Non-specular reflection of convergent beams

from liquid-solid interface

Réflexion non spéculaire de faisceaux convergents par une interface solide-liquide

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SUMMARY

The lateral shift and profile distortion of a bounded beam incident at the Rayleigh angle from a liquid onto a solid has been studied in the past for parallel or divergent beams. In a manner analogous to results obtained in optics, we show that an acoustic beam can also exhibit an angular shift when loss is present in the solid and a shift in the focal point when the angle of incidence deviates slightly from the critical angle. The extent of these shifts is given by analytical expressions for beams whose spectral width is small compared to the attenuation constant α of the Rayleigh wave. For beams having broader spectral widths, we present numerical results for the reflected fields which reveal the presence of relatively strong shifting effects.

KEY WORDS

Acoustic beams, Rayleigh angle, focal shift, angular shift.

RÉSUMÉ

Le déplacement latéral et la distorsion d'un faisceau optique borné à la réflexion sur une interface liquide-solide ont été étudiés antérieurement lorsque l'incidence est égale à l'angle de Rayleigh. Des résultats analogues à ceux de l'optique dans le cas d'un faisceau acoustique : il y a déplacement angulaire quand le solide est dissipatif, et il y a déplacement du foyer quand l'angle d'incidence diffère légèrement de l'angle critique. Nous présentons des expressions analytiques de ces déplacements dans le cas où la largeur spectrale du faisceau est petite par rapport à la constante d'atténuation α de l'onde de Rayleigh. Dans le cas de faisceaux plus larges, nous présentons des résultats numériques qui mettent en évidence des déplacements relativement importants pour des champs réfléchis.

MOTS CLÉS

Faisceaux acoustiques, angle de Rayleigh, déplacement de foyer, déplacement angulaire.

CONTENTS

1. Introduction

The reflection of an acoustic beam incident at the Rayleigh angle from a liquid onto a solid has been studied for well collimated and divergent beams [1-3]; however, the reflection of *convergent* beams, such as those produced by a concave transducer or a lens, has not yet been examined explicitly. The reflection acoustic microscope [4] employs an extreme form of a convergent beam, while beams having more modest convergence can be envisioned for NDE applications. Previous studies of acoustic-beam reflection have dealt primarily with the lateral shift resulting from coupling to the leaky Rayleigh wave [1-3]. or with the strong absorption effect produced by the losses in the solid [5,6]. These effects have also been studied for optical reflection. In addition, the study of optical beams has shown an angular shift in the direction of propagation, and a shift in the distance along the beam axis to the waist or focal plane [7,8]. The angular shift results from the variation of the magnitude $\rho(\theta)$ of the reflection coefficient $R(\theta)$ with the incidence angle θ_0, while the focal shift is due to the derivative of the phase of $R(\theta)$ with respect to θ.

The present study reviews the derivation of the lateral, focal and angular shifts for the case of a two-dimensional Gaussian beam having small angular divergence. Using the accepted pole-zero approximation [1-3] for $R(\theta)$, these shifts are evaluated for incidence near the Rayleigh critical angle θ_R. Because this leads to simple but approximate formulas, which hold only for beams with limited angular divergence, numerical calculations are presented that show the effects also for beams having greater divergence.

2. Reflection of paraxial beams

A convergent acoustic beam incident at the angle θ_0 is depicted in Figure 1. The beam is assumed to be two dimensional with no variation along the coordinate perpendicular to the plane of the paper. In the absence of the solid, the beam would be Gaussian with waist at a distance l from the origin, as indicated by the dashed lines in Figure 1. The beam that would be reflected geometrically from a rigid surface is also shown with waist at a distance l along the reflected beam axis. In addition to the x, z coordinate system,
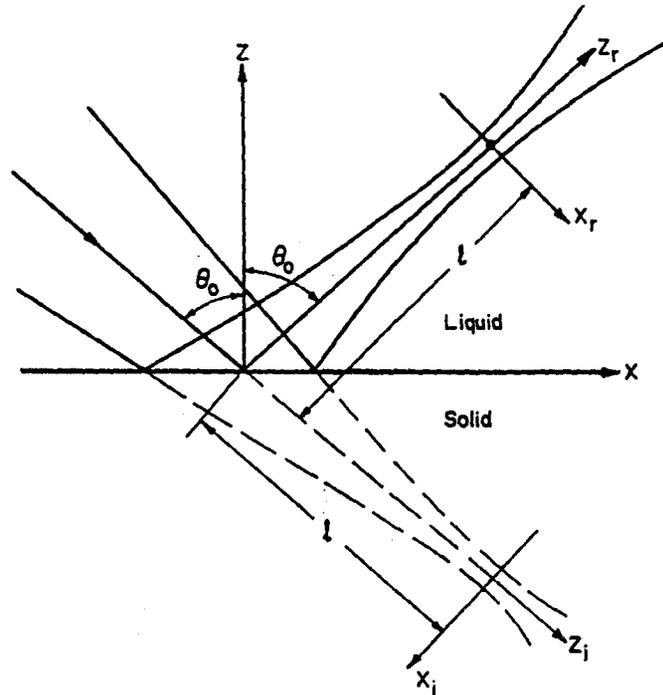
Fig. 1. — Incident and geometrically reflected beams with coordinate systems centered at the foci.

we have indicated the systems (x_i, z_i) and (x_r, z_r) for the incident and reflected beams, respectively.

For harmonic time dependence $\exp(-i\omega t)$, the pressure of the incident beam is given by:

$$(1) \qquad p_i(x_i, z_i) = \frac{w_0}{w_i} \exp[-(x_i/w_i)^2 + ikz_i].$$

Here k is the wavenumber in the liquid, w_0 is the $1/e$ half-width of the beam at its waist $z_i = 0$, and w_i is defined by:

$$(2) \qquad w_i = w_0^2 + i\, 2\, z_i/k.$$

For a well collimated incident beam, the pressure at the interface is obtained by replacing z_i by $-l$ in (2), and setting $x_i = -x\cos\theta_0$ and $z_i = -l + x\sin\theta_0$ in (1). The pressure of the reflected beam can be obtained from the Fourier transform of the incident beam multiplied by the plane wave reflection coefficient $R(k_x)$, where k_x is the wavenumber along x. Thus:

$$(3) \qquad p_r(x, z) = \frac{w_0}{\cos\theta_0} \frac{e^{-ikl}}{2\sqrt{\pi}}$$
$$\times \int_{-\infty}^{\infty} R(k_x)\, e^{-(k_x - k_i)^2\, (w_i/2\cos\theta_0)^2 + i(k_x x + k_z z)}\, dk_x,$$

where $k_i = k\sin\theta_0$ and $k_z^2 = k^2 - k_x^2$.

The integrand in (3) will have significant amplitude only over a narrow interval along the k_x axis centered at k_i. As a result, we may approximate k_z by the Fresnel expansion:

$$(4) \quad k_z \approx k \cos \theta_0 - \tan \theta_0 (k_x - k_i) - \frac{(k_x - k_i)^2}{2 k \cos^3 \theta_0}.$$

Using the identity $R(k_x) = \exp[\ln R(k_x)]$, we may also approximate $R(k_x)$ as:

$$(5) \quad R(k_x)$$
$$= R(k_i) e^{-i(k_x - k_i) L/\cos \theta_0 + i(k_x - k_i)^2 F/2 k \cos^2 \theta_0},$$

where L and F represent the following differential forms evaluated at $k_x = k_i$. Using $\rho = |R(k_x)|$ and $\Psi = \arg R(k_x)$ these forms are given by:

$$(6\,a) \qquad L = \cos \theta_0 \left(-\frac{d\Psi}{dk_x} + i \frac{1}{\rho} \frac{d\rho}{dk_x} \right),$$

$$(6\,b)$$
$$F = k \cos^2 \theta_0 \left\{ \frac{d^2 \Psi}{dk_x^2} + i \left[\left(\frac{1}{\rho} \frac{d\rho}{dk_x} \right)^2 - \frac{1}{\rho} \frac{d^2 \rho}{dk_x^2} \right] \right\}.$$

With the approximations (4) and (5), the integrand of (3) has the form of an exponential with first and second power terms in $(k_x - k_i)$ so that its solution can be obtained in closed form. The result, when expressed in terms of the (x_r, z_r) coordinates, is:

$$(7) \qquad p_r(x_r, z_r) = \frac{w_0}{w_r} R(k_i) e^{ikz_r} e^{-(x_r - L)^2/w_r^2},$$

where:

$$(8) \qquad w_r^2 = w_0^2 + i \frac{2}{k} (z_r - x_r \tan \theta_0 - F).$$

If the reflecting surface is that of a solid half space, and if θ_0 is greater than the shear-wave critical angle, then $\rho = 1$ in the absence of dissipation in the solid for k_x near k_i, so that its derivatives vanish. As a result, it is seen that L and F are real quantities. From (7) it is seen that the axis of the reflected beam is shifted to $x_r = L$, while the beam waist is shifted to the plane $z_r = L \tan \theta_0 + F$. The lateral shift of the beam axis parallel to itself, and the focal shift are depicted in Figure 2. The boundaries of the geometrically reflected beam are shown dashed, and those of the shifted beam are given by the solid curves.

When ρ varies with k_x, both L and F can have real and imaginary parts, so that $F = F' + iF''$ and $L = L' + iL''$. The imaginary part F'' causes a widening, or narrowing of the $1/e$ half width at the beam waist, whereas the focal shift is given by the real part F'. The lateral shift of the beam is given by the real part L'. To see the significance of L'', we seek the maximum of the beam amplitude by examining the minima of the real part of the exponent in (7). Substituting L' for x_r in (8), and then requiring that the

derivative of $\mathrm{Re}[(x_r - L)^2/w_r^2]$ with respect to x_r vanish leads to the condition:

$$(9) \qquad \frac{x_r - L'}{z_r - L' - F'} = \frac{2 L''}{k [w_0^2 + 2 F''/k]} = \tan(\Delta\theta).$$

This condition indicates that the direction of propagation of the beam has undergone an angular shift $\Delta\theta$ from the z_r axis.

3. Incidence near the Rayleigh critical angle θ_R

The variation of the reflection coefficient at a solid surface for angles near θ_R has been shown to be dominated by the pole at k_p and zero at k_0 that lie near $k_R = k \sin \theta_R$. This variation is embodied in the approximation:

$$(10) \qquad R(k_x) = (k_x - k_0)/(k_x - k_p).$$

Numerical investigations have found that $\mathrm{Re}(k_0)$ and $\mathrm{Re}(k_p)$ are very close to k_R. It is also found that $\mathrm{Im}(k_p)$ has a contribution α_l due to leakage or radiation into the liquid, and a contribution α_d due to dissipation in the solid. In the absence of dissipation, the pole and zero have conjugate locations so that $\mathrm{Im}(k_0) = -\alpha_l$. Dissipation in the solid causes the zero to move up in the complex plane by the distance α_d [3,5]. Thus the pole and zero locations are given approximately by:

$$(11) \qquad \begin{cases} k_p = k_R + i(\alpha_d + \alpha_l), \\ k_0 = k_R + i(\alpha_d - \alpha_l). \end{cases}$$

With the reflection coefficient given by (10) and (11), the quantity L is found to be:

$$(12) \quad L = 2 \alpha_l \cos \theta_0$$
$$\times \frac{[(k_i - k_R)^2 + (\alpha_l^2 - \alpha_d^2)] + i 2 (k_i - k_R) \alpha_d}{[(k_i - k_R)^2 + (\alpha_l + \alpha_d)^2][(k_i - k_R)^2 + (\alpha_l - \alpha_d)^2]}.$$

It is also found that:

$$(13) \qquad F = k L^2 [(k_i - k_R) - i \alpha_d]/\alpha_l.$$
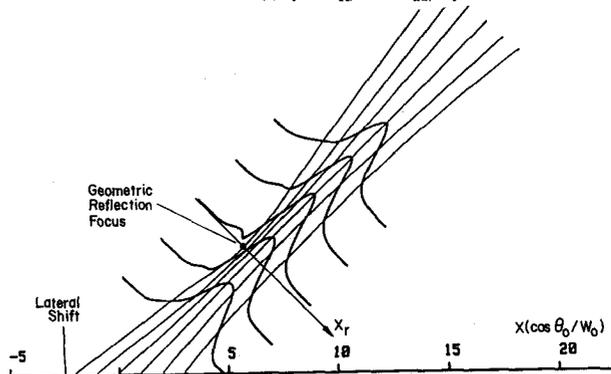
Fig. 4. — Power flow lines and amplitude profiles showing lateral shift for $\theta_0 = \theta_R = 45°$, $w_0 = 2.12 \lambda$, $\alpha_l = 0.08 k$ and $\alpha_l = 0$.
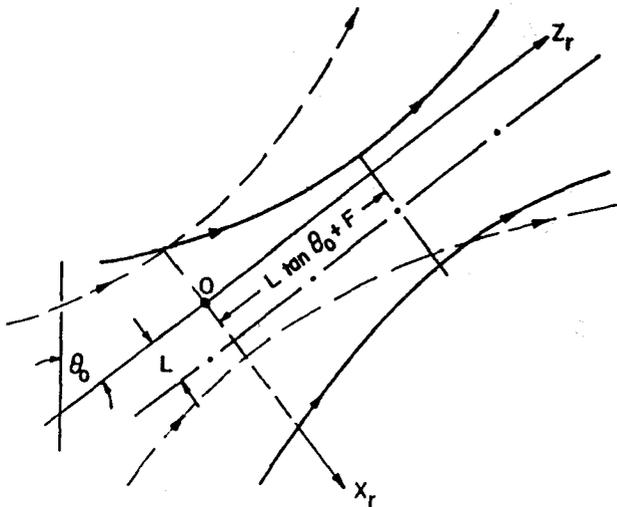
Fig. 2. — Lateral and focal shifts of the reflected beam.
Geometrically reflected beam is shown dashed.

Fig. 3. — Focal shift as a function
of the leaky wave attenuation constant.

When no dissipation is present in the substrate, $\alpha_d = 0$ and L and F are real, as discussed previously. The maximum value for L occurs when $k_i = k_R$, in which case $L = 2\cos\theta_0/\alpha_l$, which is the well known Schoch displacement. For the phase-match condition, it is seen that $F = 0$. The variation of F with k_i is depicted in Figure 3 for $\alpha_d = 0$. The focal shift can be away from the surface $(k_i > k_R)$ or towards the surface $(k_i < k_R)$. The maximum focal shift is seen to occur at

$$k_i - k_r = \pm \alpha_l / \sqrt{3},$$

at which points $|F|/L = 0.86 \, (k/\alpha_l)\cos\theta_0$. Since k/α_l is typically ten or more, the maximum focal shift is at least several times the maximum lateral shift. While the focal shift is larger than the lateral shift, well collimated beams have a large depth of focus, thus making the focal shift more difficult to observe.

When loss is present $(\alpha_l \neq 0)$ we see from (12) and (13) that L and F are complex. For incidence at the Rayleigh angle, L is real, in which case $F'' < 0$, so that (8) yields

$$\mathrm{Re}(w_r^2) > w_0^2.$$

This condition implies a widening of the beam at its waist. If the beam is incident well away from the Rayleigh angle, F'' can be positive, corresponding to a narrowing of the reflected beam at its waist. For low to moderate loss with $\alpha_d < (1/2)\,\alpha_l$, and assuming

$$w_0^2 \gg R2\,F''/k,$$

the angular shift is also maximum when $k_i - k_R \approx +\alpha_l/3$, for which

$$\Delta\theta \approx 2.6\,\alpha_d\cos\theta_0/kw_0^2\,\alpha_l^2.$$

4. Finite width beams

The foregoing analysis applies when (5) is valid over the spatial frequency range $|k_x - k_i| \leq 2\cos\theta_0/w_0$. For $R(k_x)$ of (10), this requirement implies that

$2\cos\theta_0/w_0$ be substantially less than α_l. In the far field region, the incident beam has an angular divergence width $2\theta_m \approx 4/kw_0$. Thus the results for the various shifts apply when $2\theta_m$ is substantially less than $2\alpha_l/k\cos\theta_0$. As an example, if $\theta_0 = 45°$ and α_l/k has value 0.1, then $2\theta_m$ must be less than 17°. Since α_l/k is usually less than 0.1, the angular beam width is correspondingly reduced.
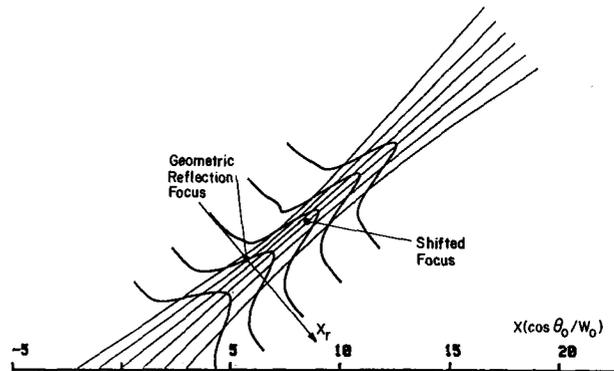
Fig. 5. — Power flow lines and amplitude profiles
showing focal shift for $\theta_0 = 48.3°$.

Using an approach similar to that of reference [1], we have calculated the power flow lines and amplitude profiles of the reflected beam for $w_0 = 2.1\,\lambda$, for which $2\theta_m = 17°$, when $\theta_R = 45°$ and $\alpha_l = 0.08\,k$. The results are plotted in Figures 4 and 5 for angles of incidence $\theta_0 = 45°$ and 48.3°. Since the half-width of the frequency spectrum for this beam is $0.11\,k$, which is greater than α_l, these parameters lie outside the range of validity of the expressions (12) and (13) for the beam shifts, so that only qualitative agreement can be expected.

For $\theta_0 = 45°$, the lateral shift L in Figure 4 is about $2.1\,\lambda$, which is in fair agreement with the value $2.8\,\lambda$ obtained from (12). A slightly smaller lateral shift is found for $\theta_0 = 48.3°$, in agreement with (12). The

separation between the power flow lines is an indication of beam width and indicates a shift of about 8λ, as opposed to the value of 14λ predicted by (13).
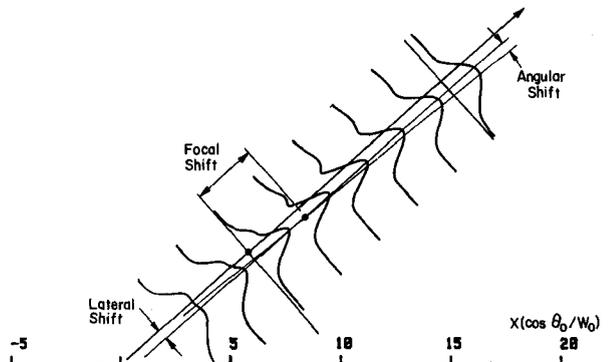
Fig. 6. — Amplitude profiles for showing angular shift $\alpha_l = 0.04\,k$, $\theta_0 = 48.3°$.

The effect of loss on the reflected beam is shown in Figure 7 for $\theta_0 = 48.3°$ and $w_0 = 2.1\lambda$. The solid has the same parameters as used in Figures 4-6, except that $\alpha_d = \alpha_l/2$. Besides the lateral and focal shifts, the propagation of the beam axis is shifted by about 2° from the specular direction, which is smaller than the value of 3.4° given by (9).

5. Conclusion

We have shown that a convergent beam reflected at a liquid-solid interface exhibits a lateral shift, a shift in the location of the focus and an angular shift in the direction of propagation. Simple expressions for these shifts were derived when the beam is incident near the Rayleigh critical angle and the angular divergence of the beam is small. For beams having greater angular divergence, numerical evaluation of the field indicates that the shifts still occur, but with reduced values.

Acknowledgement

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