

Inverse scattering

and acoustic resonance spectroscopy

Diffusion inverse et spectroscopie acoustique des résonances



Herbert ÜBERALL

Department of Physics, Catholic University of America, Washington DC 20064, USA

Professor of Physics, Catholic University of America, Washington DC 20064, USA (since 1964). Field of research: Theoretical Physics (acoustics, ultrasonics, elastic and electromagnetic waves, nuclear physics). *Acoustics*: Advancement of the theory of surface waves in acoustic scattering. Pioneering development of acoustic resonance scattering theory. *Channeling in under-ocean sound propagation*.

Elastic waves: First development of theory of resonance scattering by cavities and inclusions in solids.

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Nuclear theory: Basic development of coherent bremsstrahlung theory (editor of recent book). Development of a model of collective nuclear multipole vibrations, with application to electron scattering (author of two books). Photo-pion reactions. Collective model of ion-ion scattering. Neutrino reactions (editor of book).

SUMMARY

Echoes of acoustic or elastic waves scattered from a target carry within them the resonance features caused by the excitation of the eigenvibrations of the target. By means of a suitable background subtraction it is possible to isolate the target's spectrum of resonances. This spectrum characterizes the target just as an optical spectrum characterizes the chemical element or compound that emits it. Extracting the resonance information from the echo allows the possibility of identifying the target as to its size, shape, and composition. This is illustrated by studying the dependence of the resonance spectra of fluid targets upon changes of target shape, including variations from spheres to prolate spheroids and finite-length cylinders. The resulting "acoustic resonance spectroscopy" (a concept introduced by Derem) generates the same type of level scheme as in optics, and it may thus be used for solving some aspects of the "inverse scattering problem" (i. e., the problem of identification of the nature of the target from the returned echoes). A study along these lines shows that the eigenfrequencies of fluid-filled cavities in a solid medium (obtained by us in the complex frequency plane) are tied to resonance features in the scattering amplitude which can be analyzed to provide the material parameters (density and sound speed) of the fluid filler: the resonance spacing giving the sound speed, and the resonance widths the fluid density-hence leading to a solution of the inverse scattering problem in this case.

KEY WORDS

Acoustic resonance spectroscopy, inverse scattering, eigenvibration, target.

RÉSUMÉ

Les échos d'ondes acoustiques ou élastiques diffusés par une cible portent en eux des effets de résonances causés par l'excitation des vibrations propres de la cible. Par le moyen d'une soustraction appropriée du fond, il est possible d'isoler le spectre des résonances de la cible. Un tel spectre caractérise la cible autant qu'un spectre optique caractérise l'élément ou le composé chimique émetteur de ce spectre. L'extraction des échos de l'information sur les résonances offre la possibilité d'identifier la cible, par sa grandeur, sa forme, et sa composition. Nous illustrons cela en étudiant la dépendance des spectres de résonances de cibles fluides en changeant la forme de la cible, ceci inclus les variations entre une sphère et un sphéroïde oblong, ou un cylindre de longueur finie. Le concept de « spectroscopie acoustique des résonances » (concept introduit par Derem) a comme base le même type de schéma de niveaux que l'on trouve en optique, et il peut alors être utilisé pour résoudre en quelque sorte le « problème inverse » (c'est-à-dire l'identification de la nature de la cible en étudiant les échos). Une telle étude montre que les fréquences propres des cavités dans un milieu solide remplies d'un fluide (obtenues ici dans le plan complexe) sont liées aux effets de résonances dans l'amplitude du signal diffusé, qui peuvent être analysées afin de fournir les paramètres physiques du fluide : l'espacement des résonances donnant la vitesse du son, la largeur des résonances donnant la densité du fluide, menant ainsi à la solution du problème inverse dans ce cas.

MOTS CLÉS

Spectroscopie des résonances acoustiques, diffusion inverse, vibrations propres, cible.

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1. Introduction

In 1979, A. Derem wrote [1]: "L'analyse de la diffusion des ondes acoustiques par un cylindre élastique immergé, fait surgir une véritable *spectroscopie acoustique*". This was said following an analysis of effects of the resonances of elastic objects, after their excitation by incident sound, as they appear in the scattered echoes, and it seems to be the first formulation of this concept in conjunction with elastic resonance frequency schemes. (The previous term "ultrasonic spectroscopy", and the related early experiments of Gericke [2, 3], concern the general notion of the ultrasonic spectra reflected from scatterers in solids, and the information on these scatterers contained in the spectrum, not referring specifically to any resonances.) The new concept was brilliantly justified experimentally shortly afterwards by Maze, Ripoché *et al.* [4, 5] in their pioneering experiments which exhibited in a direct fashion the mechanical eigenvibrations of a

submerged elastic cylinder, together with the "level scheme" of the corresponding resonances excited by incident sound, straightforwardly extracted from the reflected echo [6]. Figure 1.1 shows the resonances in the reflected sound of an aluminum cylinder in water, as contained in the total echo (top), and as extracted experimentally (bottom) [7]. The bottom curve represents the "acoustic spectrum" of the cylinder, which appears strikingly similar to the optical spectrum, or level scheme, of atoms or chemical compounds.

These facts then raise the question whether, as in the optical case, such a measured spectrum can provide information on the source of the spectrum (here, the scattering object). This led to a study of the sensitivity of the acoustic level scheme for internal [8, 9] or external resonances [10, 11] to changes in the shape of the target. As to the information which can be extracted from the spectrum regarding the consistency of the target, we have carried out illustrative calculations on fluid-filled spherical [12, 13] or cylindrical [14] cavities in a solid, or on fluid cylinders in a fluid [15], which showed that in general, for a cavity of known shape, the sound velocity in the filler fluid can be obtained from the frequency values of the resonances (together with the size of the cavity), and the density of the filler fluid from the width of the resonances. Finally, the determination of the physical properties of ocean bottom layers has also been shown to be obtainable [16-18] from the effects of layer resonances contained in the acoustic echo.

The proposed solution of inverse scattering problems by utilizing target resonances represents, of course, only one particular approach towards the solution of inverse problems [19]. Other notable techniques in ultrasonics are the mentioned use of ultrasonic spectroscopy [20], or a method of analysis to determine the geometry of material defects based on the Kirchhoff approximation [21, 22].

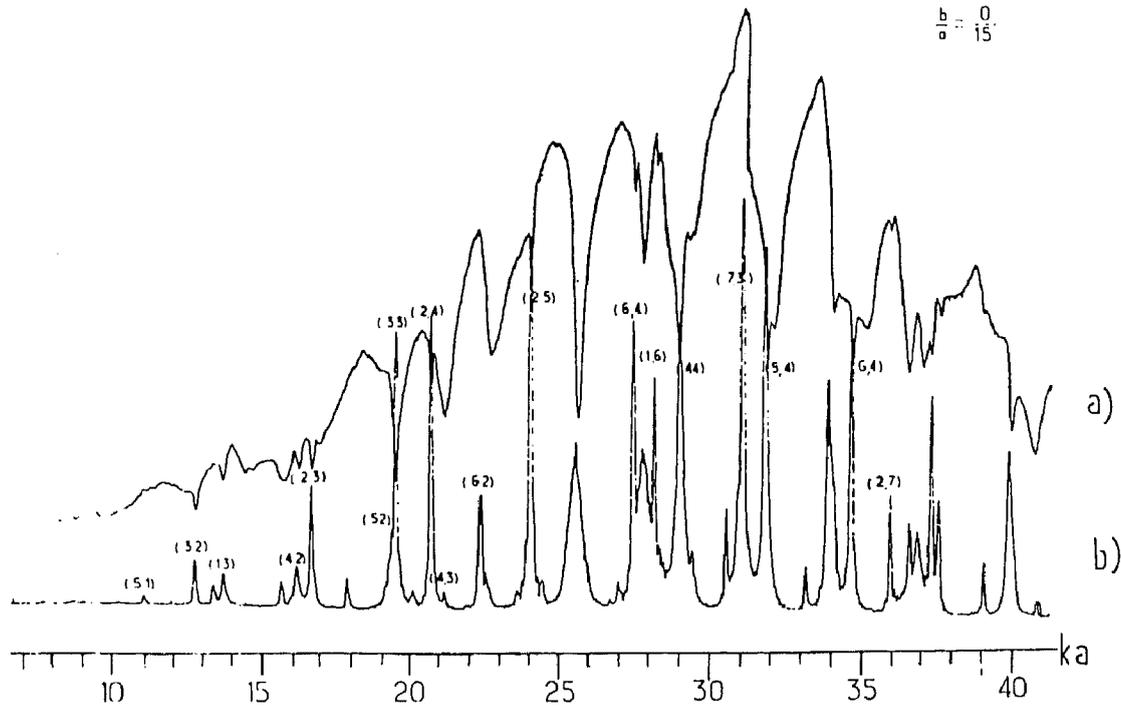


Fig. 1. 1. — Acoustic backscattering amplitude (total: top; resonant: bottom) vs. frequency for a solid aluminum cylinder in water.

2. Acoustic resonance level scheme

A calculation of the eigenfrequencies for prolate fluid spheroids and finite-length fluid cylinders in vacuum, which will approximate the internal resonance frequencies in another, much less dense fluid (except for the imaginary parts of the frequencies, which vanish for ambient vacuum) was carried out [8, 9] in order to gauge the sensivity of the level scheme under changes in shape. For the spheroids, the calculation was carried out by subjecting spheroidal wave functions to the appropriate (Dirichlet) boundary conditions on the inner surface; for finite-length cylinders, exact solutions are available. Figure 2. 1 shows the eigenfrequency "level scheme" for spheres and infinite cylinders as limiting cases, as well as finite-length cylinders and spheroids (axes ratios $b/a = 1.11\dots$ and 4.0). The striking "splitting" (lifting of degeneracy) of the limiting-case levels for the smaller symmetry of finite non-spherical bodies, the azimuthal quantum number n no longer being degenerate as for the sphere case, was explained later [11, 23] in terms of the formation of helical surface waves with discrete pitch angles.

For the case of penetrable objects in a fluid or solid medium (including soft or rigid objects where only external resonances appear), one may extend the level scheme of Figure 2. 1 to a plot of resonance frequencies in the complex frequency plane; see, e. g., Figures 4. 1 or 5. 1 of Reference [23].

3. Examples of inverse scattering

It has been shown analytically [12, 13] for the example of a fluid-filled spherical cavity in a solid that a determination of the resonance frequencies can determine the sound velocity in the fluid; one of the resonance width the density of the fluid. Numerical calculations for a fluid-filler cylinder in steel [14] bear this out. Figure 3. 1 shows the dependence of the reduced resonance frequency spacing Δx_{nl} ($x = ka$, $k =$ wave number in the fluid, $a =$ cavity radius) on the ratio c_1/c_2 of internal sound speed c_1 to external p -wave speed c_2 , for different values of internal-to-external density ratio, n being the mode number and l the order of the resonance frequency within the n -th mode. The near-linear dependence on c_1/c_2 , and the insensitivity to ρ_1/ρ_2 indicates the feasibility of determining c_1/c_2 from the resonance spacing.

In Figure 3. 2, we plot the resonance width Γ_{0l} of the $n=0$ resonances vs. $\rho_1 c_1^2/\rho_2 c_2^2$, for different values of c_1/c_2 . This illustrates the feasibility of determining ρ_1/ρ_2 from the resonance widths.

4. Conclusion

The foregoing represents an elaboration on the concept of Acoustic Resonance Spectroscopy, first pronounced by Derem [1], as a means of determining

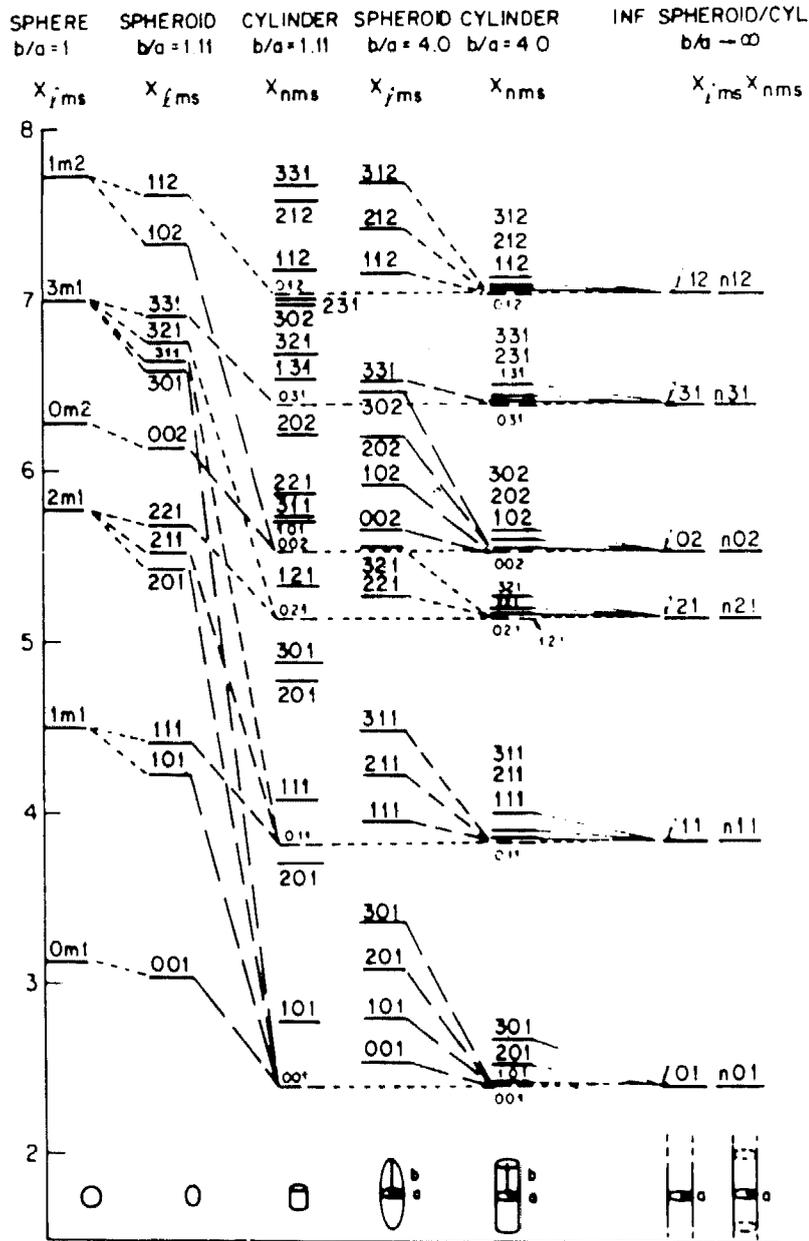


Fig. 2. 1. — Eigenfrequency "level scheme" of fluid spheres, cylinders and spheroids in vacuum.

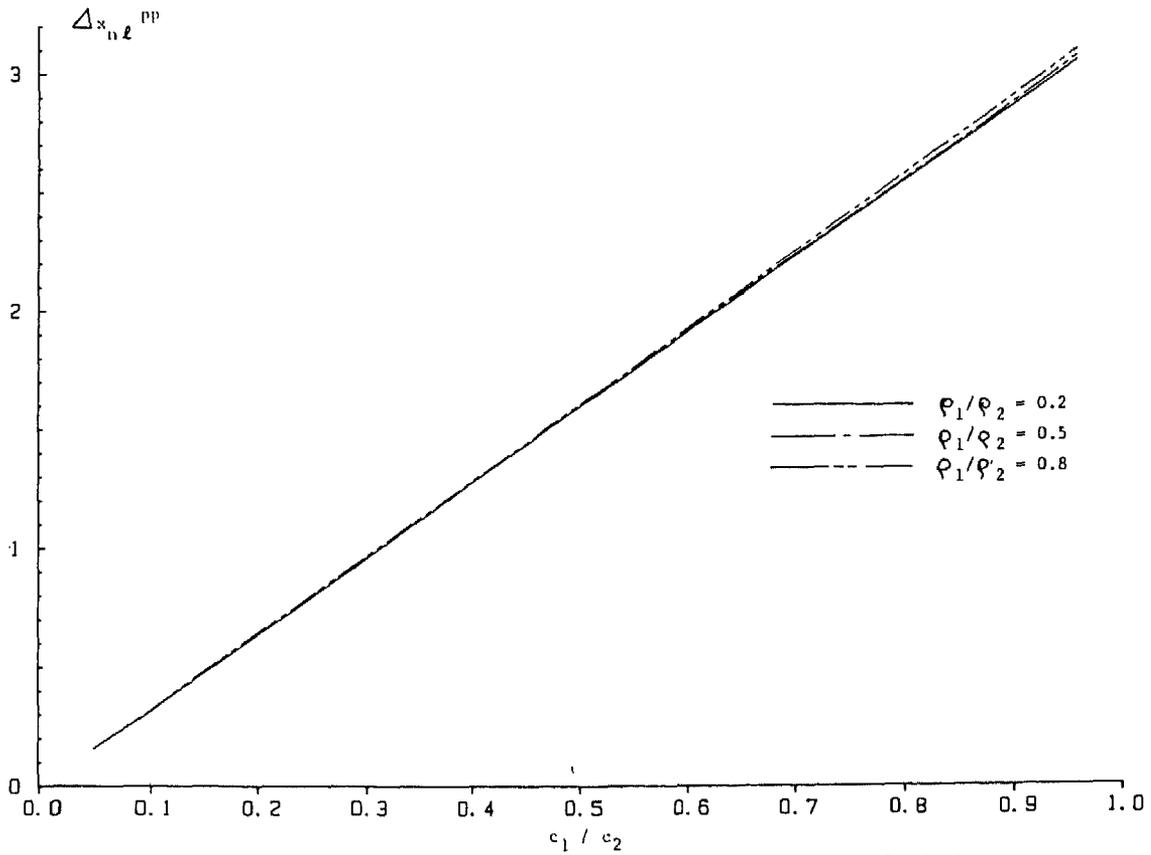


Fig. 3.1. — Spacing of the eigenfrequencies of a cylindrical fluid-filled cavity in steel, vs. p -wave speed ratio, for various density ratios.

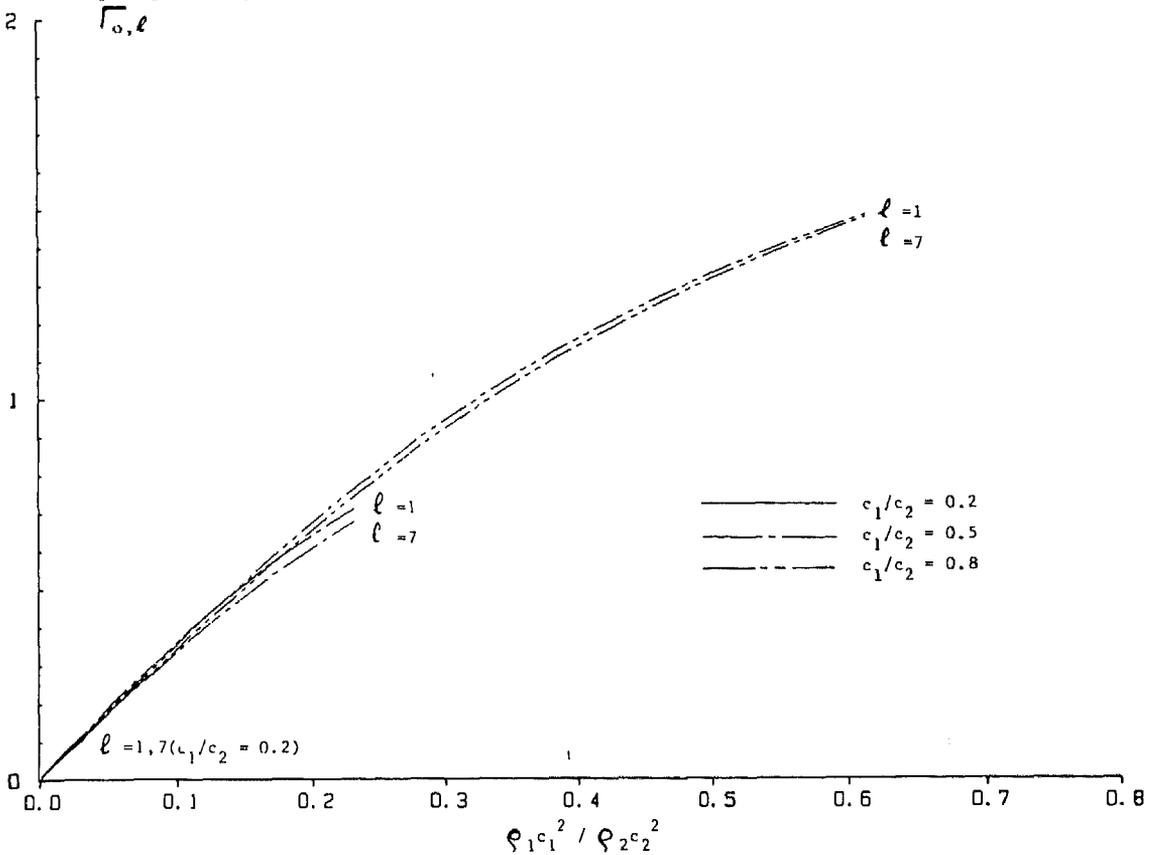


Fig. 3.2. — Resonance widths of a cylindrical fluid-filled cavity in steel, vs. ρc^2 ratio, for various p -wave speed ratios.

the properties of acoustic targets from the information on the target resonances which is contained in the observed echo returns. The feasibility of this concept, although yet to be generally demonstrated for cases of practical interest [24], appears evident for targets of large impedance contrast with the environment (so that pronounced resonances result). It is also obvious, though, that an application of this approach has to proceed, at this time, with complete understanding of the physics of the scattering process. If this caution is disregarded, failure may easily occur. After sufficient development of the approach, one may ultimately expect, however, that automated systems for experimental acoustic resonance spectroscopy will have been devised.

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