin 2\pi\tau f]\, df .$$

The experimental verification of this estimate is presented in what follows.

Stationary Gaissian noise of approximately white spectrum in the band 100 Hz-10 kHz was applied to the inputs of a two-channel system according to Figure 1. In the band occupied by the signal the amplitude frequency characteristics of the system were flat, $|H_a(f)| \equiv |H_b(f)| \equiv A_0 = 1$, $A_n(f) \equiv 0$, and the difference-phase characteristic $\phi(f)$, and the phase delay characteristic $\tau_f(f)$ were as in Figure 2. The power spectrum density

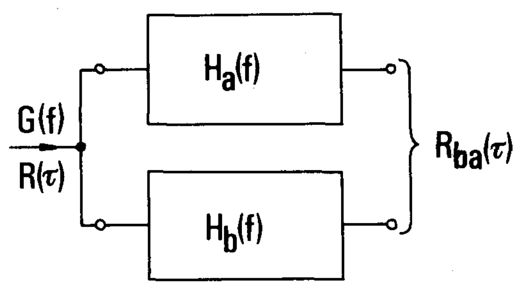

Figure 1. — Model of the system.

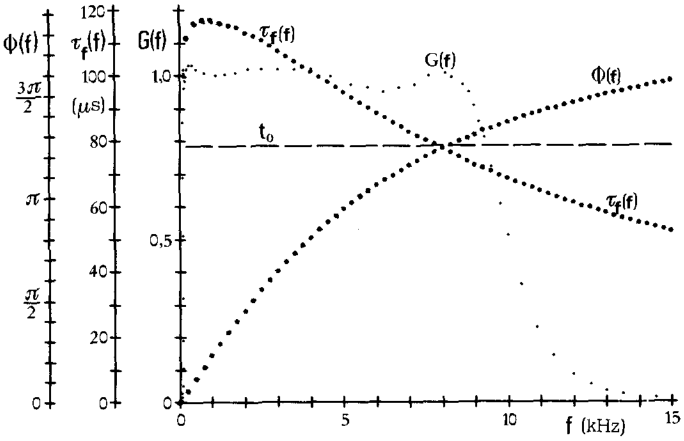

Figure 2. — Experimental verification : data on the system and the signal.

$G(f)$ of the input signal with an arbitrary amplitude scale is also given in Figure 2. From the given data, the position of the maximum t_0 denoted in Figure 2 was calculated by means of equations (8)/(9). As can be observed, t_0 differs appreciably from the estimate (10).

The input signal auto-correlation function $R(\tau)$ and the output cross-correlation function $R_{ba}(\tau)$ obtained by the measurement are given in Figure 3.

They were derived using efficient averaging, so that random errors are scarcely noticable in these results as well as in the data derived subsequently.

In Figure 3 also the estimate of t_0 is shown. As can be observed, t_0 predicts the position of the maximum very accurately.

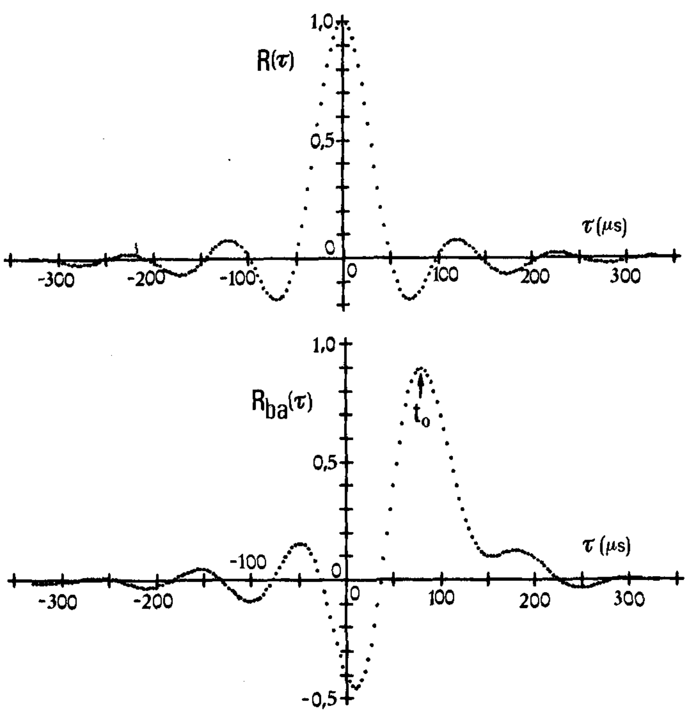

Figure 3. — Correlation functions obtained by the measurement.

From the given t_0, $R(\tau)$, $R_{ba}(\tau)$, and $A_0 = 1$, the measured correlation loss defined by equation (3) was calculated. The result is presented by the dotted line in Figure 4. The solid line curve in this figure is the result of the calculation according to equation (16) with $G(f)$ as given in Figure 2, and with $\phi_n(f)$ derived according to equation (6) from the calculated value t_0 and $\phi(f)$ as given in Figure 2. As may be observed the results of the measurement and the calculation are well matched. Thus, the theoretical estimate of the correlation loss induced by the phase characteristics nonlinearity, which is usually dominant, is experimentally verified.

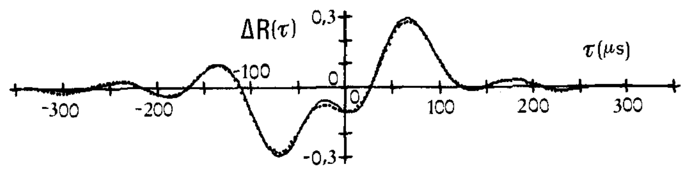

Figure 4. — Comparison of the experimental results (dotted line) and the theoretical prediction (solid line).

5. Conclusions

Loss of signal correlation induced by non-ideal characteristics of a linear transfer system is derived; the result is experimentally verified.

The loss is described by a change in the signal correlation function due to nonlinear phase and non-constant amplitude characteristics of the channels in a two-channel system. The appropriate definitions of the nonlinear viz. non-constant components appear to be signal-dependent. If small, these components act independently.

The relative change in the correlation function maximum is bounded by the sum of the maximum deviation of the product of the amplitude characteristics from a constant, and the half of the maximum squared deviation of the phase difference characteristic from a linear one. The resulting maximum amplitude of the correlation function cannot be higher than the original value.

If the amplitude characteristics are almost constant, so that there is only phase distortion, the maximum amplitude can only be decreased. In a white spectrum case, this decrease amounts to half squared phase-nonlinearity averaged over the frequency band covered by the signal.

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