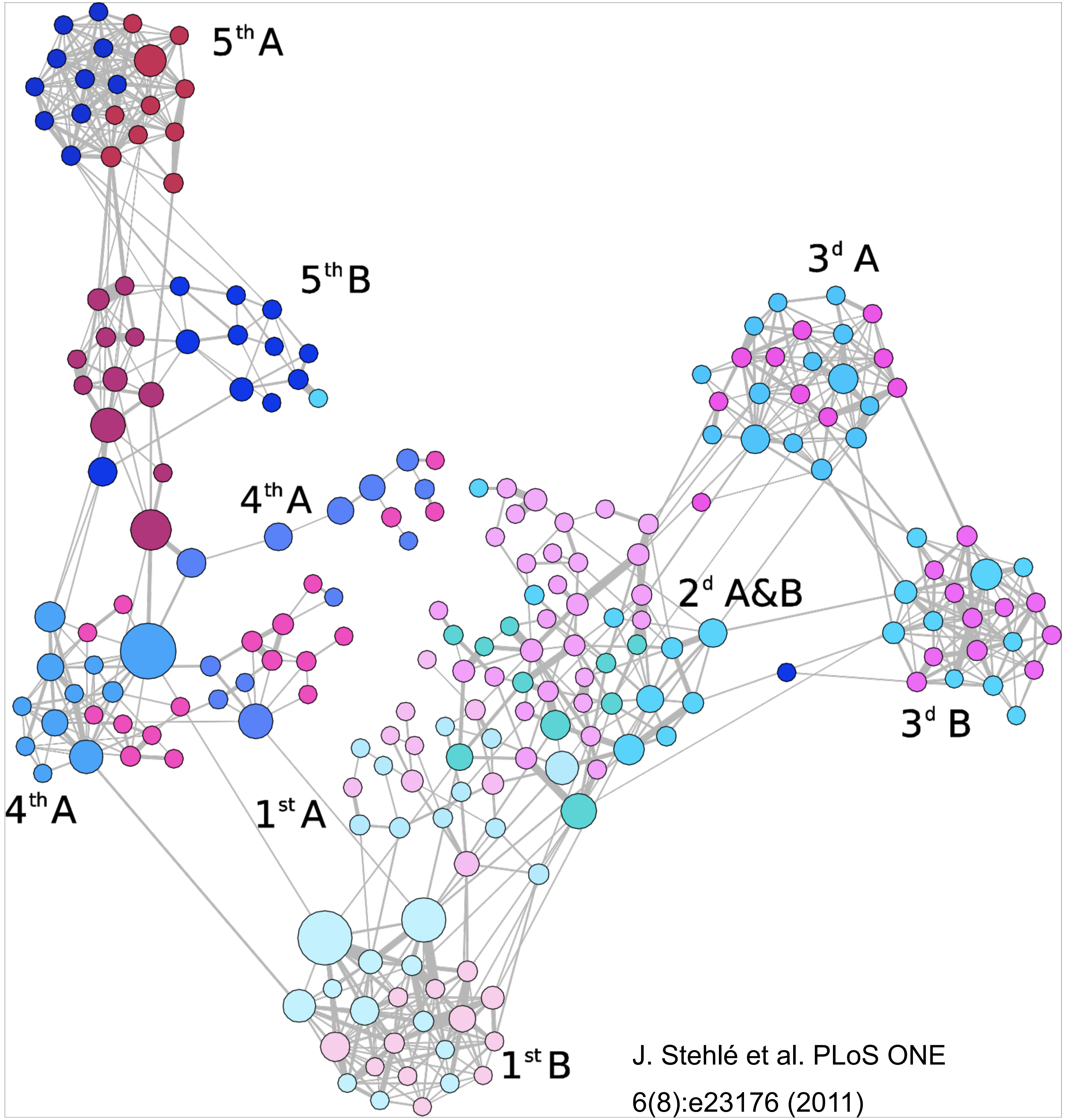
Outline of the lectures

I. Networks: definitions, statistical characterization, correlations, structures, hierarchies…
II. Modeling frameworks
III. Resilience, vulnerability
IV. **Temporal networks**

- From static to temporal
  - examples
  - representations, aggregation
- Paths
- Structure
  - statistics, burstiness, persistence,
  - motifs
  - cores, rich-club
  - timescales
- Models; Null models

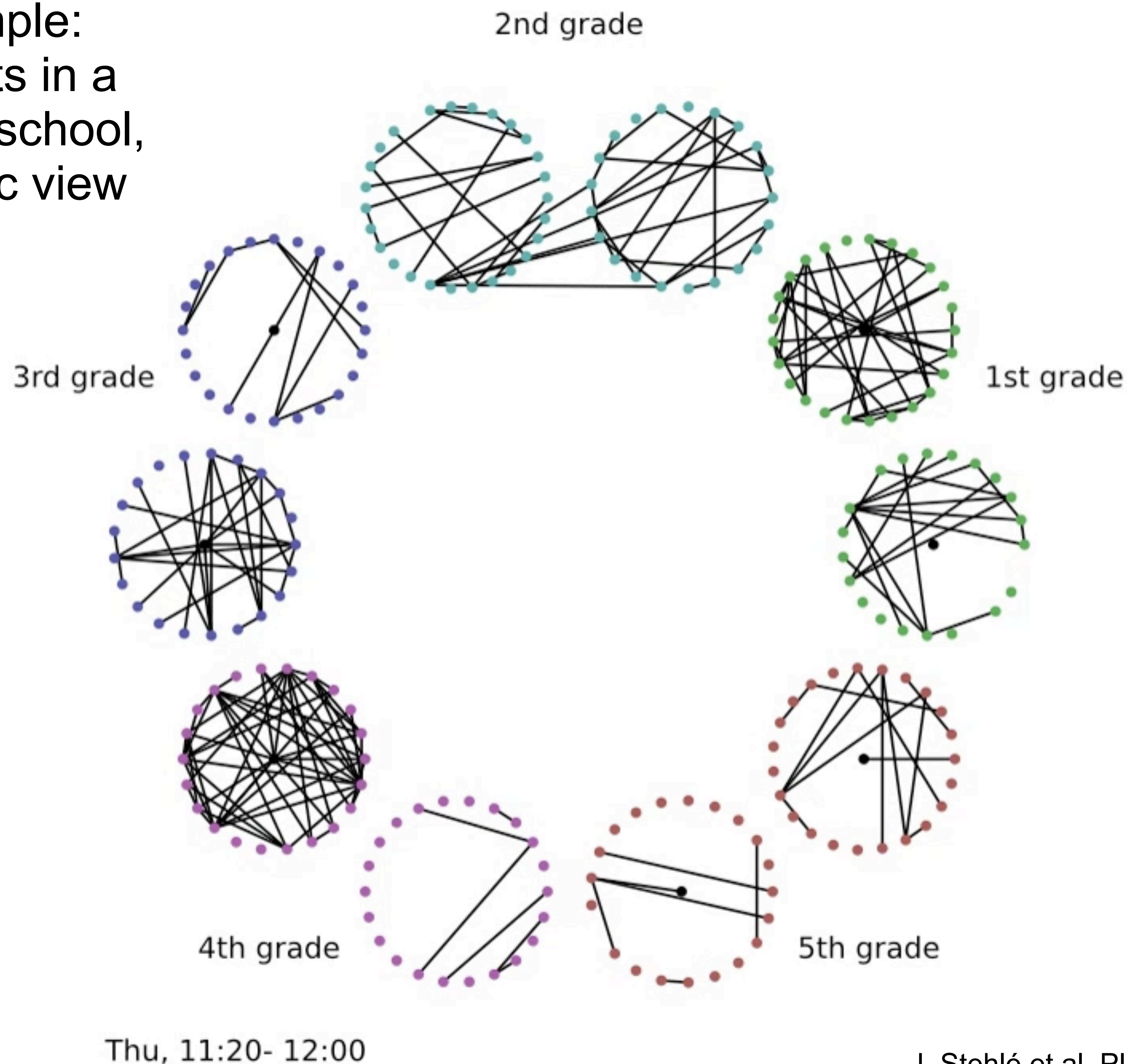
>Networks change over time

Example: contacts in a primary school, static view



J. Stehlé et al. PLoS ONE 6(8):e23176 (2011)

Example: contacts in a primary school, dynamic view



J. Stehlé et al. PLoS ONE 6(8):e23176 (2011)

Exemples of temporal networks

- Social networks
  - contacts
  - friendships
  - collaborations
- Communication networks
  - cell phone data
  - online social networks (Twitter, etc)
- Transportation networks
  - air transportation
  - transport of goods
- Biological networks

Temporal networks

Networks= (often) dynamical entities

(communication, social networks, online networks, transport networks, etc…)

- Which dynamics?
- Characterization?
- Modeling?
- Consequences on dynamical phenomena? (e.g. epidemics, information propagation…)
- Link temporal+topological structure and function

Time-varying networks: often represented by aggregated views

- Lack of data
- Convenience

Back to square one

Physics Reports

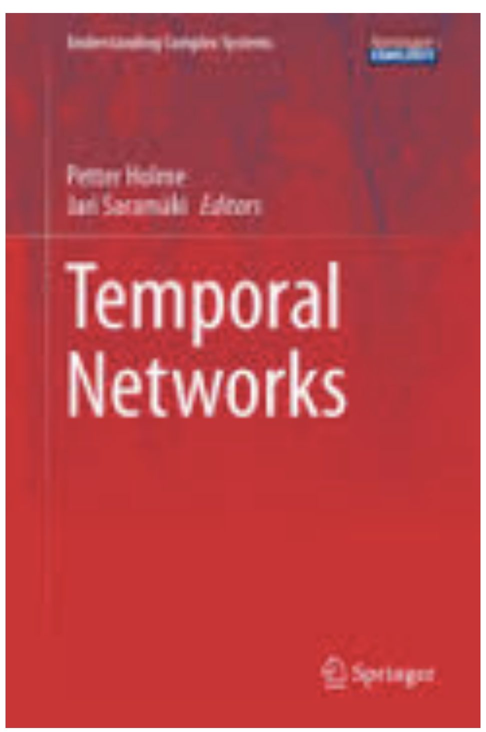
Volume 519, Issue 3, October 2012, Pages 97–125

Temporal networks

Petter Holme[a, b, c], Jari Saramäki[d]

Show more

http://dx.doi.org/10.1016/j.physrep.2012.03.001



The European Physical Journal B

September 2015, 88:234

Modern temporal network theory: a colloquium

Authors | Authors and affiliations

Petter Holme

Colloquium

First Online: 21 September 2015

DOI: 10.1140/epjb/e2015-60657-4

Cite this article as:
Holme, P. Eur. Phys. J. B (2015) 88: 234. doi:10.1140/epjb/e2015-60657-4

Part of the following topical collections:

- Topical issue: Temporal Network Theory and Applications

Definition: temporal network

Temporal network: T=(V,S)

- V=set of nodes
- S=set of event sequences assigned to pairs of nodes

$$s_{ij} \in S: \; s_{ij} = \{(t_{ij}^{s,1}, t_{ij}^{e,1}) \cdots (t_{ij}^{s,\ell}, t_{ij}^{e,\ell})\}$$

Other representation:
time-dependent adjacency matrix:
a(i,j,t)= 1 <=> i and j connected at time t

Representations of temporal networks

Contact sequences

Time	ID1	ID2
2	2	4
2	1	5
3	2	4
3	1	6
4	2	3
5	2	4
5	1	4
8	4	6

…….

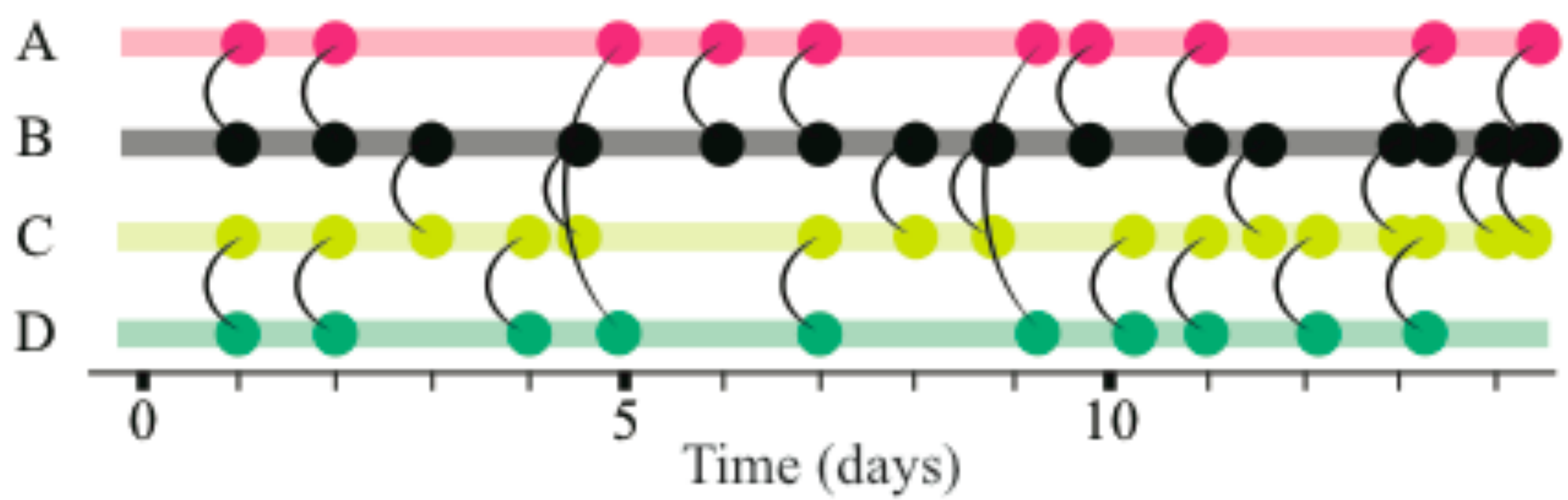
Contact intervals

ID1	ID2	Time interval
2	3	[1,5]
2	1	[2,4]
4	6	[5,9]
1	3	[7,15]
5	3	[7,9]

…….

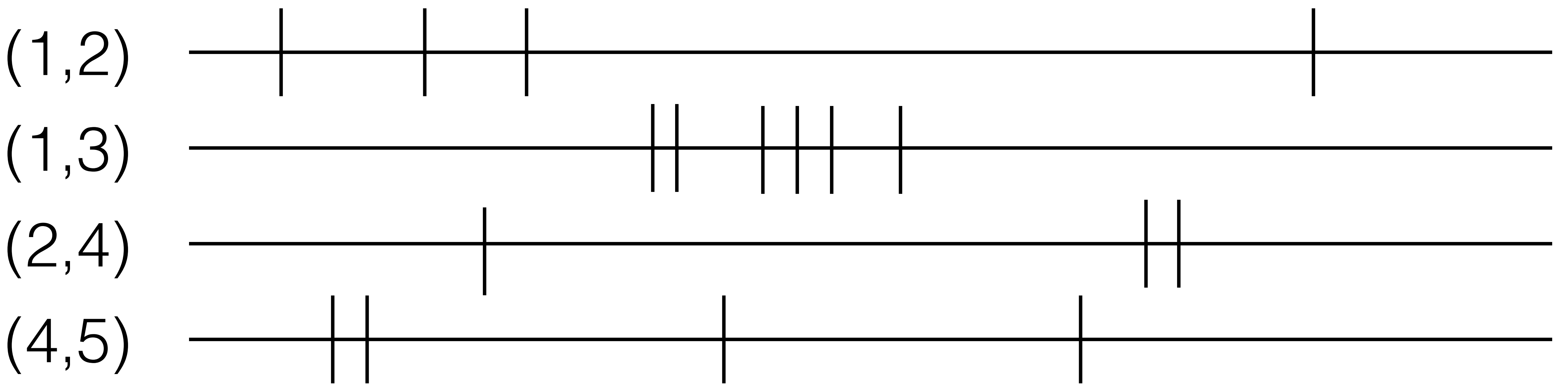
Representations of temporal networks

Timelines of nodes



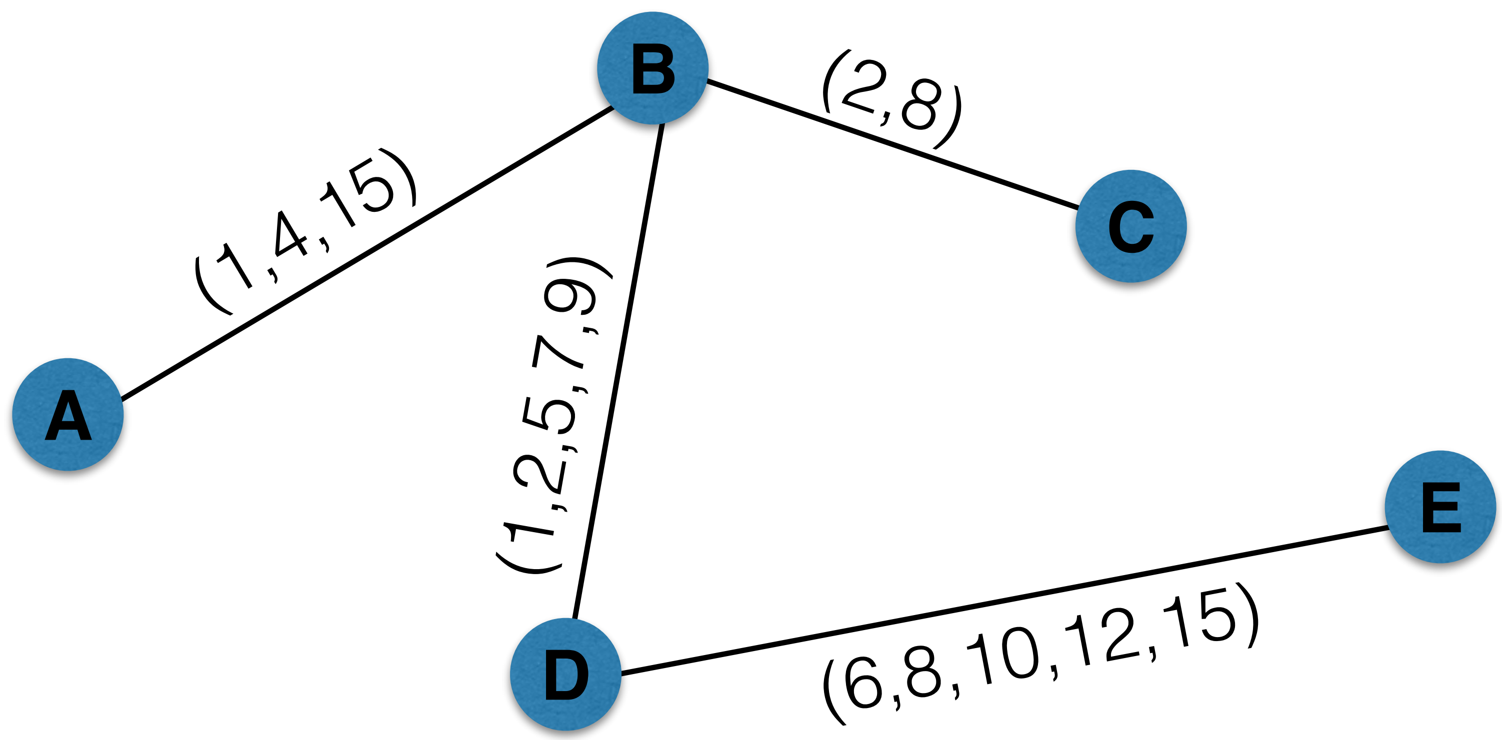
Representations of temporal networks

Timelines of links

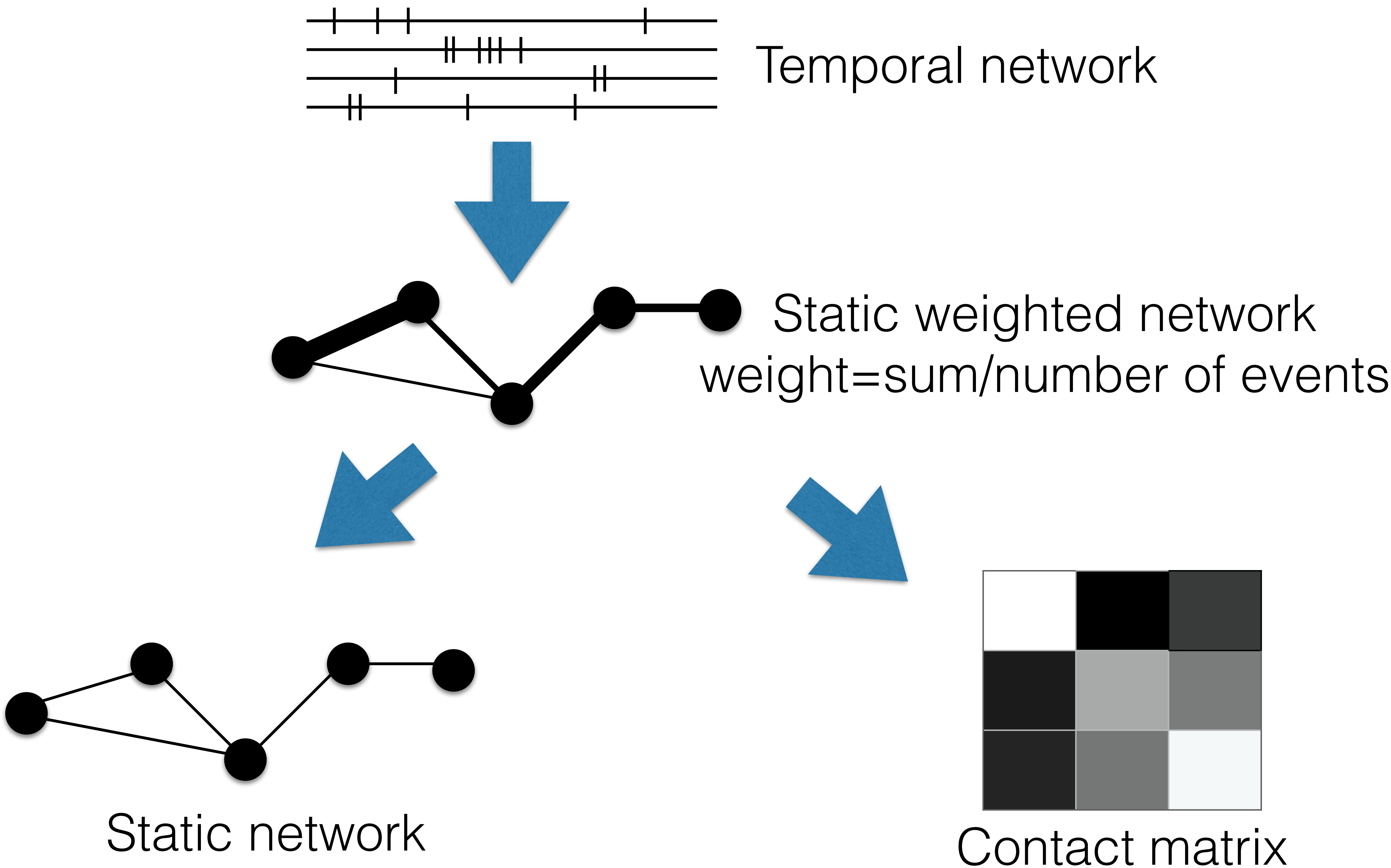


Representations of temporal networks

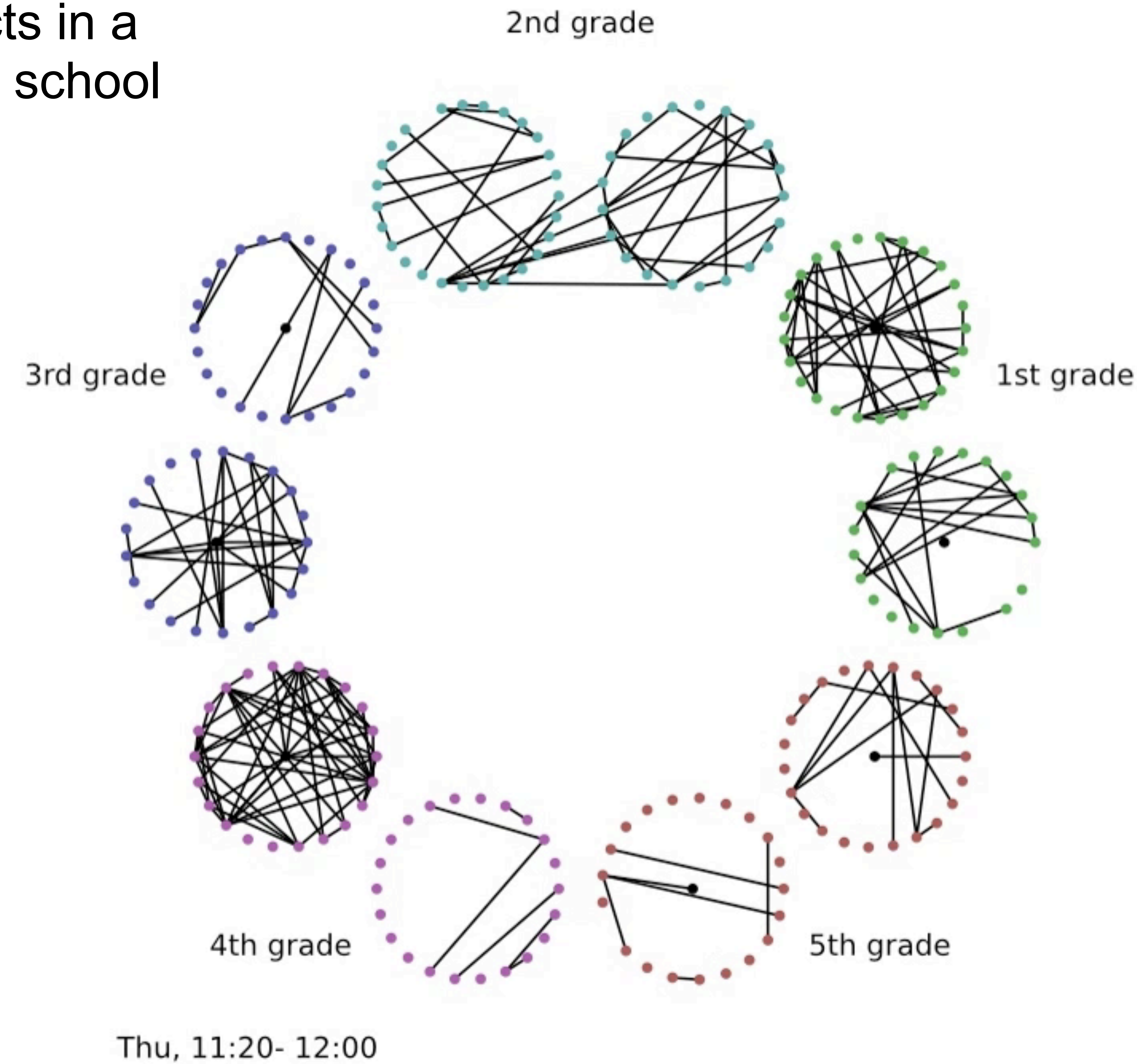
Annotated graph



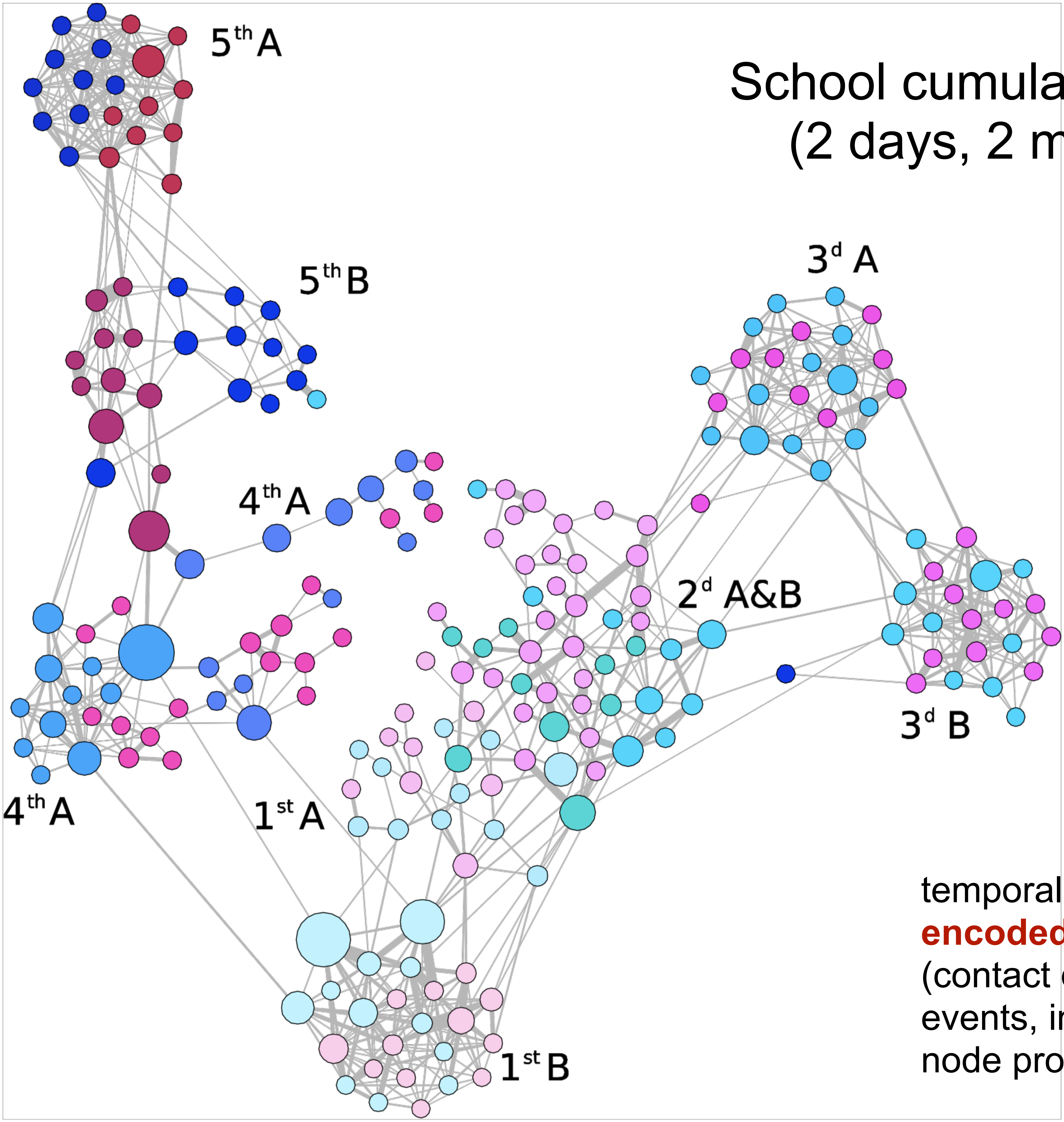
Aggregation of temporal networks



contacts in a primary school



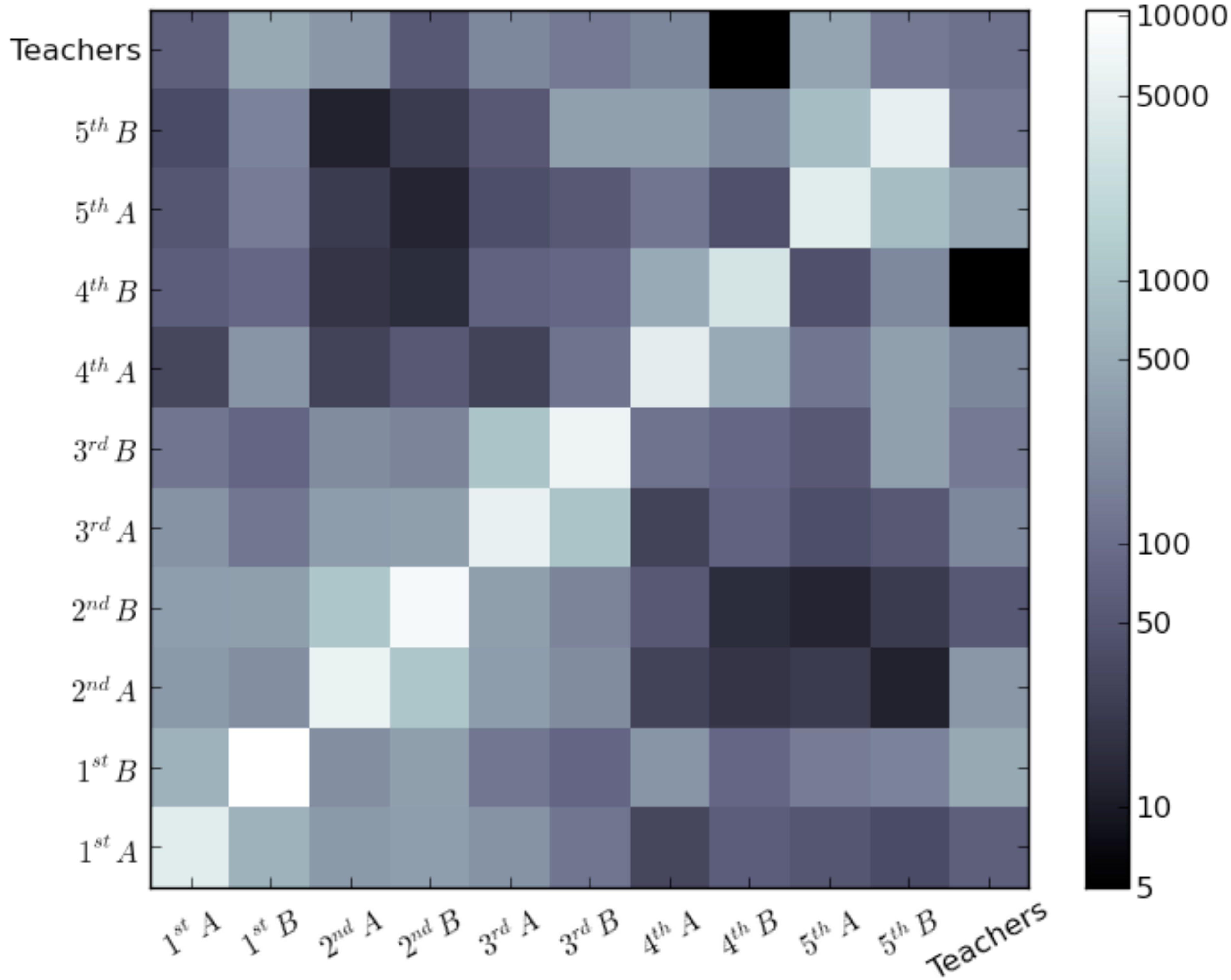
School cumulative f2f network (2 days, 2 min threshold)



J. Stehlé et al. PLoS ONE 6(8):e23176 (2011)

temporal information:
encoded in edges' weights
(contact duration(s), number of events, intermittency, etc...) and node properties

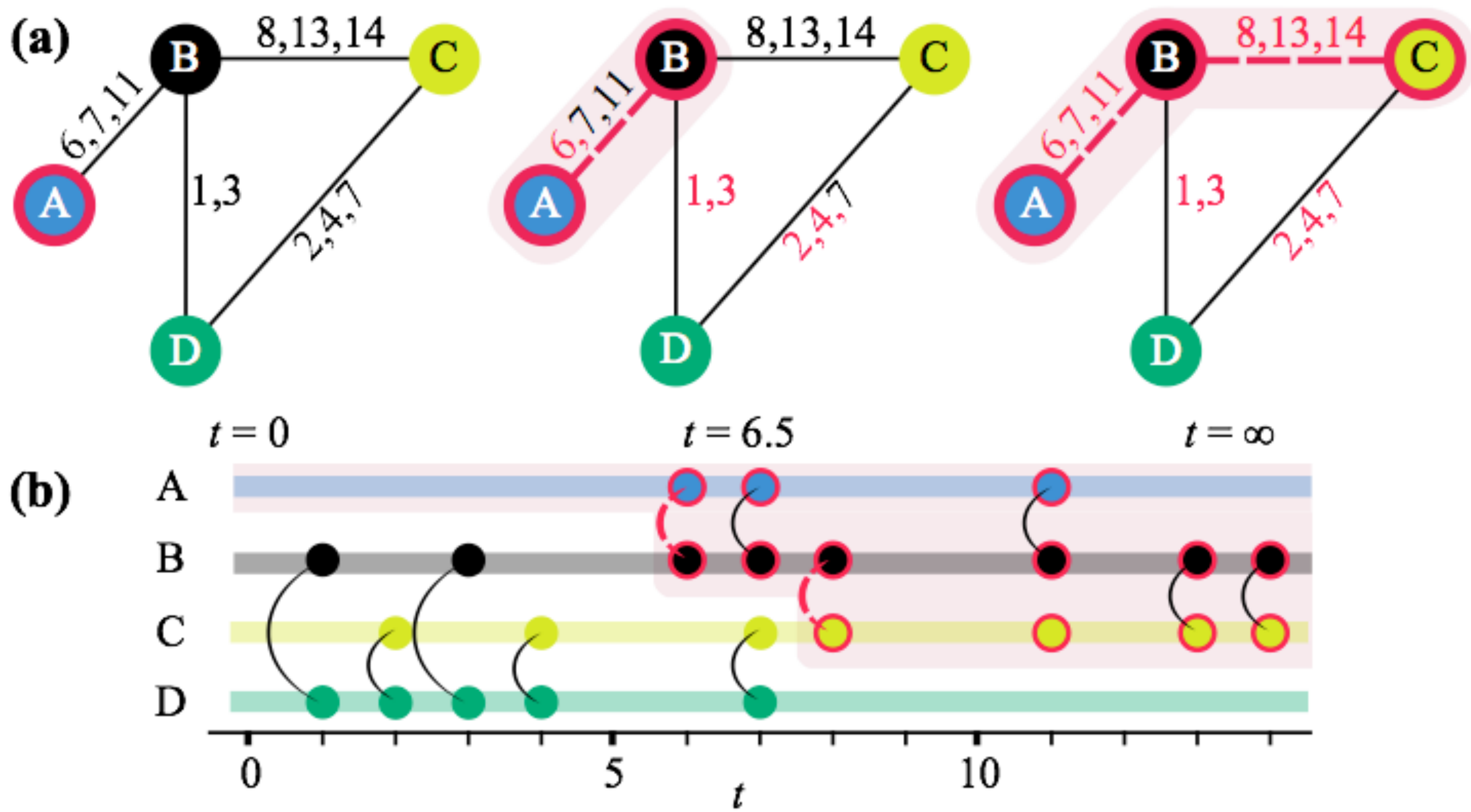
contact matrices



Average values: little information

J. Stehle, et al.
High-Resolution Measurements of Face-to-Face Contact Patterns in a Primary School
PLoS ONE 6(8), e23176 (2011)

Temporality matters: reachability issue



Review Holme-Saramaki, Phys. Rep. (2012), arXiv:1108.1780

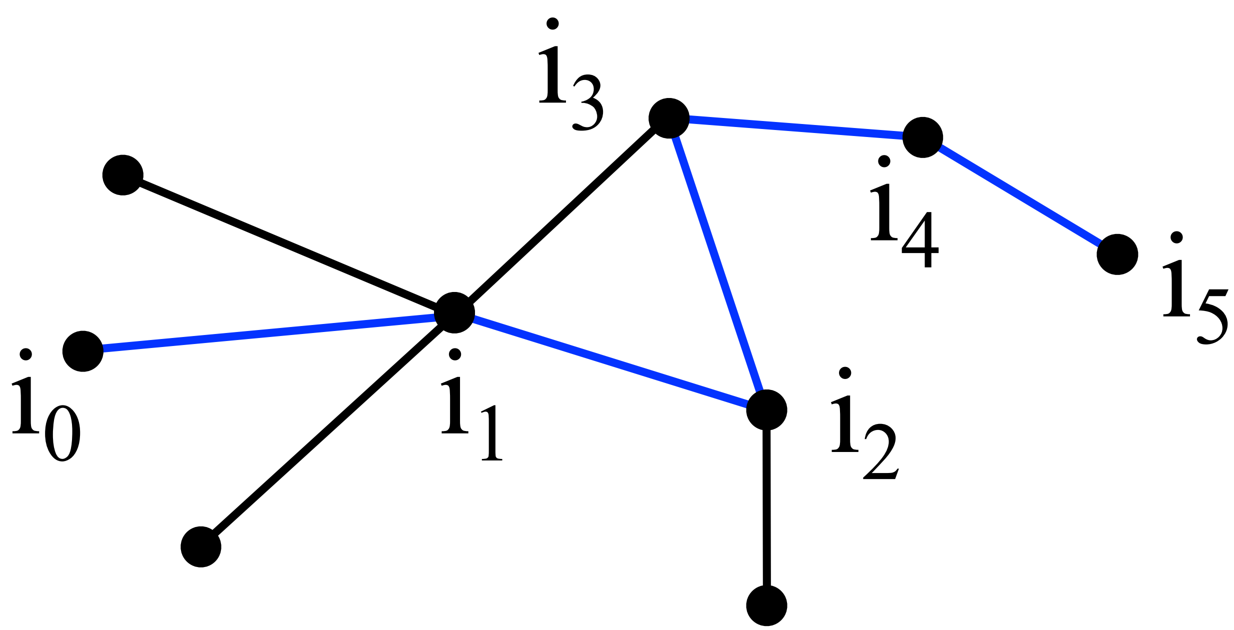
>Paths in temporal networks

Paths in static networks

G=(V,E)

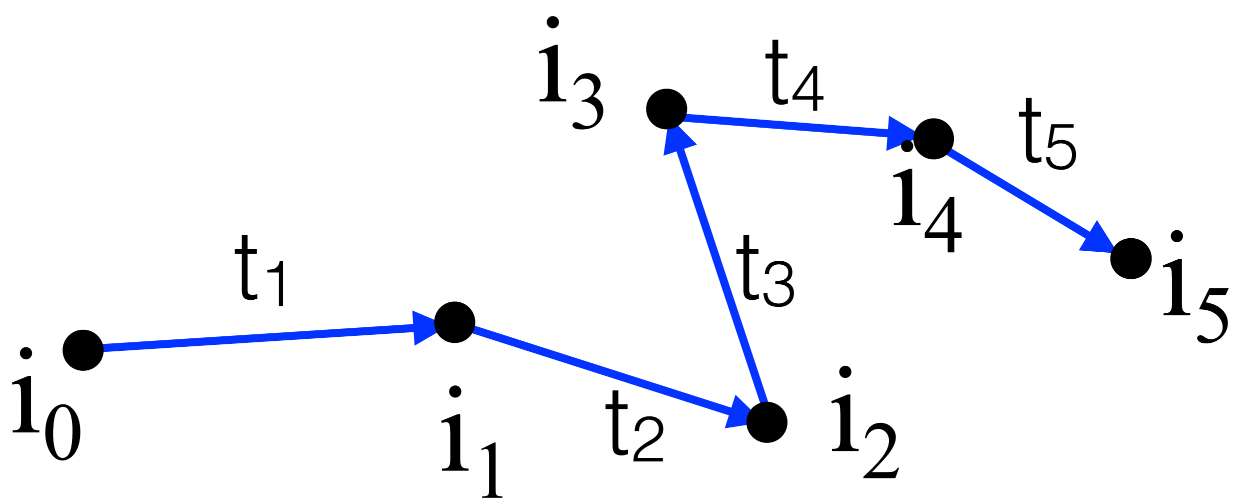
Path of length n = ordered collection of

- n+1 vertices $i_0, i_1, \ldots, i_n \in V$
- n edges $(i_0,i_1), (i_1,i_2) \ldots, (i_{n-1},i_n) \in E$



Notions of shortest path, of connectedness

Time-respecting paths in temporal networks



Sequence of ***events***

Path = $\{(i_0,i_1,t_1),(i_1,i_2,t_2),\ldots,(i_{n-1},i_n,t_n) \mid t_1 < t_2 < \ldots < t_n)\}$

Length of path: n

Duration of path: $t_n - t_1$

Notions of ***shortest*** path and of ***fastest*** path

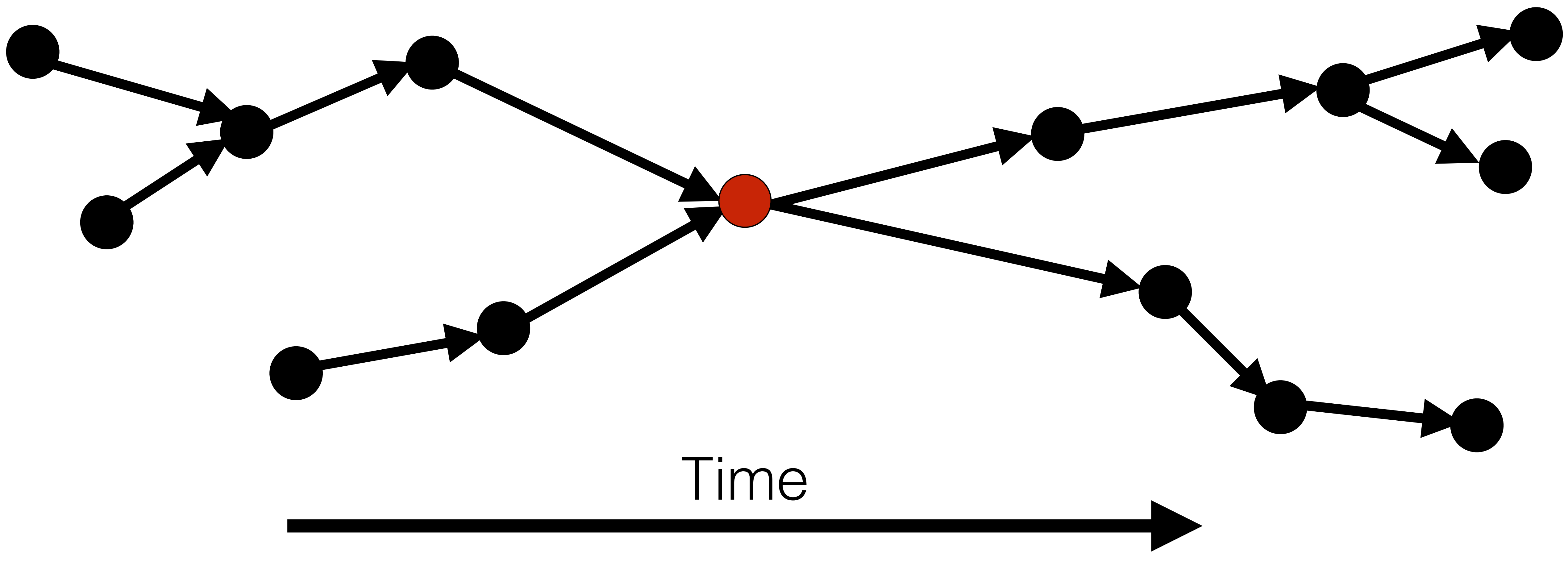
Reachability

Node i at time t

"Light cone"

Source set:
set of nodes that
can reach i at time t

Reachable set:
set of nodes that
can be reached from i
starting at time t



Time-respecting paths in temporal networks

- **Not always reciprocal**: the existence of a path from i to j does not guarantee the existence of a path from j to i

- **Not always transitive**: the existence of paths from i to j and from j to k does not guarantee the existence of a path from i (to j) to k

- **Time-dependence**:
  - there can be a path from i to j starting at t but no path starting at $t' > t$
  - shortest and fastest paths can differ
  - length of shortest path can depend on starting time
  - duration of fastest path can depend on starting time
  - there can be a path starting from i at t_0, reaching j at t_1, and another path starting form i at $t'_0 > t_0$ reaching j at $t'_1 > t_1$, with $t'_1 - t'_0 < t_1 - t_0$ (i.e., smaller duration but arriving later), and/or of shorter length (smaller number of hops)

Centrality measures in temporal networks

Temporal betweenness centrality
Temporal closeness centrality

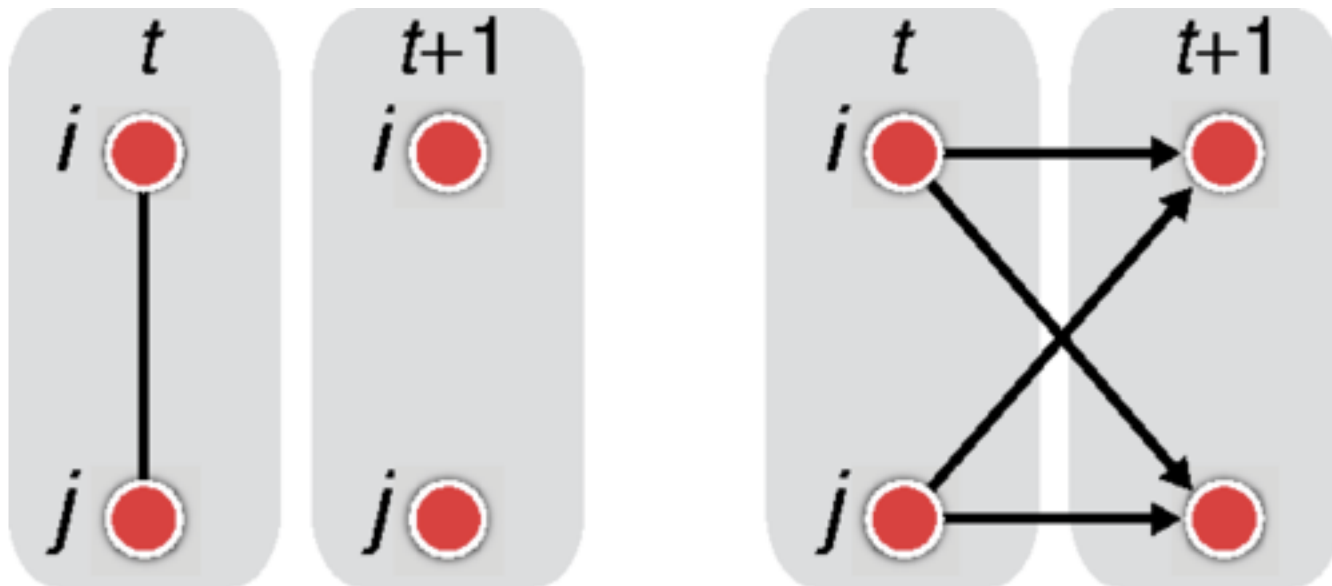
https://www.cl.cam.ac.uk/~cm542/phds/johntang.pdf

Coverage centrality

Takaguchi et al., European Physical Journal B, 89, 35 (2016)
http://arxiv.org/abs/1506.07032

>Two static representations of temporal networks

From temporal to multilayer: supra-adjacency



$$\begin{pmatrix}0&1\\1&0\end{pmatrix} \quad \begin{pmatrix}0&0\\0&0\end{pmatrix} \qquad \begin{pmatrix}0&0&1&1\\0&0&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

We now define a more convenient representation of the coupled dynamics adopting the multilayer approach introduced in Ref. [33]. We map the temporal network to the tensor space $\mathbb{R}^N \otimes \mathbb{R}^T$, where each node is identified by the pair of indices (i, t), corresponding to the node label i and the time frame t, respectively. A multilayer representation of the temporal network can be introduced through the following rules:

(i) Each node, at time t, is connected to its future self at $t+1$.

(ii) If i is connected to j at time t, then we connect i at time t to j at time $t+1$, and j at time t to i at time $t+1$.

$$A_{ij}^{tt'} = \delta^{t,t'+1}\left(\delta_{ij} + A_{ij}^{t'}\right)$$

causal paths in temporal network

=

paths in directed static representation

Analytical Computation of the Epidemic Threshold on Temporal Networks

Eugenio Valdano, Luca Ferreri, Chiara Poletto, and Vittoria Colizza

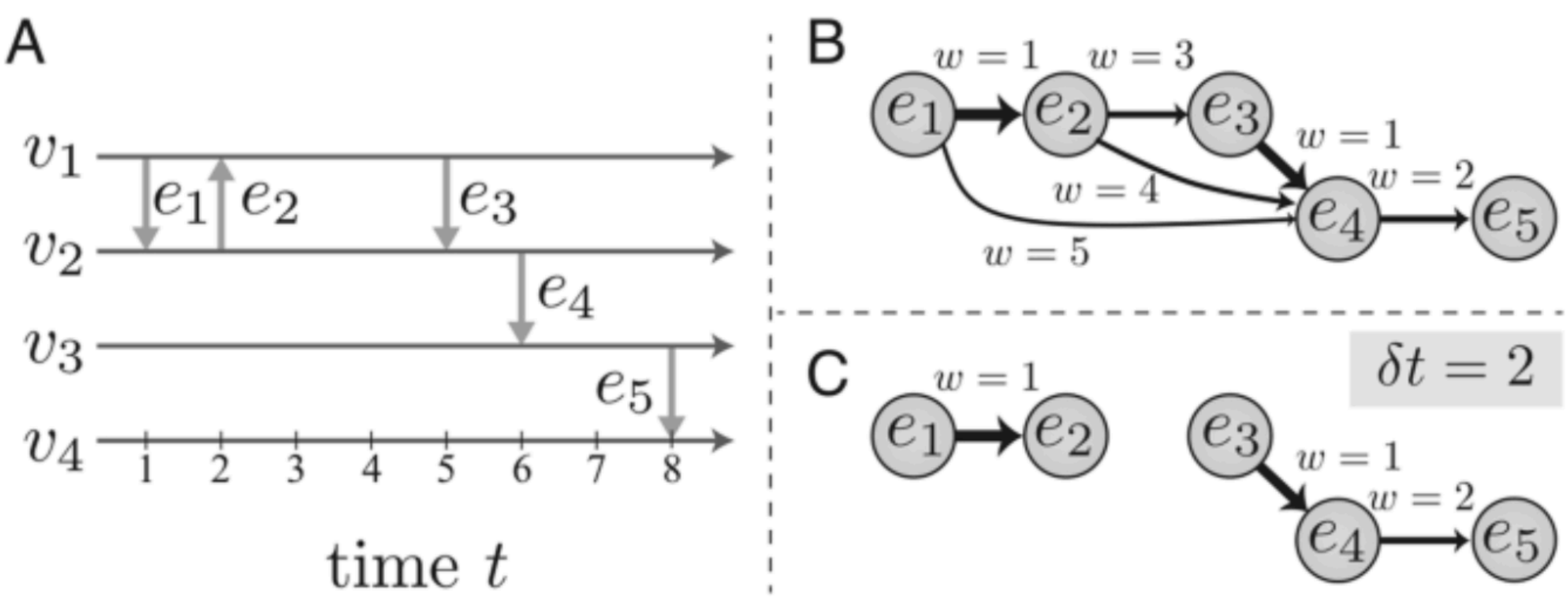
Phys. Rev. X **5**, 021005 – Published 8 April 2015

Mapping temporal-network percolation to weighted, static event graphs

Mikko Kivelä, Jordan Cambe, Jari Saramäki & Márton Karsai

Scientific Reports **8**, Article number: 12357 (2018) | Cite this article

Figure 1



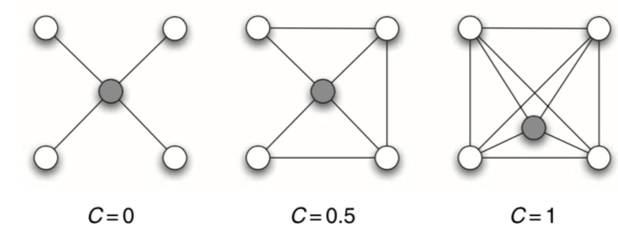
Constructing and thresholding the weighted event graph. (**a**) The time line of a temporal network with four nodes $v_1 – v_4$ and five events $e_1 – e_5$. (**b**) The weighted event graph representation of the temporal network. (**c**) The thresholded event graph, containing only pairs of events with a maximum time difference of $\delta t = 2$.

>Structures
in temporal networks

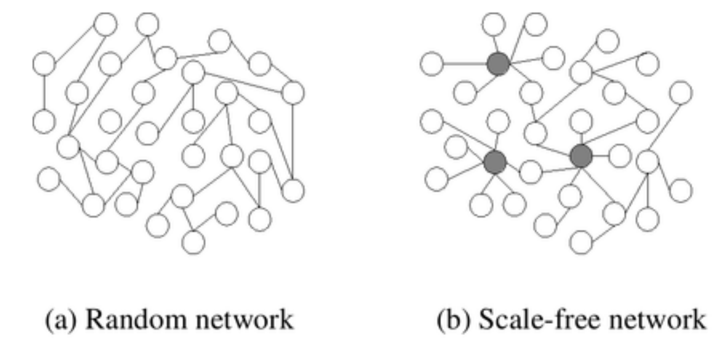
Structures in static graphs

Various scales

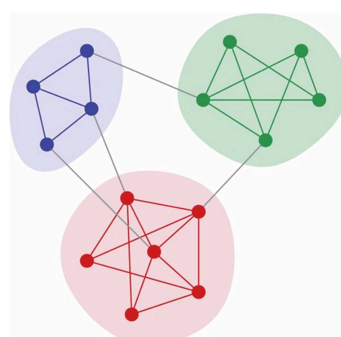
- Clustering coefficient, local cohesiveness



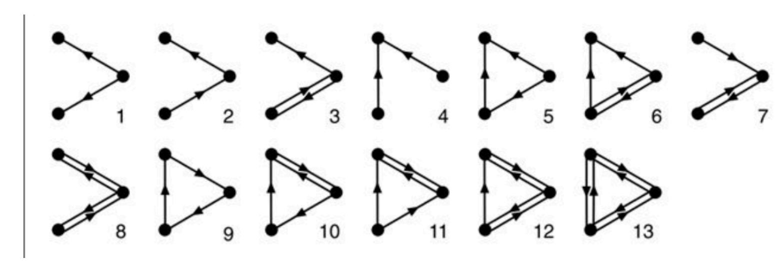
- Heterogeneities (degree distribution, hubs…)



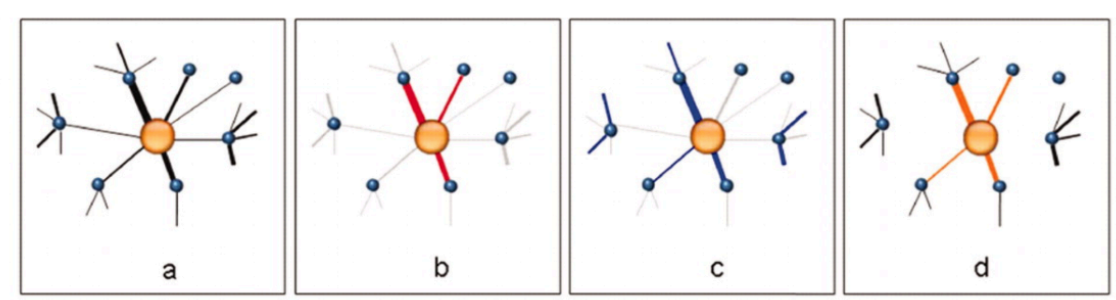
- Communities



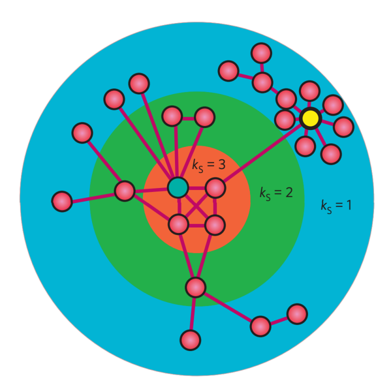
- Motifs (small subgraphs more frequent than expected)



- Backbones (most “relevant” parts of a network)



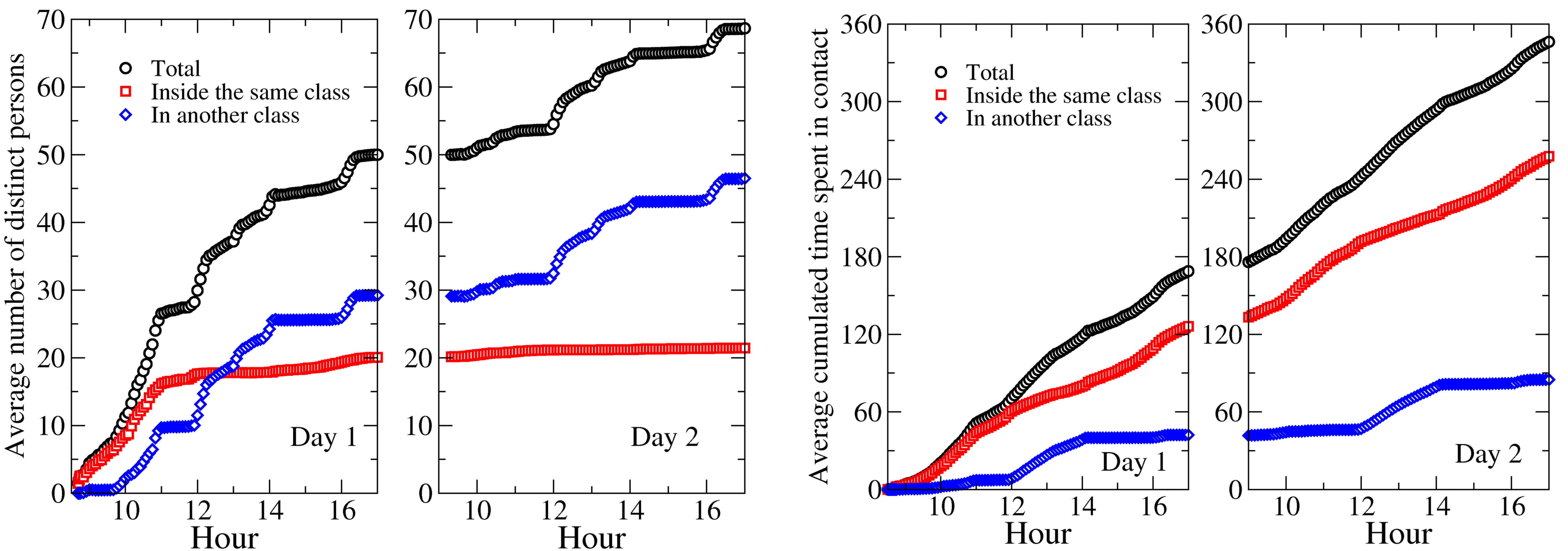
- Hierarchies (e.g., k-core decomposition)



- …

Temporal networks

- Properties of networks aggregated on different time windows: P(k), P(w), P(s), etc…
- Evolution of averaged properties when time window length increases (e.g., <k>(t), <s>(t))



example: face-to-face contacts in a primary school

J. Stehlé et al. PLoS ONE 6(8):e23176 (2011)

Temporal properties in temporal networks

Temporal statistical properties

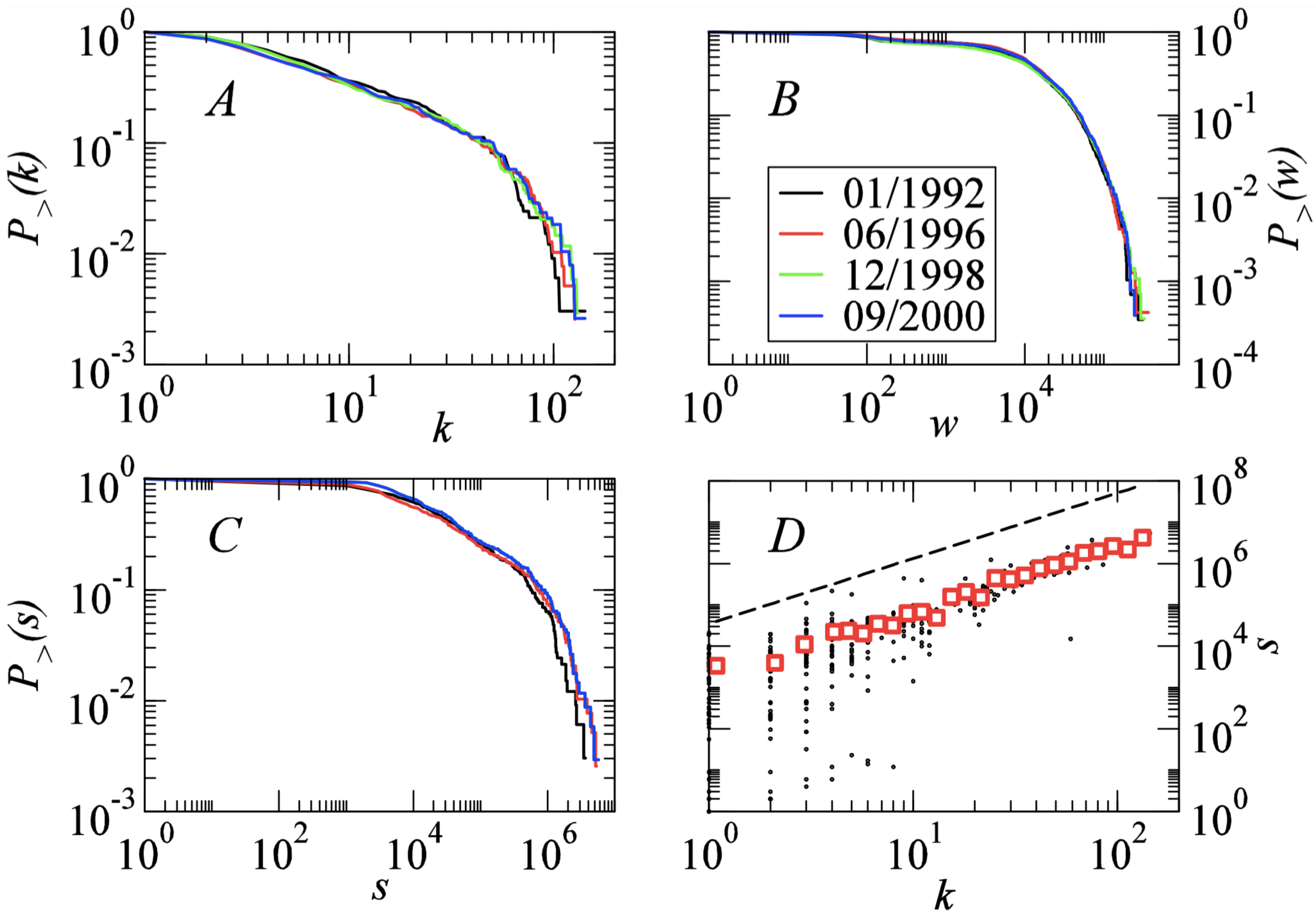
- distribution of contact numbers P(n),
- distribution of contact durations P(τ),
- distribution of inter-contact times P(Δt)

for nodes and links

Stationarity of distributions
(Non-)stationarity of activity

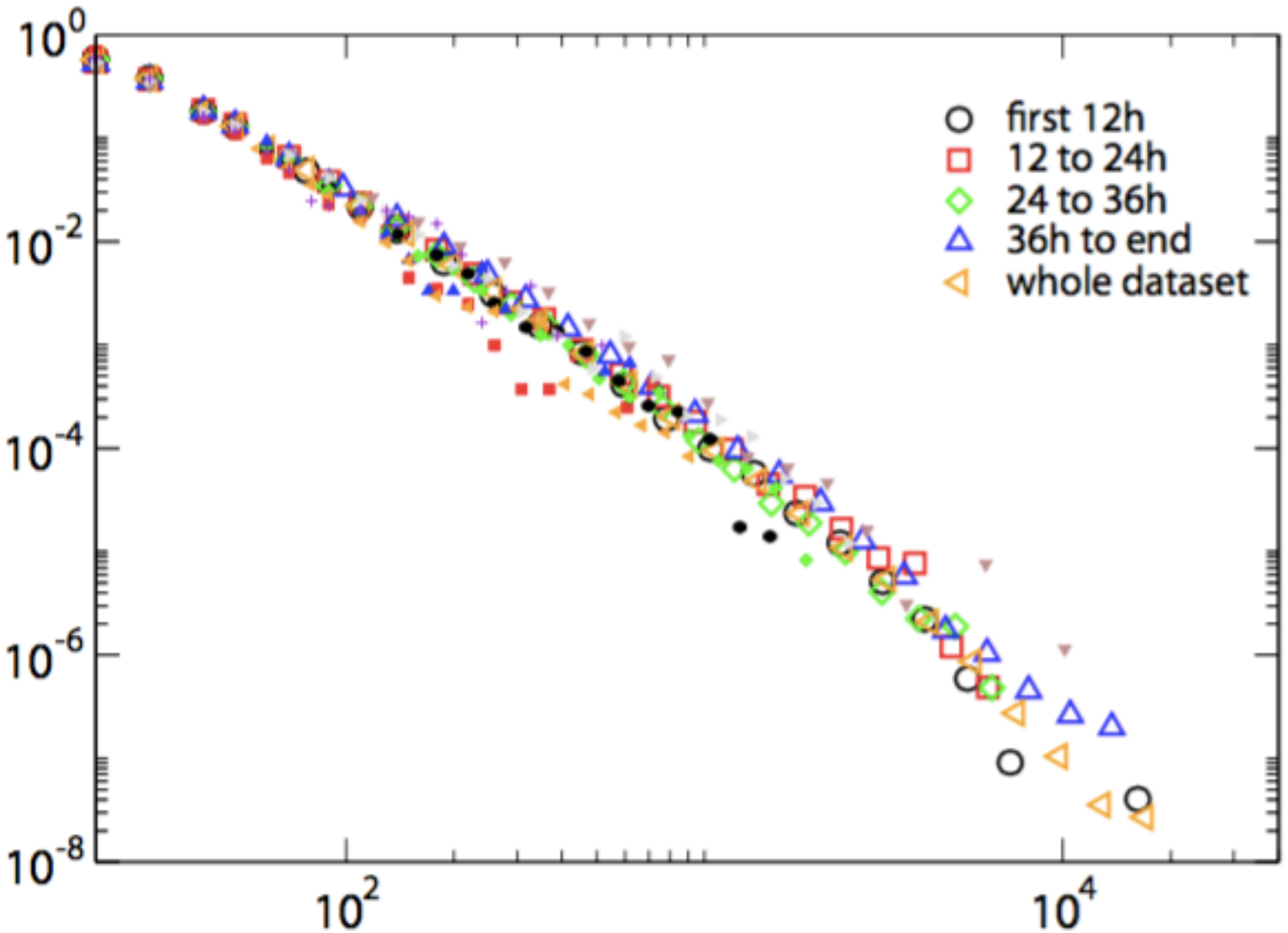
Burstiness

Example: US airport network



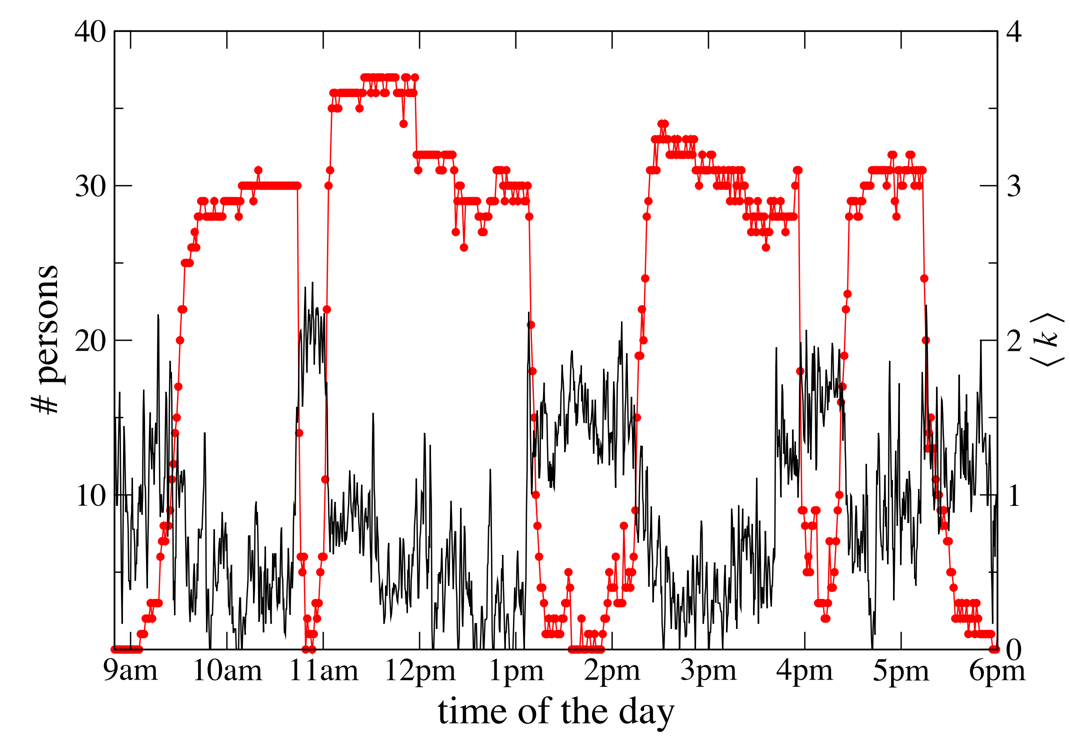
Stationarity of distributions

Example: face-to-face contacts

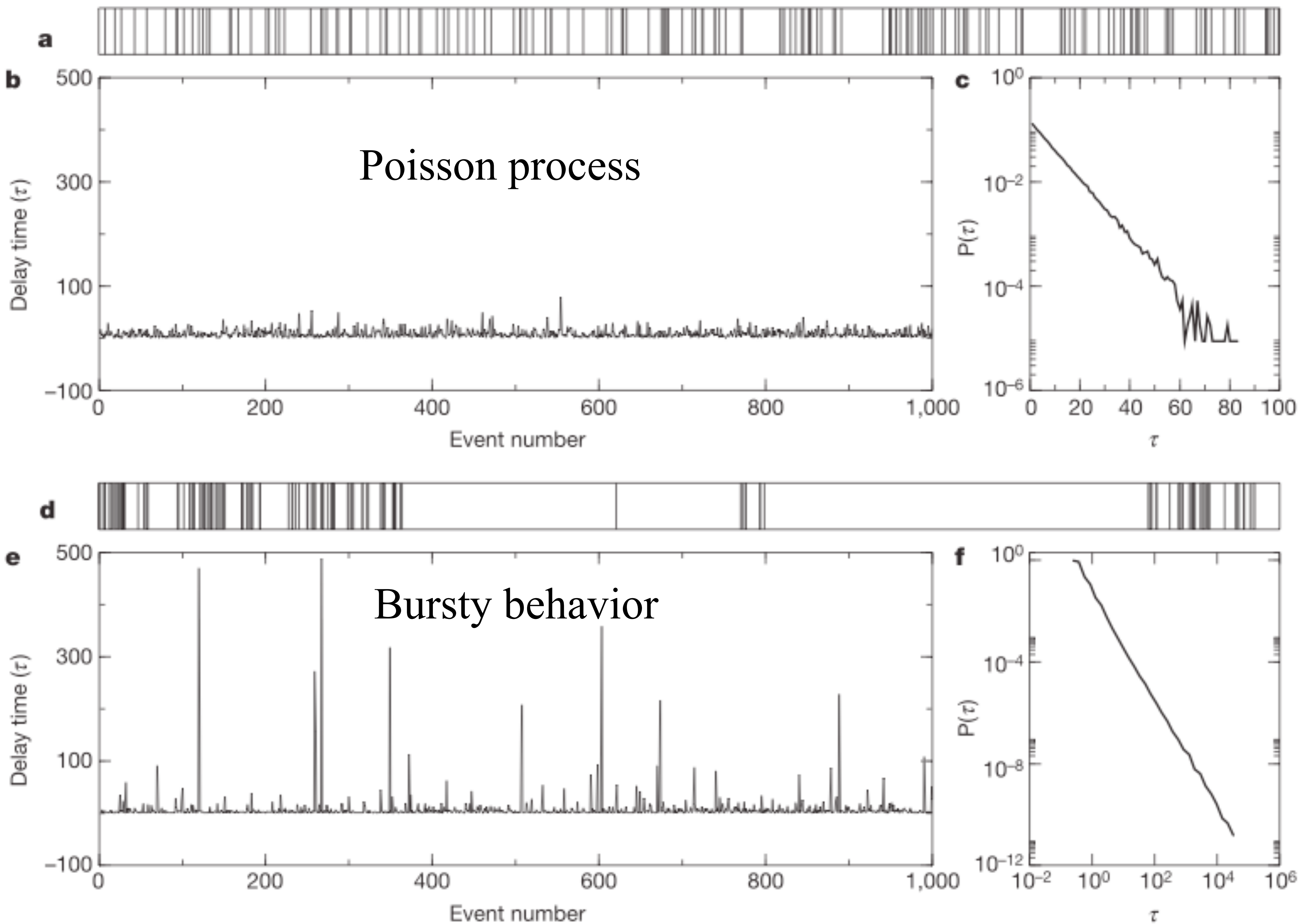


Contact duration

Stationarity of distributions



Burstiness

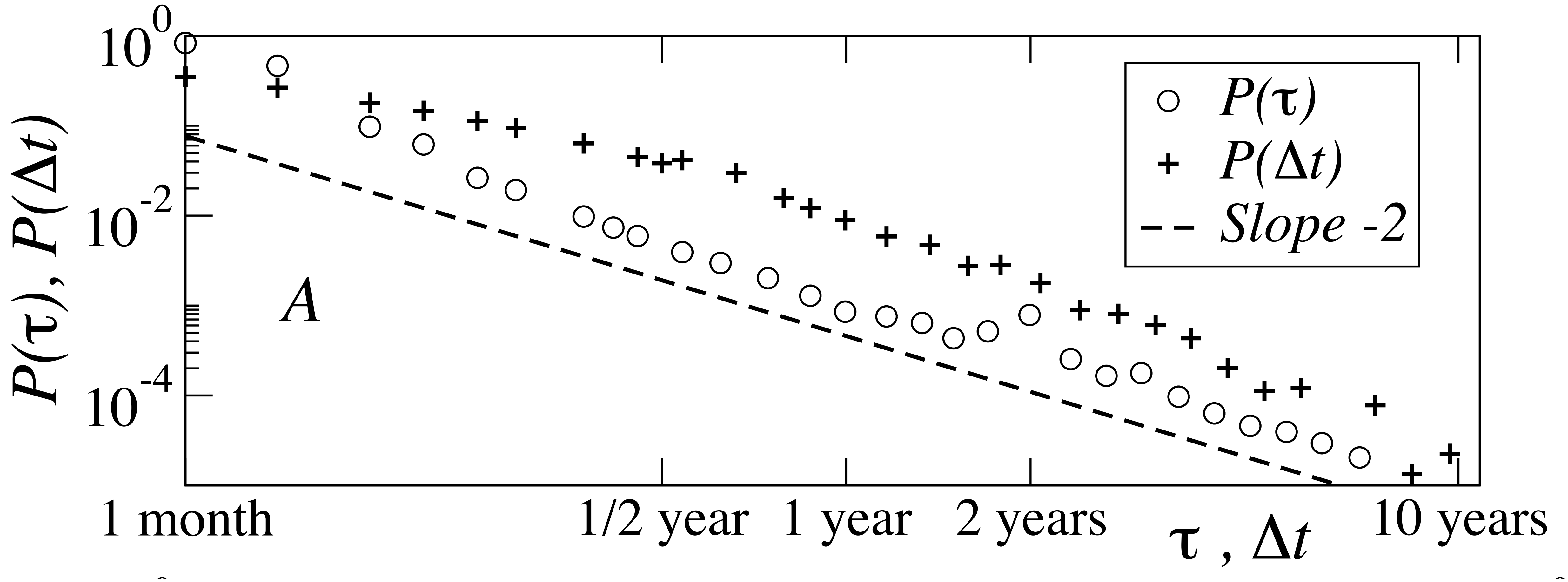


A.-L. Barabási, Nature (2006)

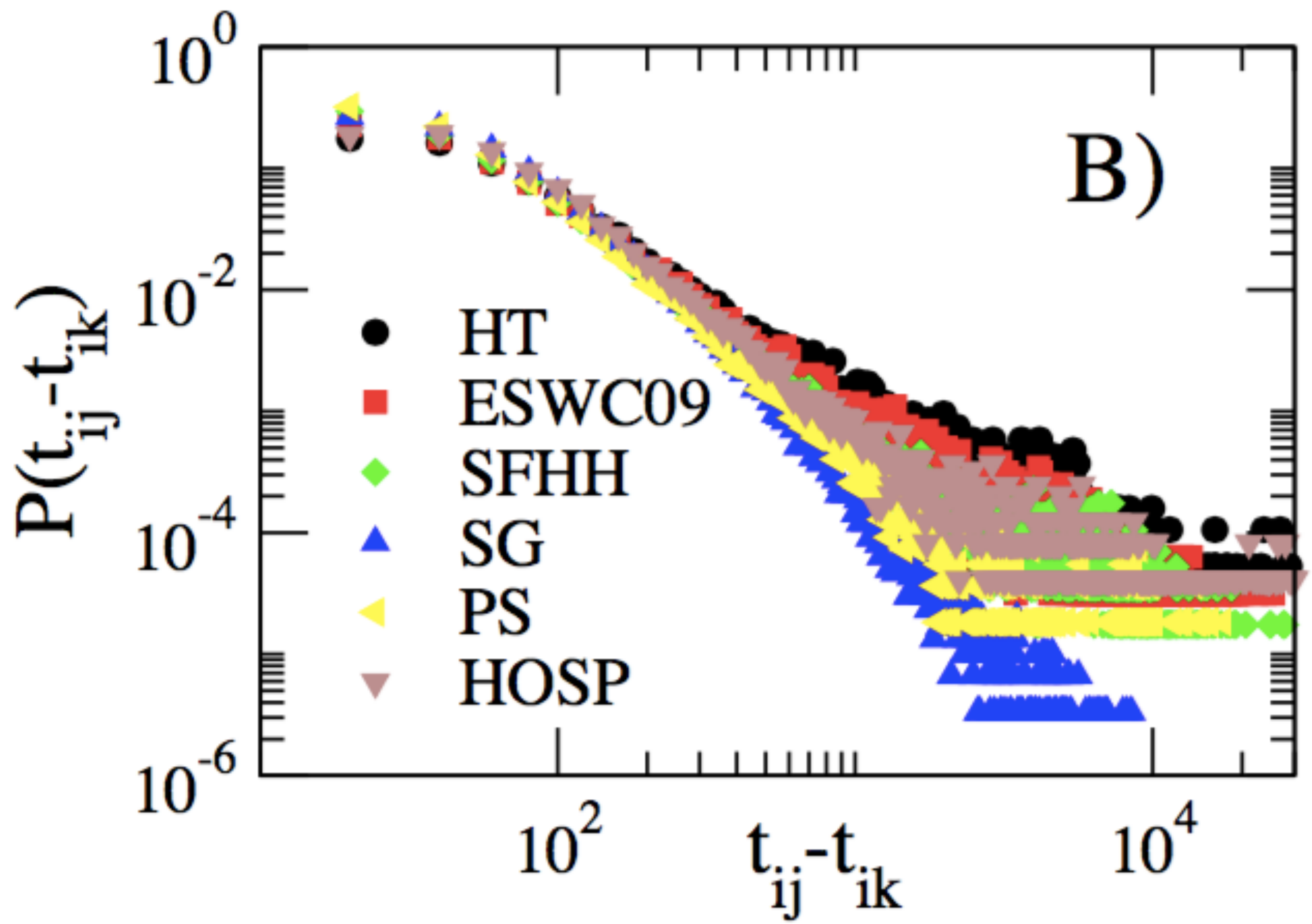
Example: US airport network

τ = duration of a link

Δt = interval between active periods of a link



Example: face-to-face contacts



Inter-contact duration

Measure of burstiness

$$B = \frac{\sigma_\tau - m_\tau}{\sigma_\tau + m_\tau}$$

where m_τ is the mean and σ_τ the std deviation of the inter-event time distribution

Poisson: B = 0 (exponential distribution)

Periodic: B = -1 (Delta distribution)

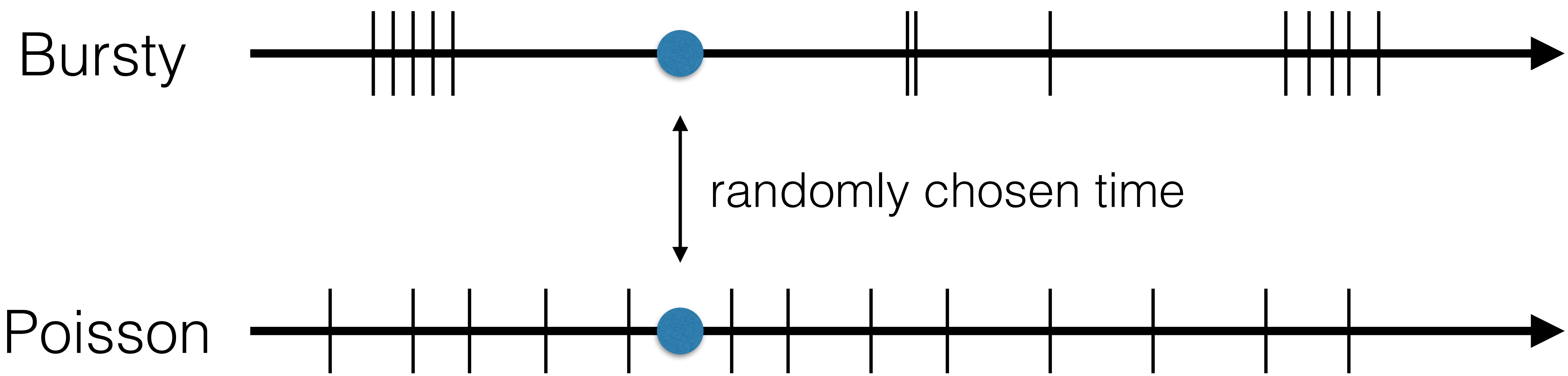
Broad distribution: B = 1 (if σ_τ diverges)

Burstiness = clustering of events in time

Goh, Barabási, EPL 2008

See also: Karsai et al., Sci Rep 2, 397 (2012)

Consequence of burstiness



Bursty timeline implies larger waiting time with higher probability
=> typically slower diffusion (if no correlations)

Structures: Persistence of patterns

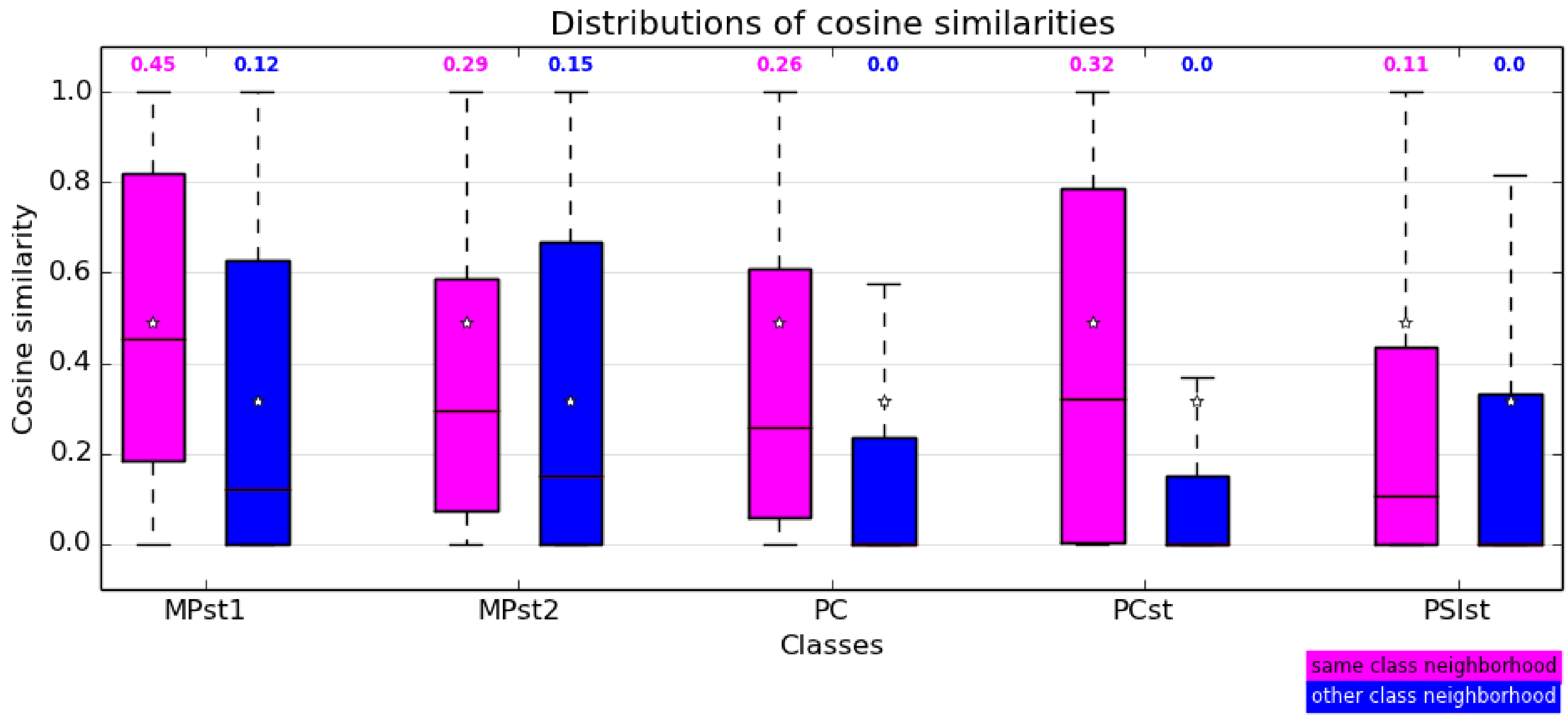
Compare successive snapshots or time-aggregated networks:

- correlation between (weighted) adjacency matrices
- correlation between contact matrices
- local similarity of neighbourhoods

$$\sigma_i = \frac{\sum_j w_{ij,(1)} w_{ij,(2)}}{\sqrt{\sum_j w_{ij,(1)}^2 \sum_{i,j} w_{ij,(2)}^2}}$$

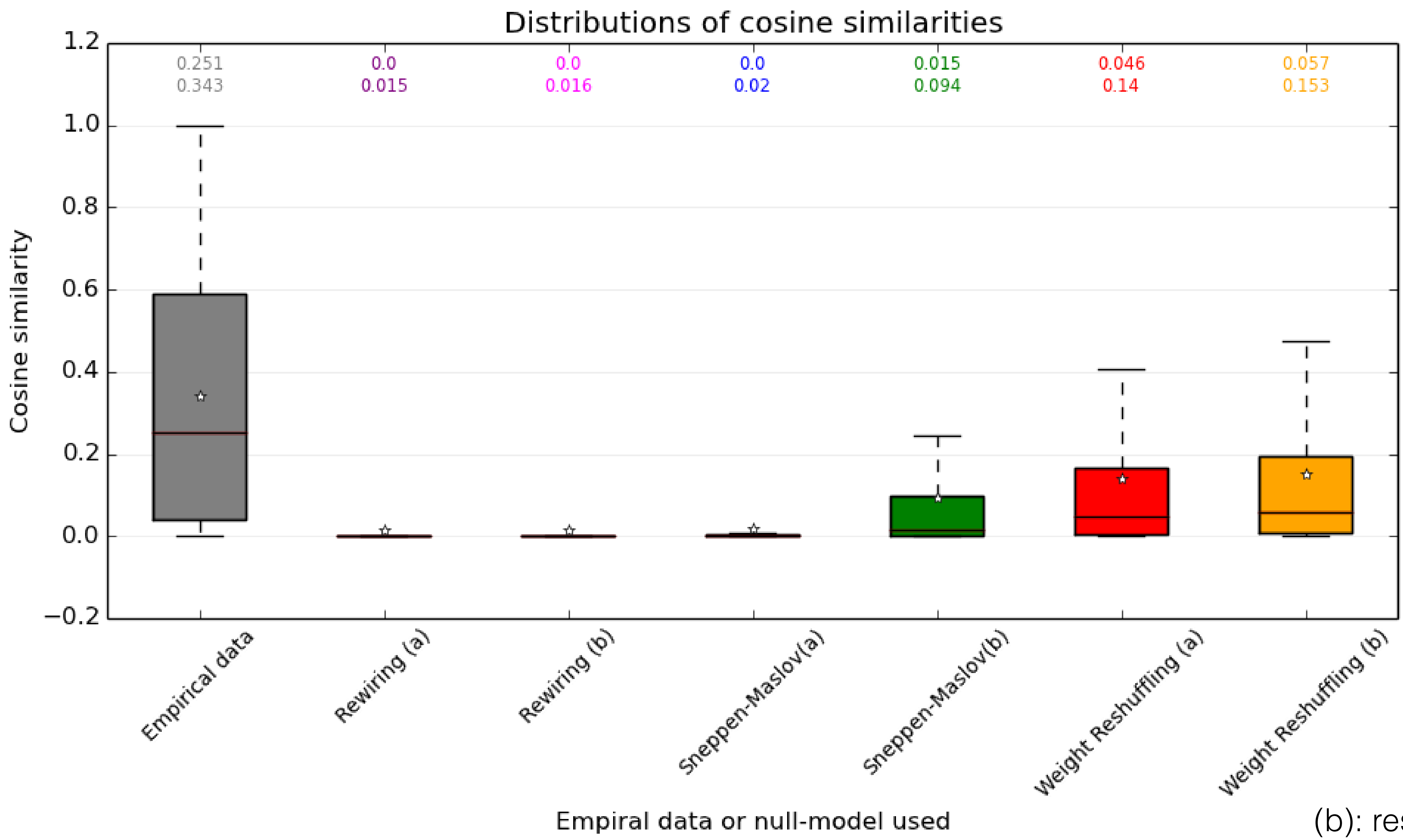
Persistence of patterns

Example: contacts in a high school,
neighbourhood similarities between different days



NB: need to compare with null model(s)
importance for spreading processes

Example: contacts in a high school, neighbourhood similarities between different days, vs different null models



(b): respecting class structure

SocioPatterns

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WELCOME

SocioPatterns is an interdisciplinary research collaboration formed in 2008 that adopts a data-driven methodology to study social dynamics and human activity.

Since 2008, we have collected longitudinal data on the physical proximity and face-to-face contacts of individuals in numerous real-world environments, covering widely varying contexts across several countries: schools, museums, hospitals, etc. We use the data to study human behaviour and to develop agent-based models for the transmission of infectious diseases.

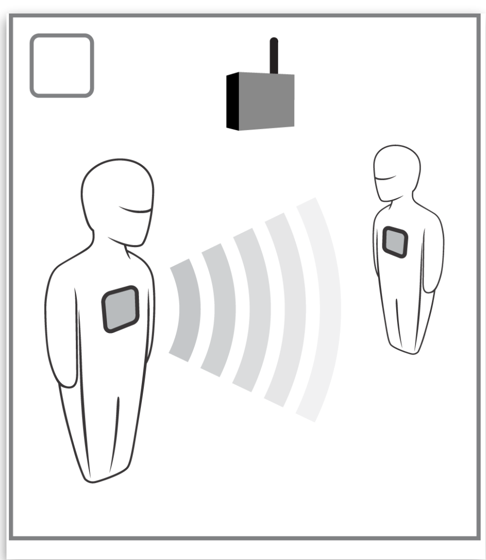
We make most of the collected data freely available to the scientific community.

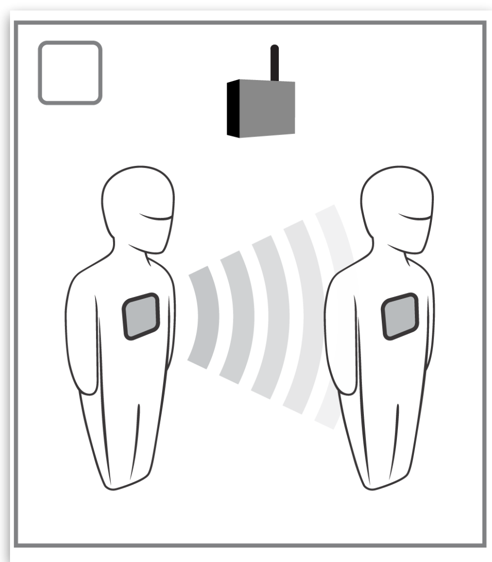
NEWS

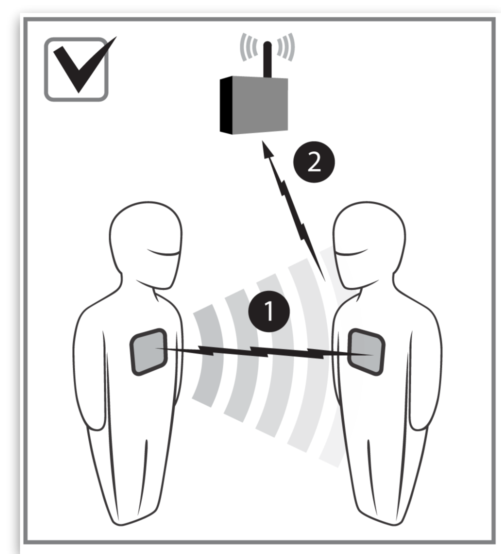
New data sets published: co-presence and face-to-face contacts

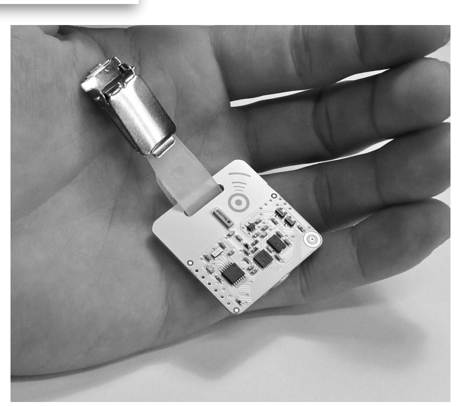
Through a publication in EPJ Data Science, we have released several new data sets of different types. These datasets can be found on Zenodo.

On the one hand, we have released new temporally resolved data on face-to-face



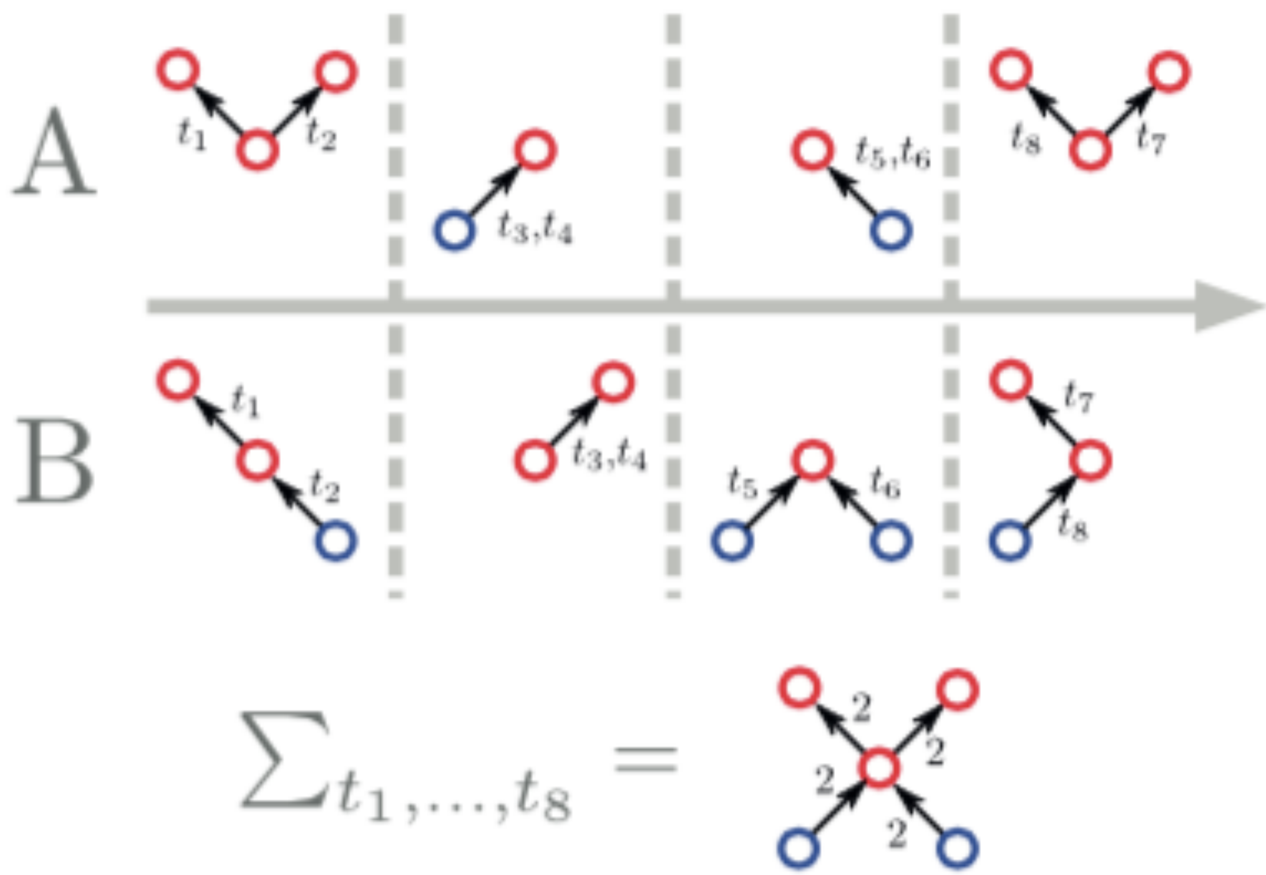






>Structures in temporal networks: Temporal motifs

Same aggregated network, different temporal sequences

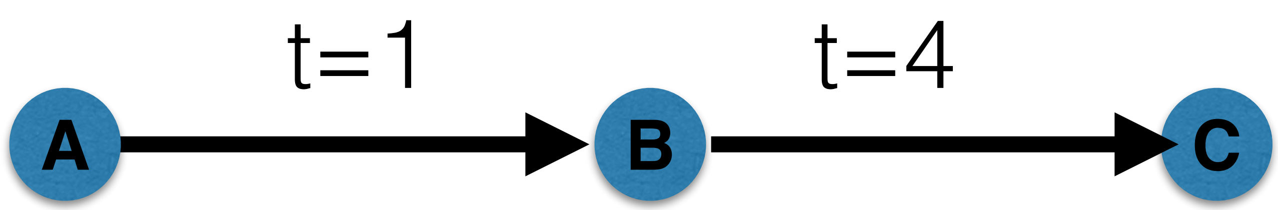


detection of temporal patterns?

L. Kovanen et al., J. Stat. Mech. (2011) P11005

L. Kovanen et al., PNAS (2013)

Δt-adjacent events



events Δt-adjacents for Δt=4

events Δt-adjacent:

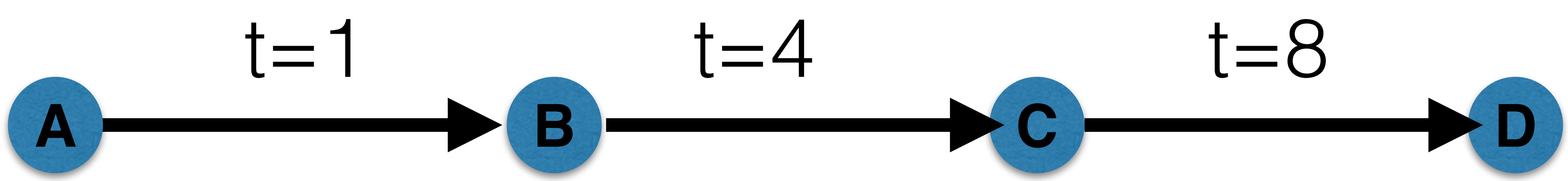
- at least one node in common
- at most Δt between end of 1st event and start of 2nd event

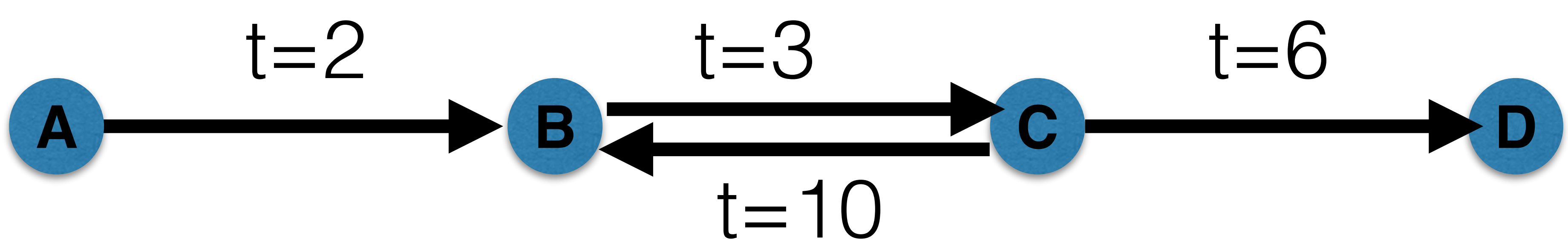
L. Kovanen et al., J. Stat. Mech. (2011) P11005

Δt-connected events

=events connected by a chain of Δt-adjacent events

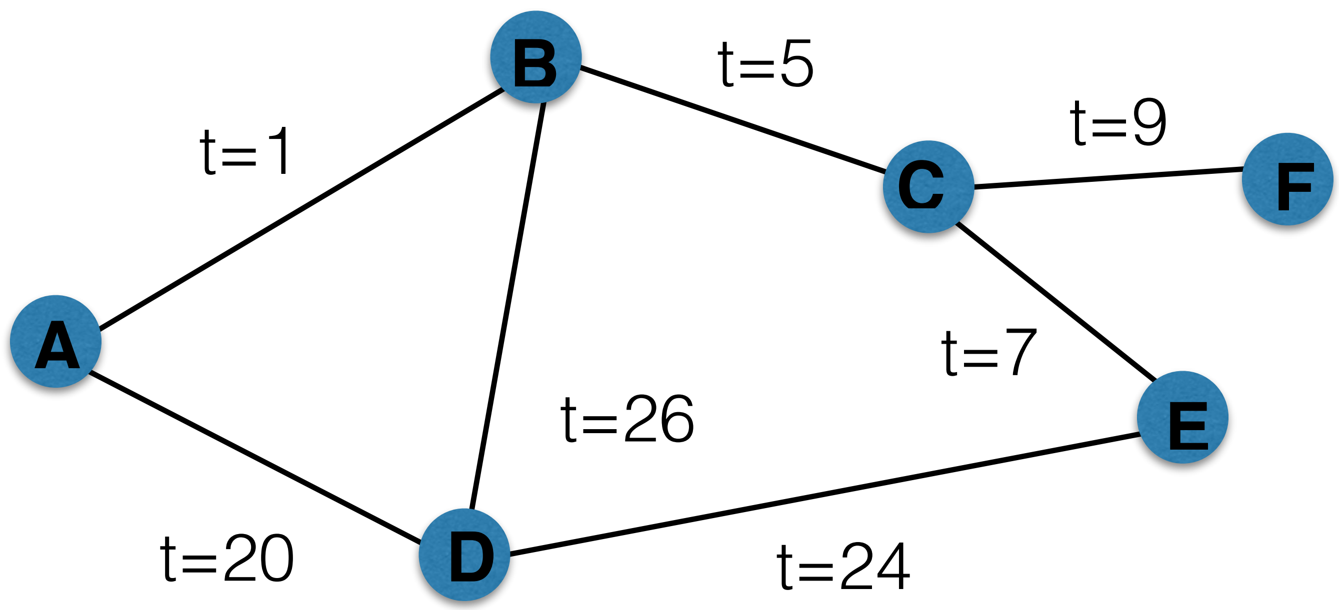
Examples (Δt=5):



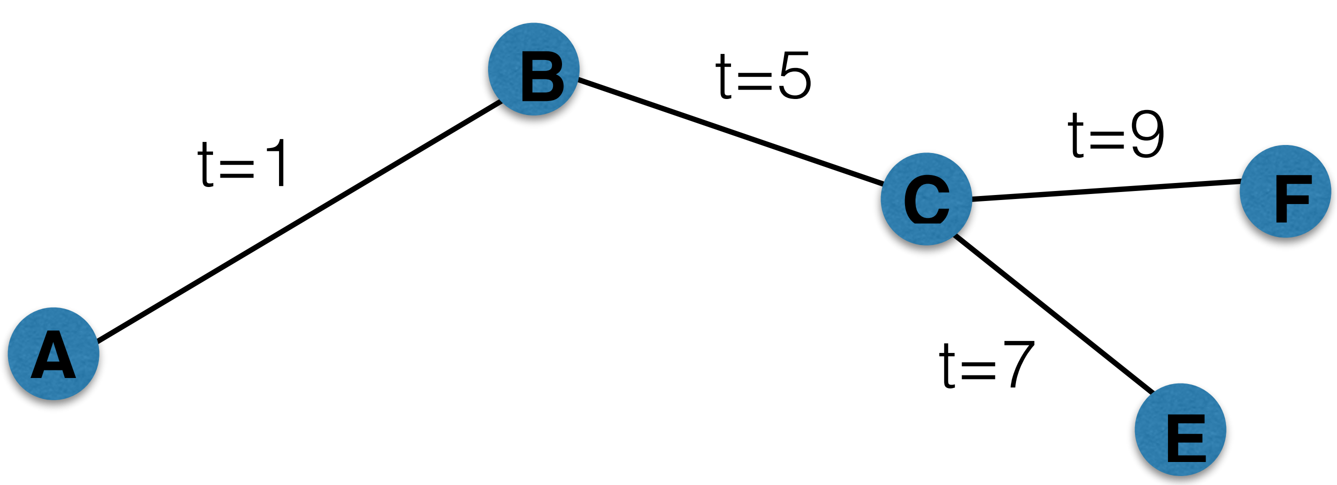


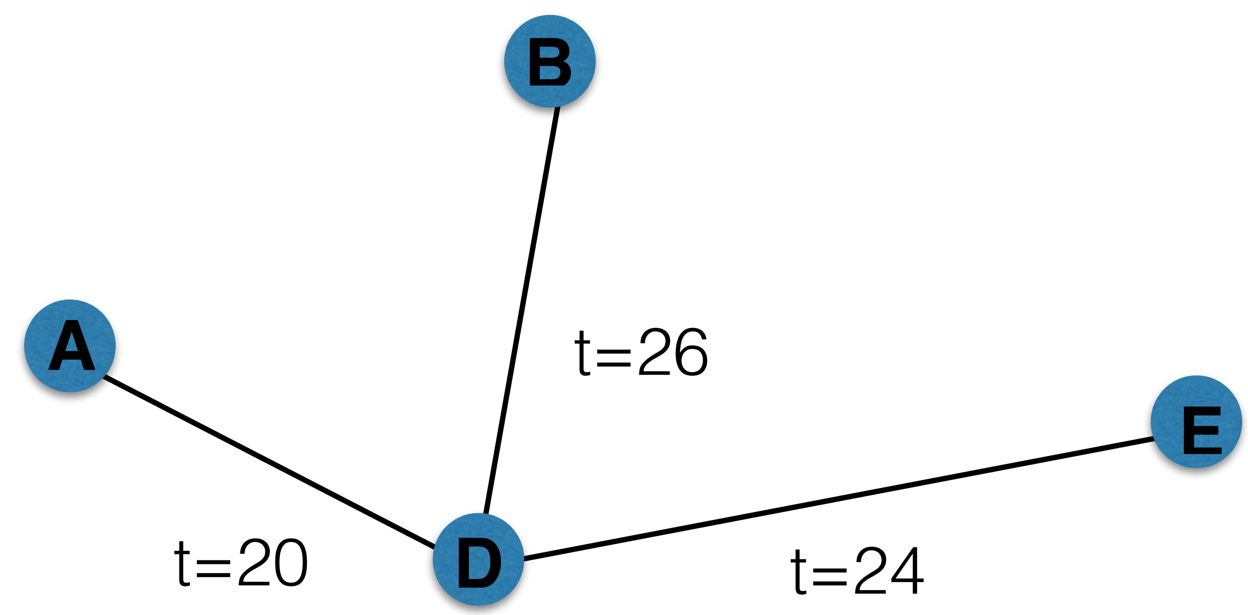
L. Kovanen et al., J. Stat. Mech. (2011) P11005

Temporal subgraph = set of Δt-connected events



Maximal temporal subgraphs (Δt=5):





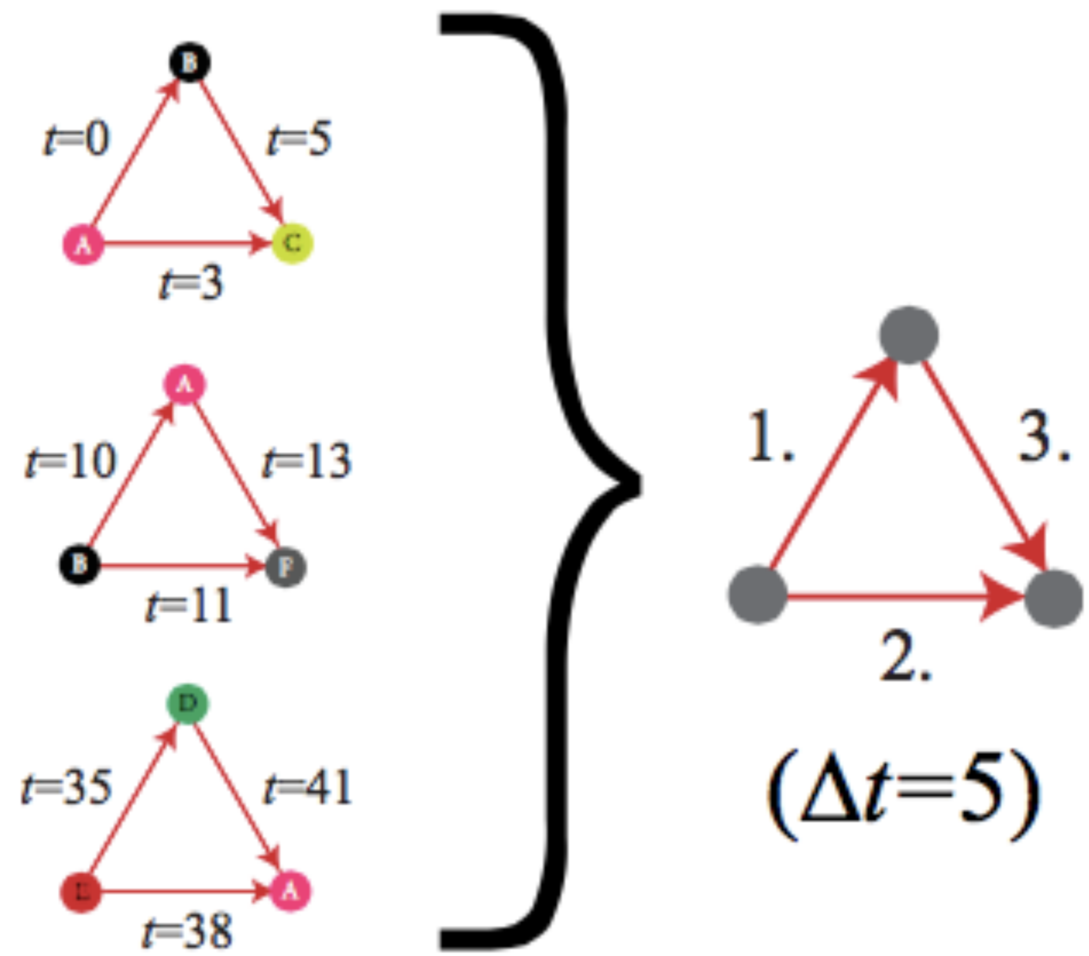
L. Kovanen et al., J. Stat. Mech. (2011) P11005

Temporal motifs

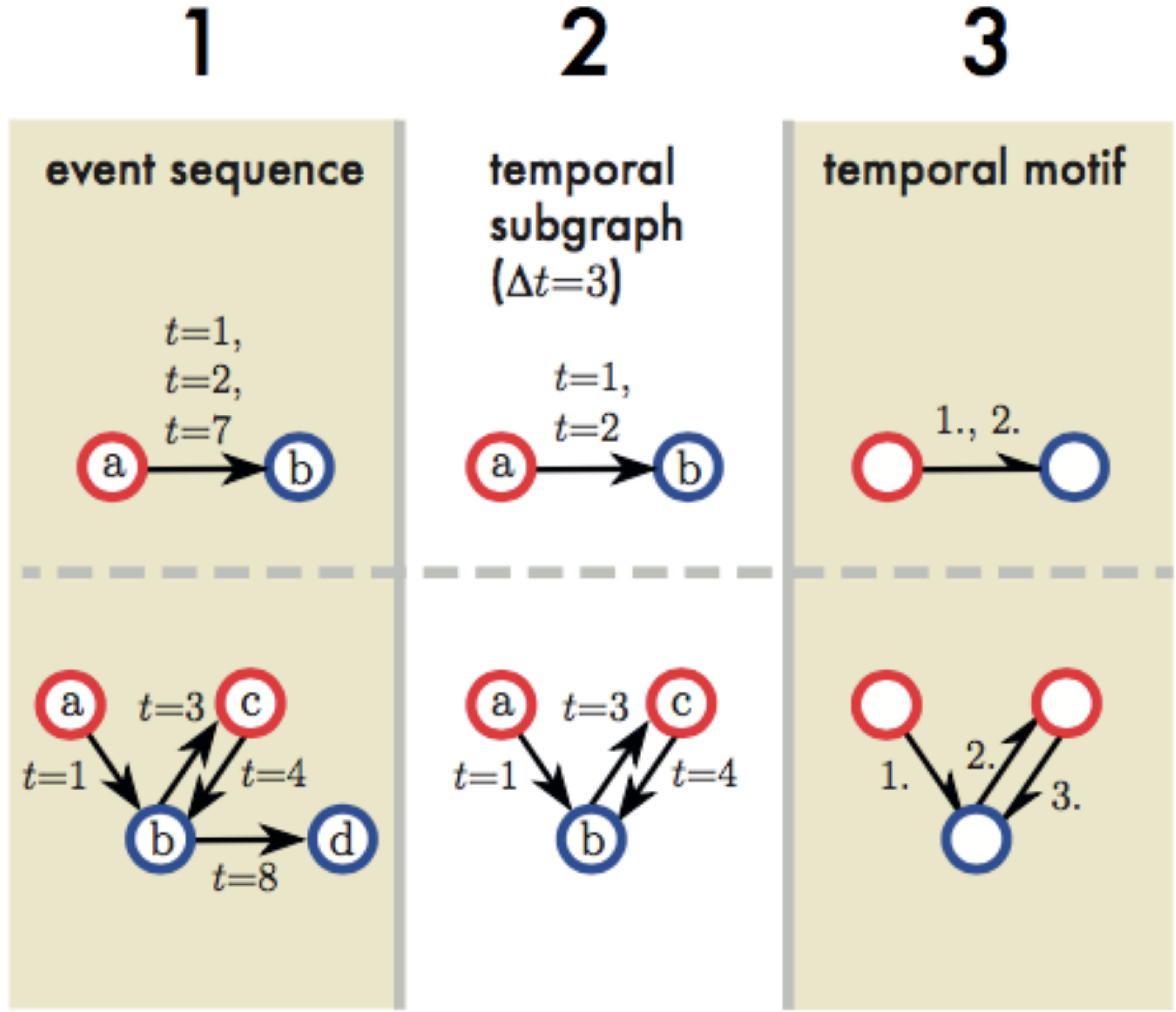
= Equivalence classes of valid temporal subgraphs

Valid: no events are skipped at each node

Equivalence classes: forget identities of nodes and exact timing



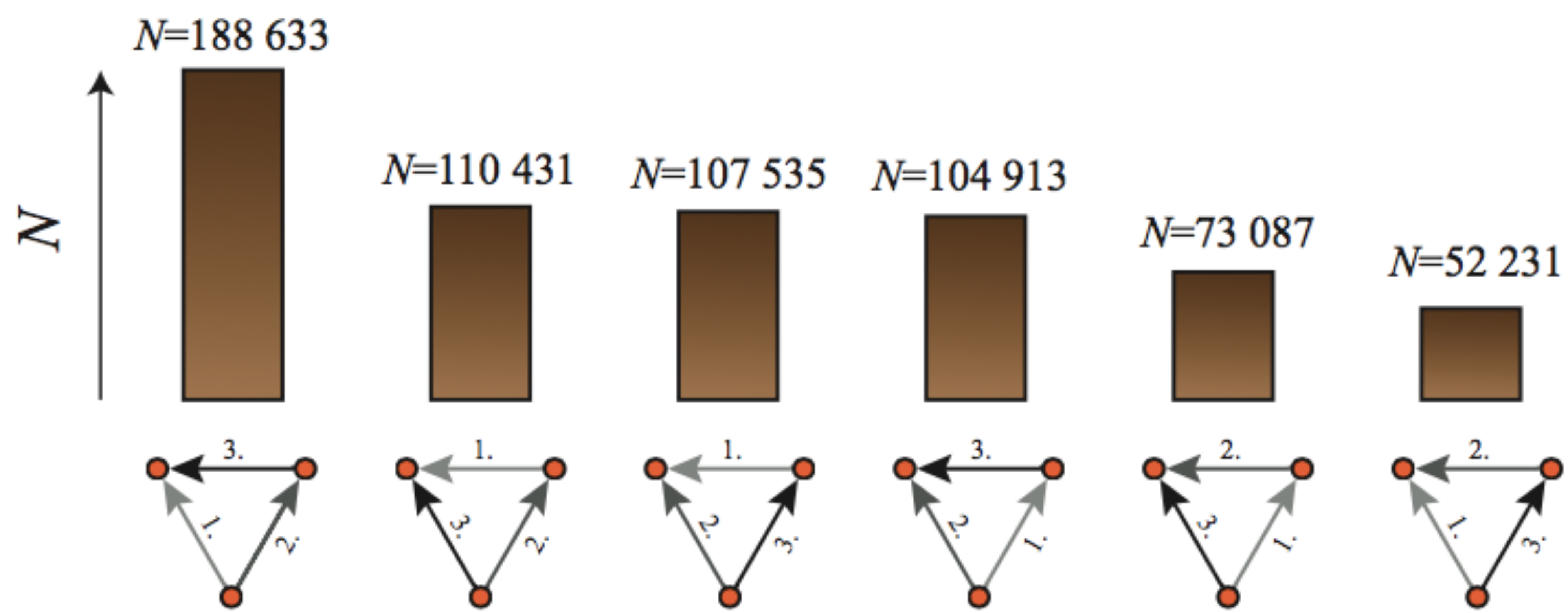
J. Saramäki



which motifs are most frequent?
metadata on nodes, compare with null models, …

Temporal motifs reveal homophily, gender-specific patterns, and group talk in call sequences
L. Kovanen et al., PNAS (2013)

Structure: temporal motifs



Motifs in mobile phone call networks

Kovanen et al. (2012)

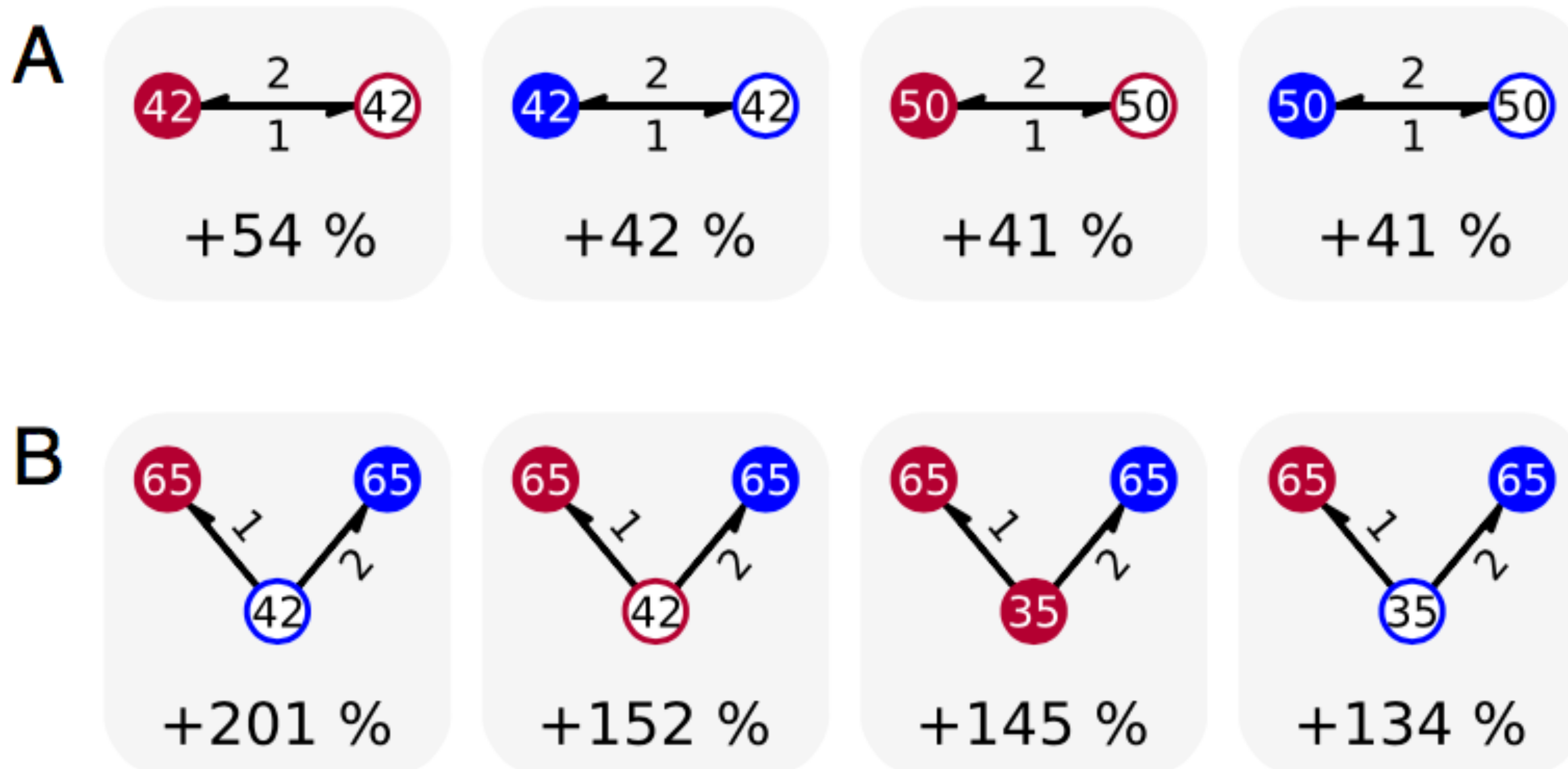


Fig. 4. The most common temporal motifs exhibit shared properties. (*A*) The four most common returned-call motifs. The numbers inside the nodes denote the age group (18–26, 27–32, 33–38, 39–45, 46–55, or 56–80; the value shown is the weighted average rounded to closest integer). The open nodes denote postpaid and filled prepaid customers; red denotes female, and blue, male. The arrows denote events, and the numbers next to them show their temporal order. In all four cases, the first call takes place from the prepaid (filled node) to the postpaid (open node) customer. The number below each motif shows the relative occurrence compared with the null model. (*B*) The four most common out-star motifs. In all four cases, the two receivers have the same age, a pattern that is typical for the most common out-stars.

Temporal motifs reveal homophily, gender-specific patterns, and group talk in call sequences
L. Kovanen et al., PNAS (2013)

>Structures: Egocentric temporal motifs

Home › Data Mining and Knowledge Discovery › Article

Open Access | Published: 12 November 2021

An efficient procedure for mining egocentric temporal motifs

Antonio Longa, Giulia Cencetti, Bruno Lepri & Andrea Passerini

Data Mining and Knowledge Discovery **36**, 355–378 (2022) | Cite this article

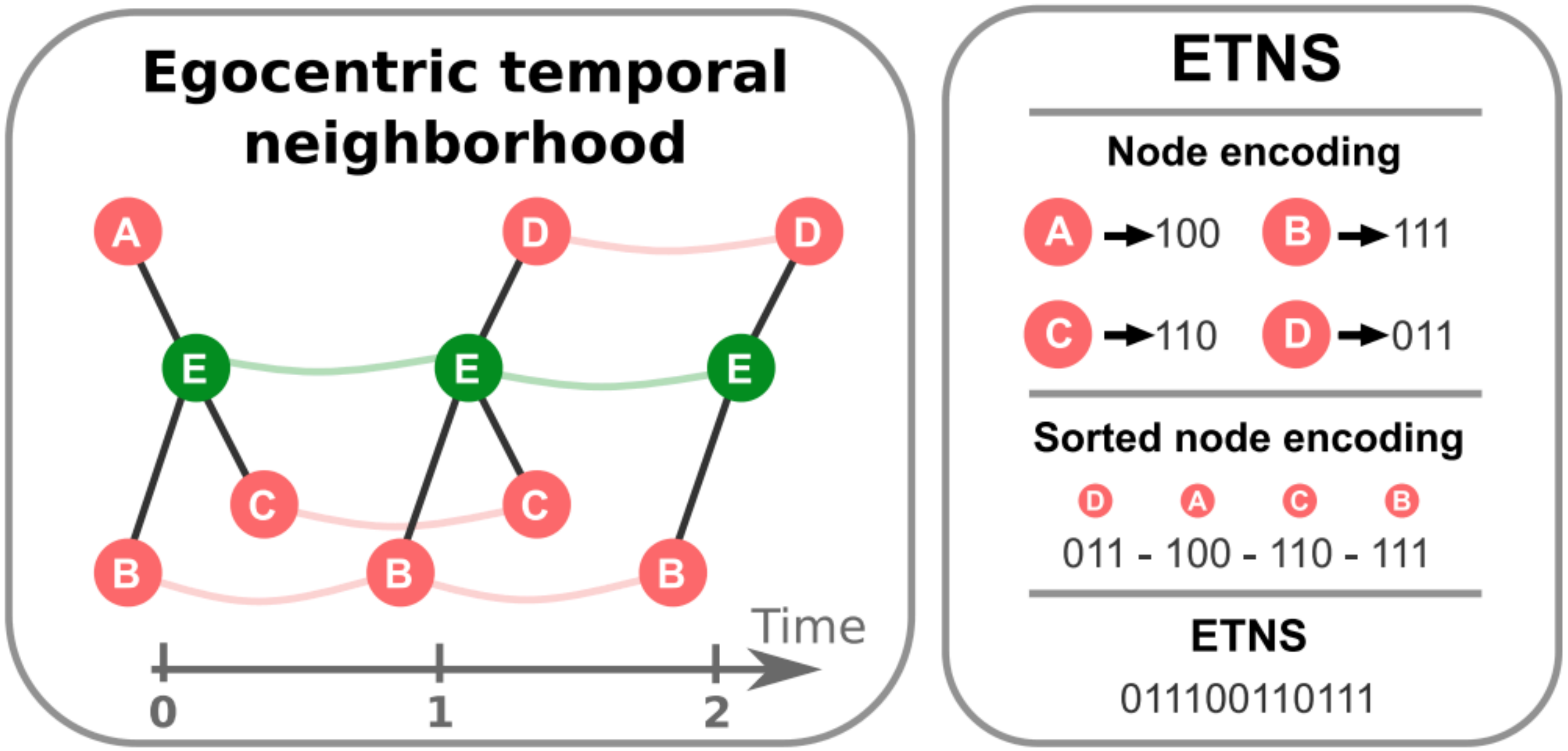


Fig. 1 Graphical summary of the procedure for extracting egocentric temporal motifs. The left panel shows the *egocentric temporal neighborhood* of the ego node E (in green), with temporal order two and initial time instant zero. Black edges connect the central node with its neighbors (in red) at each time step, while green (resp. red) edges connect consecutive occurrences of the central (resp. a neighboring) node along the time sequence. The right panel shows how the corresponding *egocentric temporal neighborhood signature* (ETNS) is computed. Each neighboring node is encoded into a bit vector indicating the time slots when it is present. The node encodings are lexicographically sorted first and then concatenated to generate the signature

ETN and ETNS

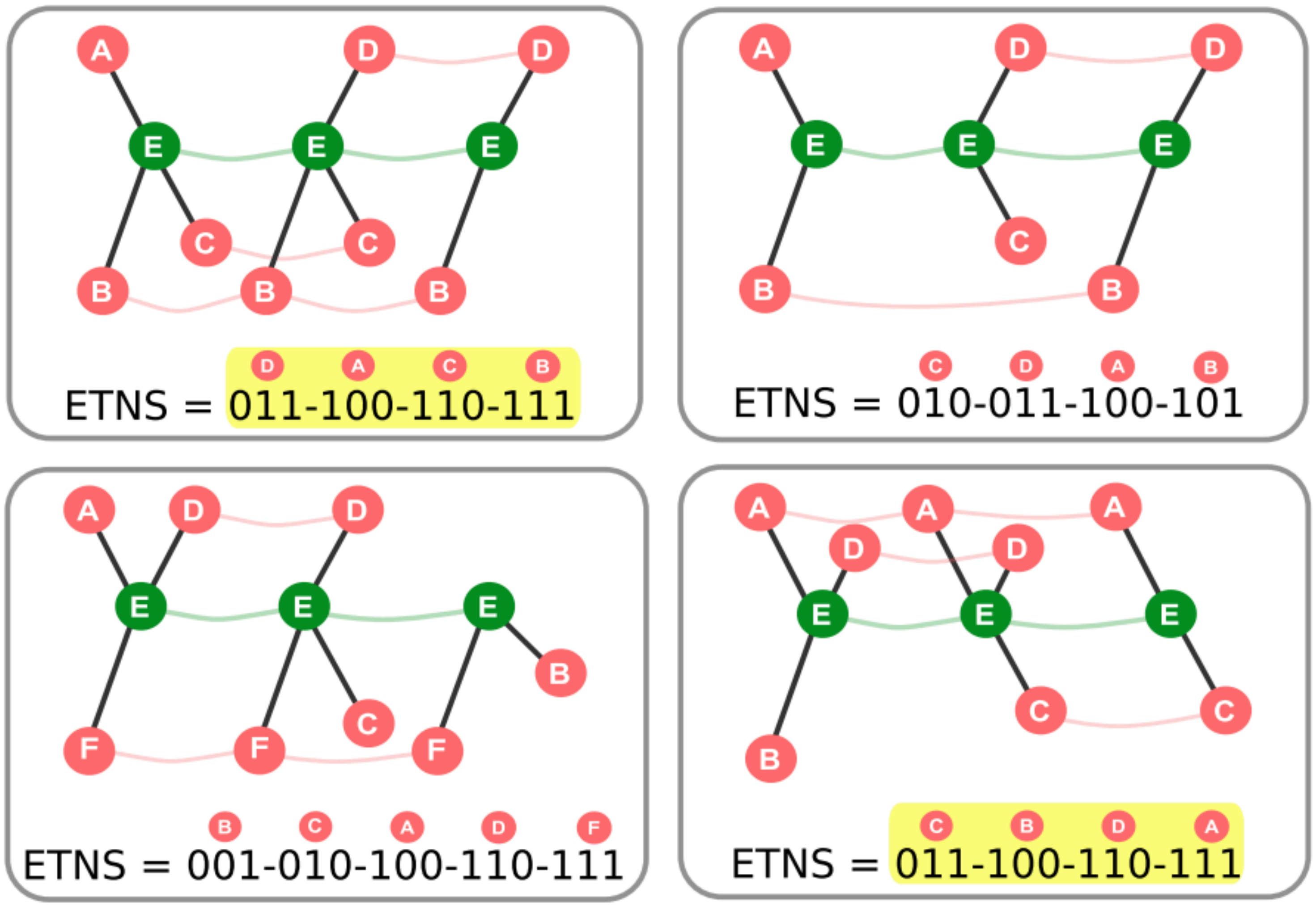


Fig. 3 Examples of ETN and ETNS for different temporal graphs with $k = 2$. The two highlighted ETNS are identical and correspond to isomorphic ETN

Motifs=ETNS more frequent than expected in a null model

[Submitted on 18 May 2022]

Neighbourhood matching creates realistic surrogate temporal networks

Antonio Longa, Giulia Cencetti, Sune Lehmann, Andrea Passerini, Bruno Lepri

Temporal networks are essential for modeling and understanding systems whose behavior varies in time, from social interactions to biological systems. Often, however, real-world data are prohibitively expensive to collect or unshareable due to privacy concerns. A promising solution is `surrogate networks', synthetic graphs with the properties of real-world networks. Until now, the generation of realistic surrogate temporal networks has remained an open problem, due to the difficulty of capturing both the temporal and topological properties of the input network, as well as their correlations, in a scalable model. Here, we propose a novel and simple method for generating surrogate temporal networks. By decomposing graphs into temporal neighborhoods surrounding each node, we can generate new networks using neighborhoods as building blocks. Our model vastly outperforms current methods across multiple examples of temporal networks in terms of both topological and dynamical similarity. We further show that beyond generating realistic interaction patterns, our method is able to capture intrinsic temporal periodicity of temporal networks, all with an execution time lower than competing methods by multiple orders of magnitude.

>Structures in temporal networks: Span cores

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Mining (maximal) Span-cores from Temporal Networks

Full Text: PDF Get this Article

Authors:
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2018 Article

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SCIENTIFIC REPORTS

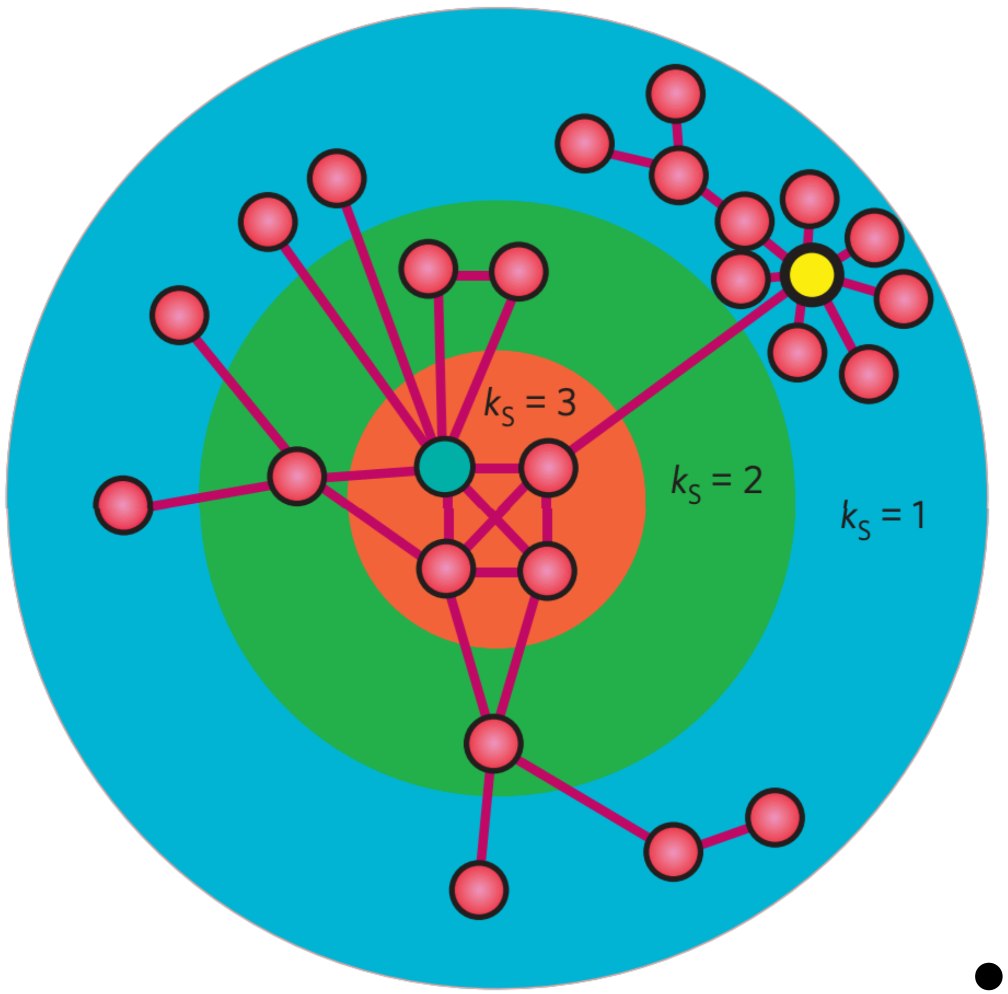
Article | Open Access | Published: 27 July 2020

Relevance of temporal cores for epidemic spread in temporal networks

Martino Ciaperoni, Edoardo Galimberti, Francesco Bonchi, Ciro Cattuto, Francesco Gullo & Alain Barrat

Scientific Reports **10**, Article number: 12529 (2020) | Cite this article

Reminder: k-core decomposition for static networks



(picture from Kitsak et al., Nat Phys 2010)

graph G=(V,E)

k-core of graph G: maximal subgraph such that for all vertices *in this subgraph* have degree at least k

- vertex i has shell index k iff it belongs to the k-core but not to the (k+1)-core
- k-shell: ensemble of all nodes of shell index k

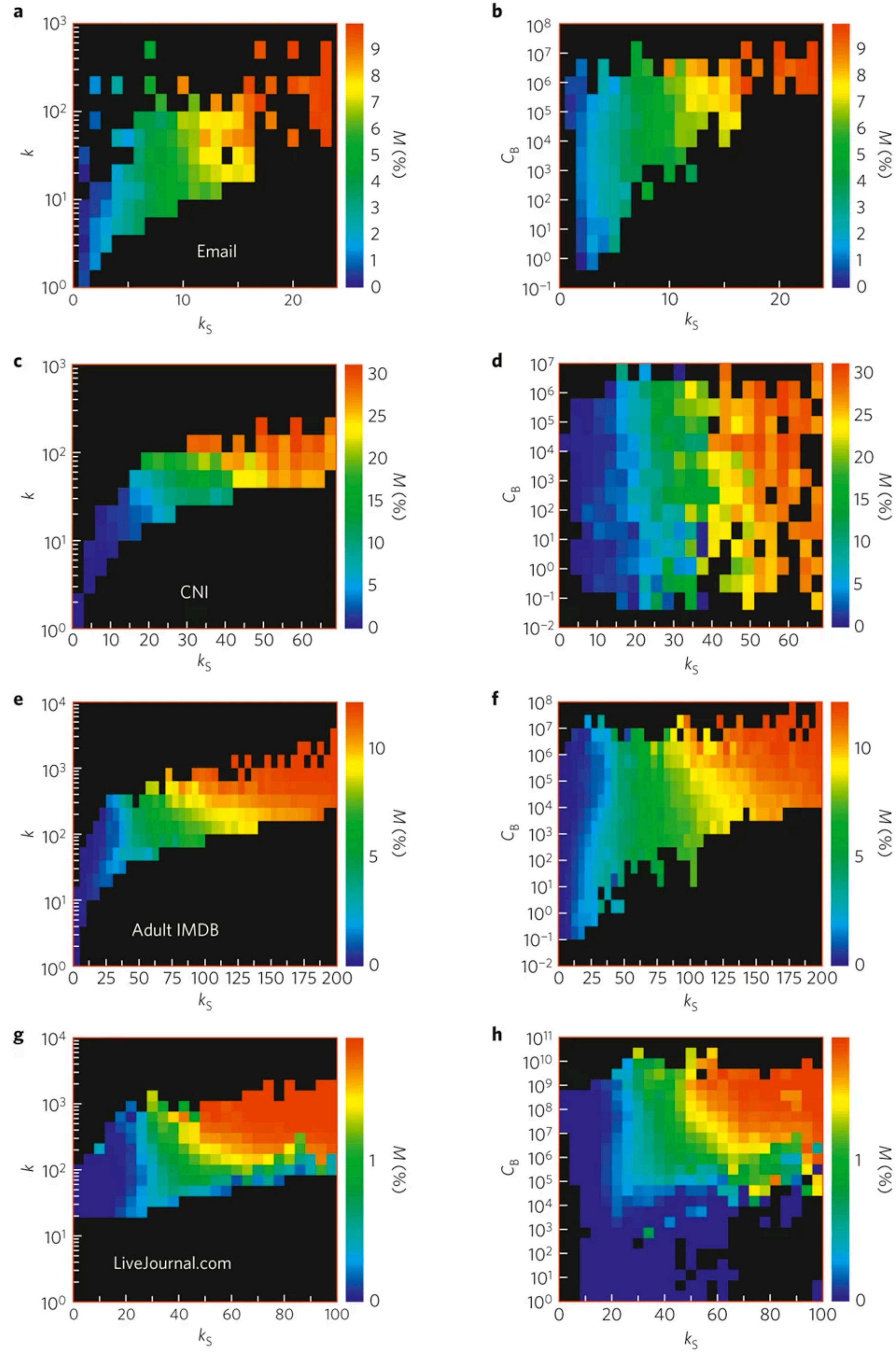
nature physics

Published: 29 August 2010

Identification of influential spreaders in complex networks

Maksim Kitsak, Lazaros K. Gallos, Shlomo Havlin, Fredrik Liljeros, Lev Muchnik, H. Eugene Stanley & Hernán A. Makse

Nature Physics **6**, 888–893(2010) | Cite this article



Size of an outbreak as a function of the seed's properties

=> largely determined by coreness

What are “cores” in temporal networks?

Temporal cores = cohesive structures that

- have a certain level of ***simultaneous cohesiveness*** (“coreness”)
- exist on a ***certain time interval,*** i.e., have a ***duration***

Span-core: definition

Temporal network G, set of vertices V,
temporal interval $T = [0,1,\ldots,t_{max}]$

Set of edges at time t: E_t

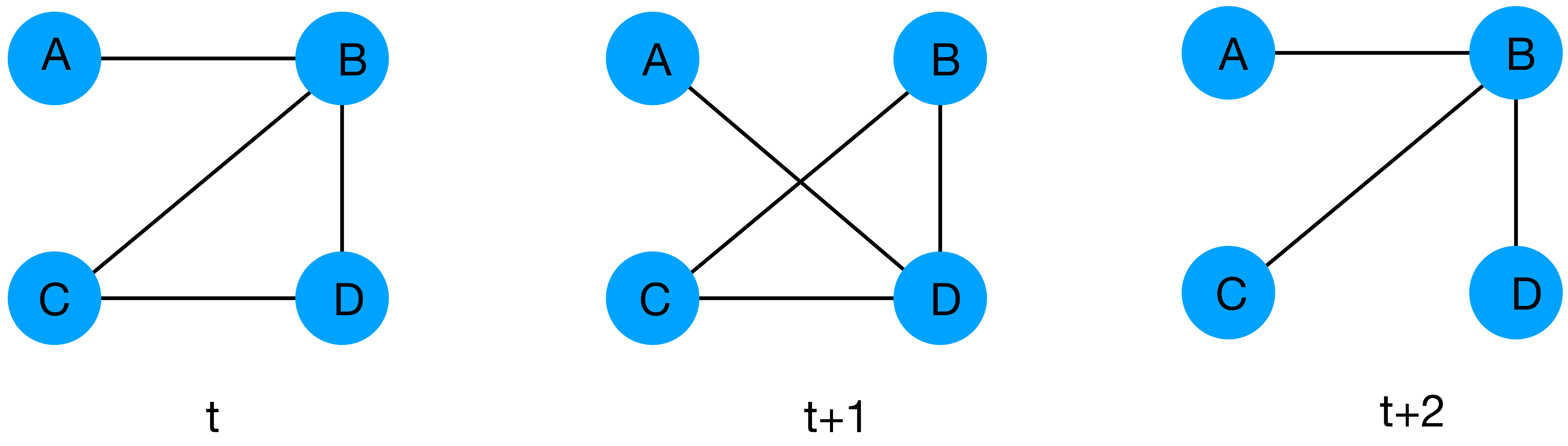
Set of edges active **at all times** t of an interval $\Delta : E_\Delta = \bigcap_{t \in \Delta} E_t$

Temporal degree of a node within a subgraph S during Δ :

$$d_\Delta(S,u) = |\{v \in S \mid (u,v) \in E_\Delta[S]\}|$$

*=number of nodes **in S** to which u is linked **at all times** during* Δ

Galimberti, Barrat, Bonchi, Cattuto, Gullo, CIKM 2018, arXiv:1808.09376



Temporal degrees on [t,t+1] and [t,t+2]:

d(A,[t,t+1]) = 0, d(A,[t,t+2]) = 0

d(B,[t,t+1]) = 2, d(B,[t,t+2]) = 2

d(C,[t,t+1]) = 2, d(C,[t,t+2]) = 1

d(D,[t,t+1]) = 2, d(D,[t,t+2]) = 1

Span-core: definition

Definition 2 (((k, Δ))-core). *The (k, Δ)-core of a temporal graph $G = (V, T, \tau)$ is (when it exists) a maximal and non-empty set of vertices $\emptyset \neq C_{k,\Delta} \subseteq V$, such that $\forall u \in C_{k,\Delta} : d_\Delta(C_{k,\Delta}, u) \geq k$, where $\Delta \sqsubseteq T$ is a temporal interval and $k \in \mathbb{N}^+$.*

i.e., all nodes of the core have **degree at least k within the core** during the temporal interval,
and have **at least k "constant" neighbors in the core** during that interval

Definition 3 (Maximal Span-core). *A span-core $C_{k,\Delta}$ of a temporal graph G is said* maximal *if there does not exist any other span-core $C_{k',\Delta'}$ of G such that $k \leq k'$ and $\Delta \sqsubseteq \Delta'$.*

Galimberti, Barrat, Bonchi, Cattuto, Gullo, CIKM 2018, arXiv:1808.09376

Extracting the span-cores

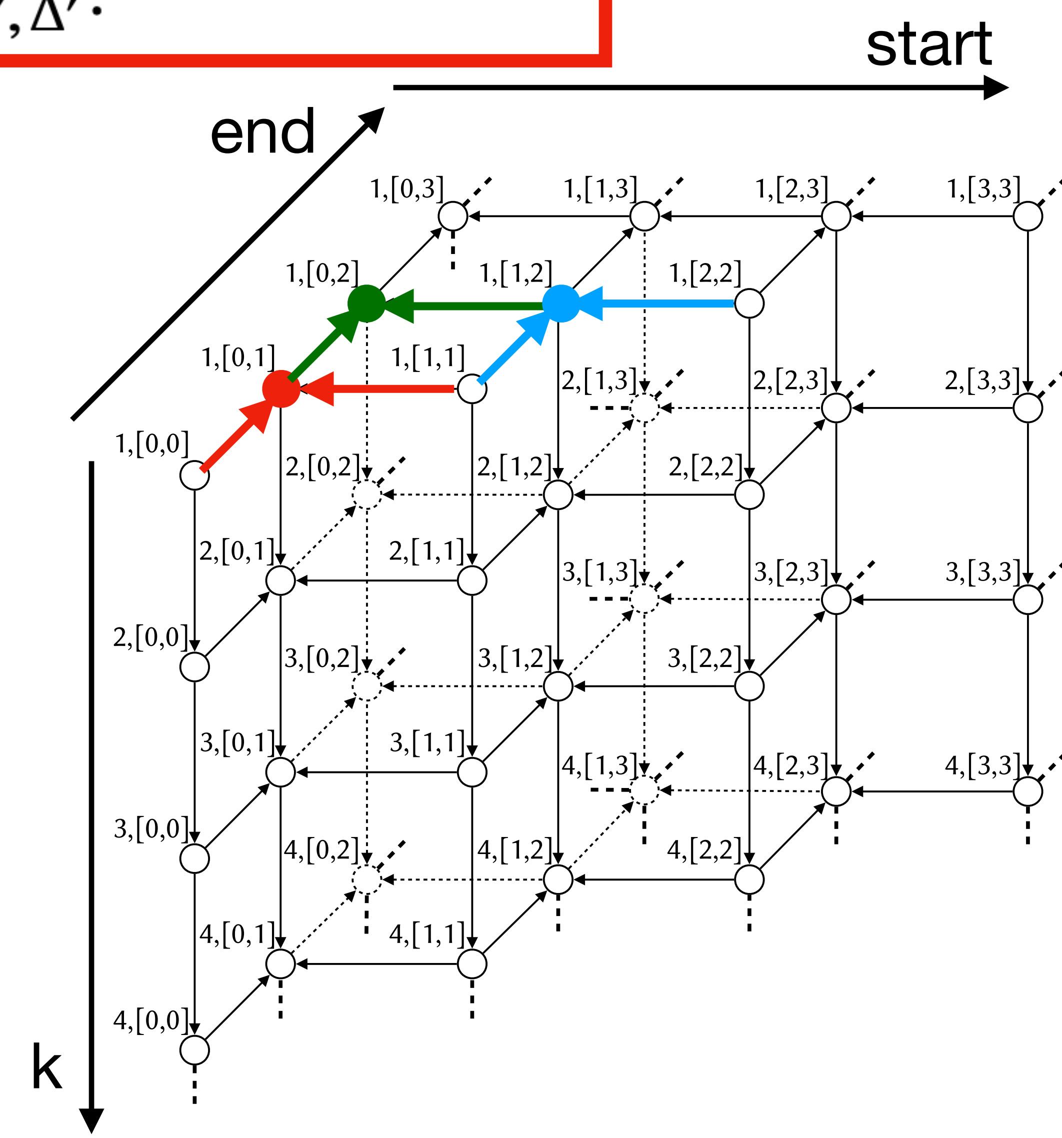
Efficient algorithms

https://github.com/egalimberti/span_cores

- Exploit the containment property:

> **Proposition 1 (Span-core containment).** *For any two span-cores $C_{k,\Delta}$, $C_{k',\Delta'}$ of a temporal graph G it holds that*
>
> $$k' \leq k \wedge \Delta' \sqsubseteq \Delta \Rightarrow C_{k,\Delta} \subseteq C_{k',\Delta'}.$$

- generate temporal intervals of increasing size
- start the decomposition from intersection of previously found cores



Examples:

core k=1 on interval [0,1] is obtained by starting from the intersection of **core 1 on interval [0,0] and core 1 on interval [1,1]**

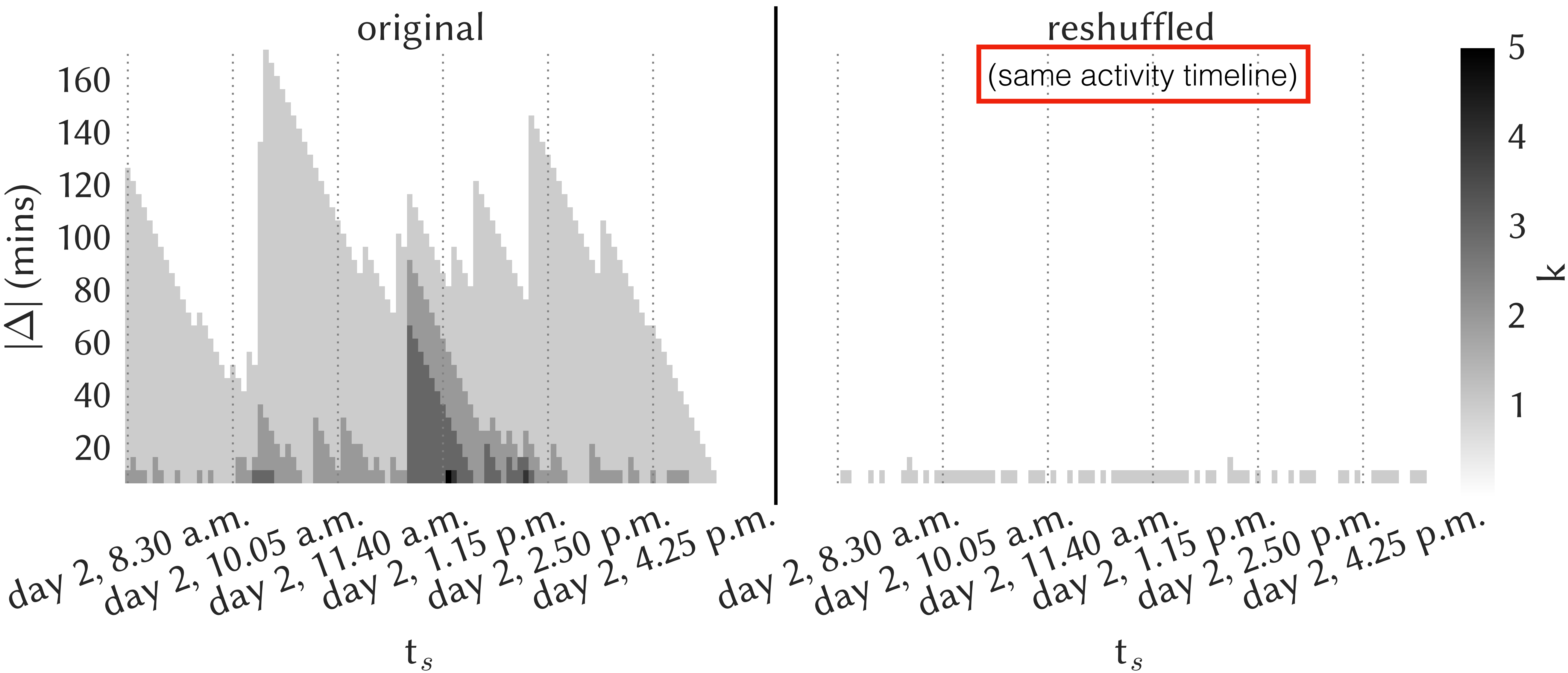
core k=1 on interval [0,2] is obtained by starting from the intersection of **core 1 on interval [0,1] and core 1 on interval [1,2]**

Galimberti, Barrat, Bonchi, Cattuto, Gullo, CIKM 2018, arXiv:1808.09376

Span-core: examples

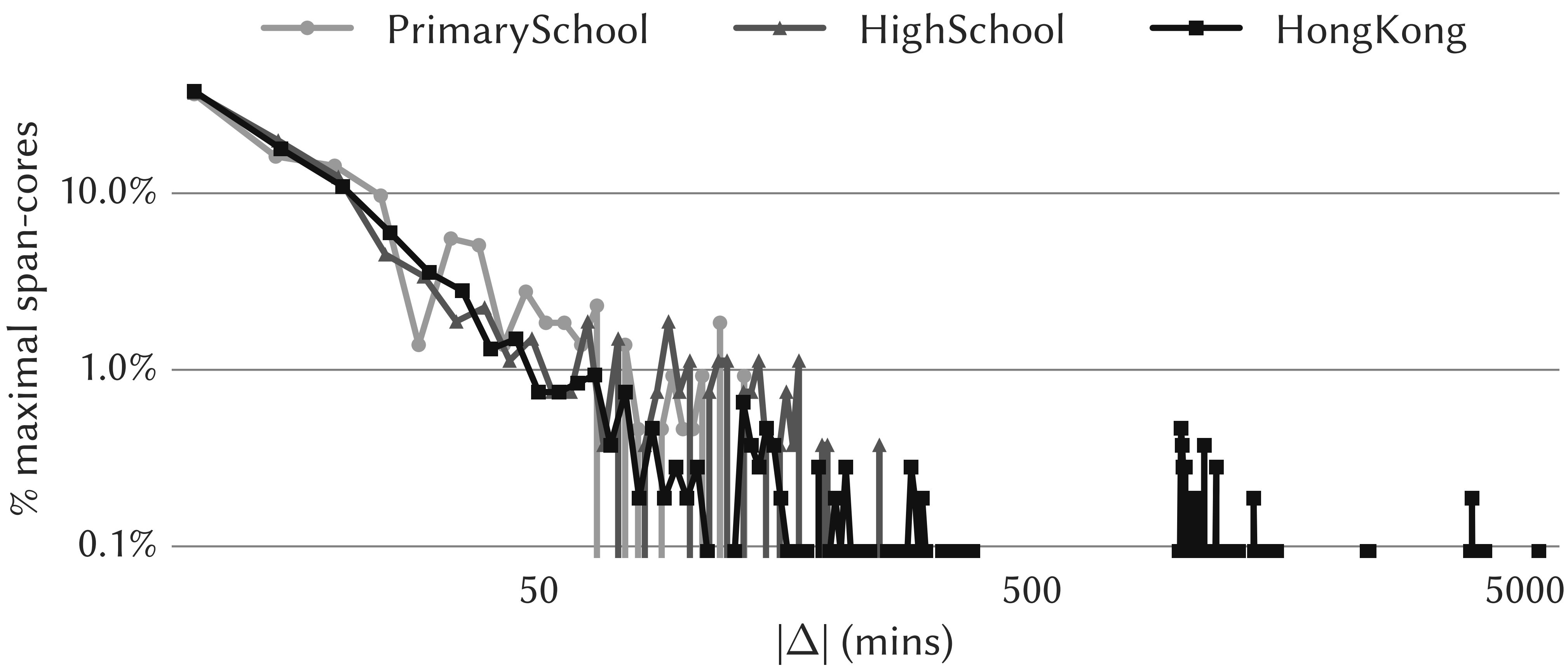
https://github.com/egalimberti/span_cores

Primary school data:
Order of the span-cores as a function of their starting time and of the temporal span length



Galimberti, Barrat, Bonchi, Cattuto, Gullo, CIKM 2018, arXiv:1808.09376

Span-core: examples



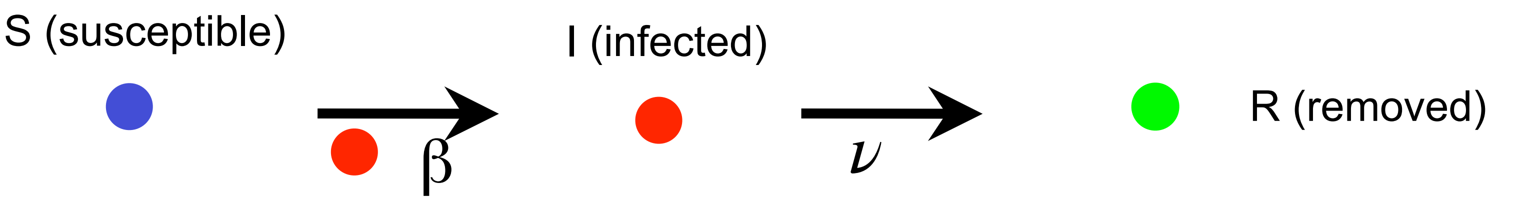
Distributions of maximal span-cores durations

Galimberti, Barrat, Bonchi, Cattuto, Gullo, CIKM 2018, arXiv:1808.09376

Span cores and spreading processes

Procedure - SIR case

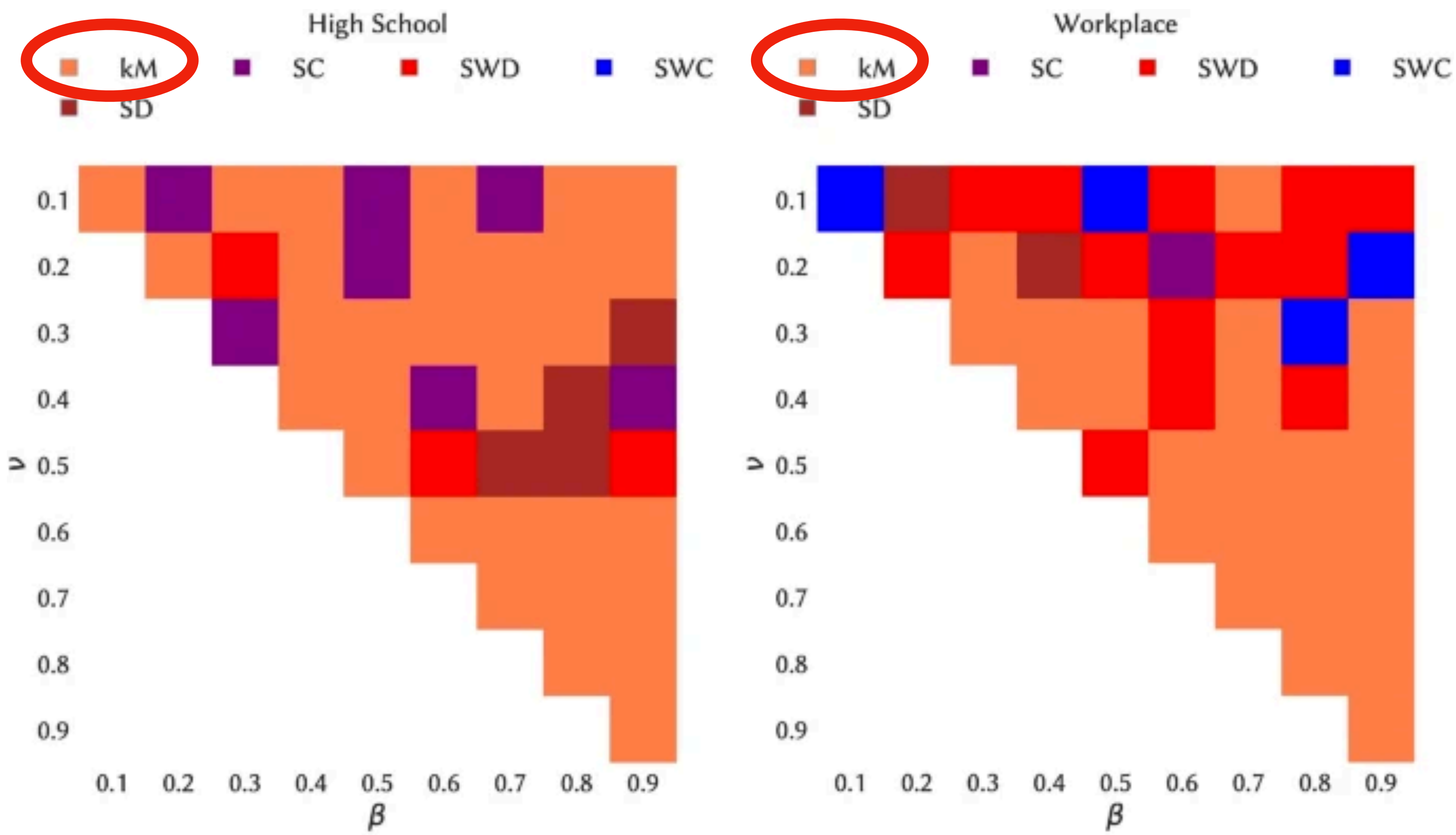
1. consider an SIR process with seed in maximal span cores
2. compare outbreak size with random seed
3. compare with other seeding strategies



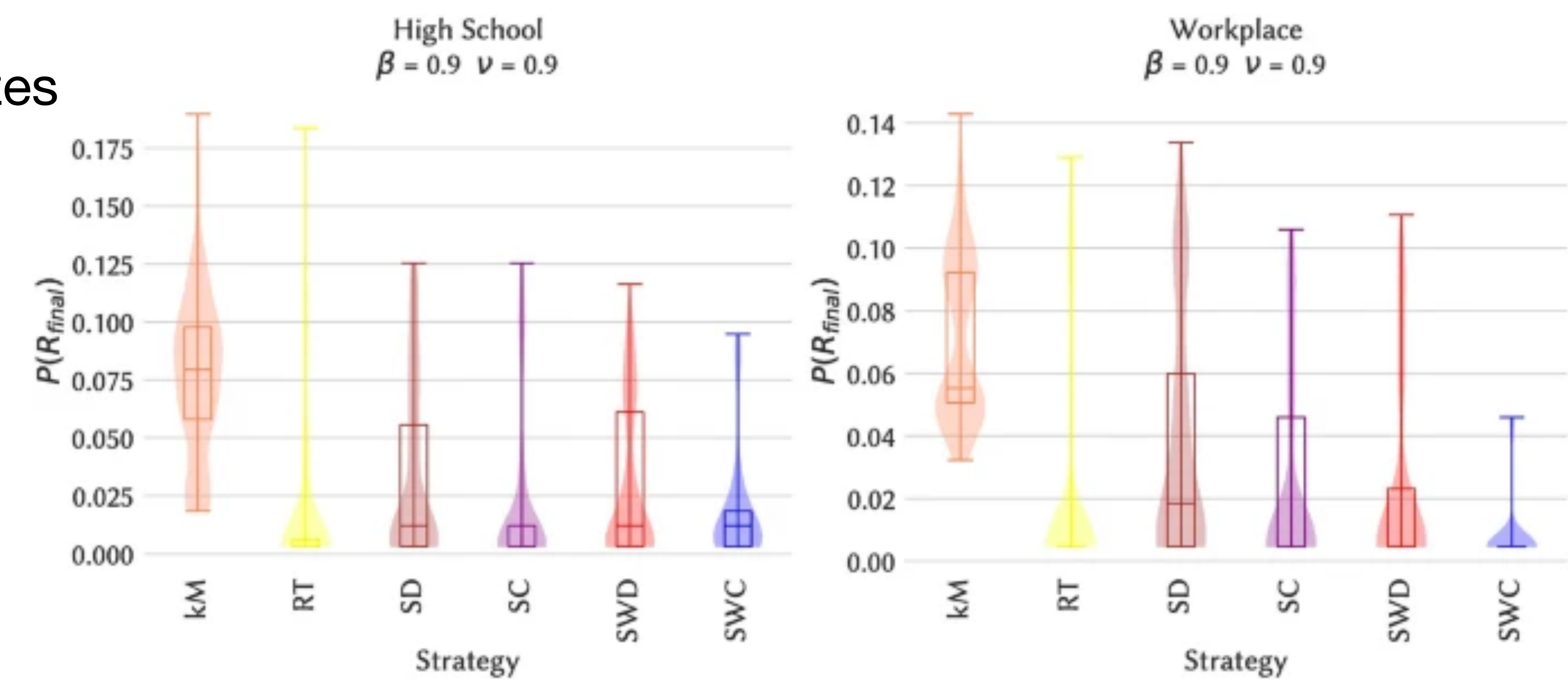
Span cores and spreading processes

Best seeding strategy

kM: cores with largest order
(i.e., most cohesive structures)



Distributions of outbreak sizes



Span-cores

- Generalization of static core decomposition
- Effective algorithms to find (maximal) span-cores
- Uncovers strongly connected structures together with their duration/span
- Uncovers structures not trivially related to activity
- Broad distributions of durations
- Detection of anomalies in data
- Relevance in spreading processes

➡Need to take span-core structures into account in temporal network analysis and modelling

>Finding structures: rich club

nature physics

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Article | Published: 13 June 2022

The temporal rich club phenomenon

Nicola Pedreschi, Demian Battaglia & Alain Barrat

Nature Physics (2022) | Cite this article

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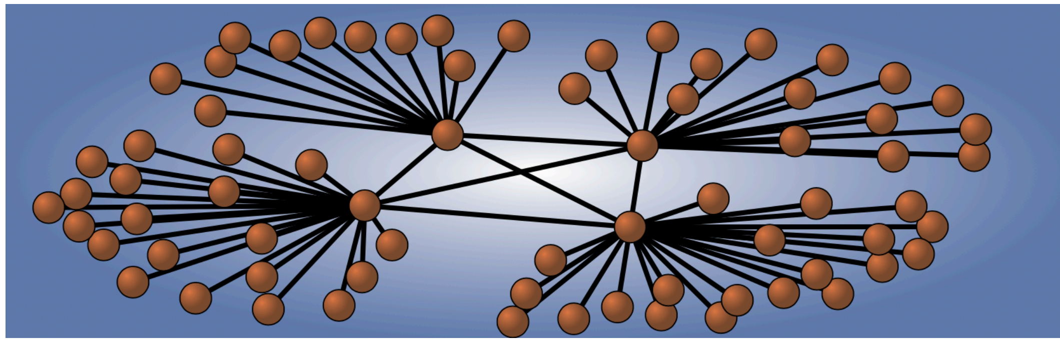
Reminder: rich club for static networks

Are "rich" nodes (large degree) more inter connected (than chance), i.e., forming a "***rich club***"?



$S_{>k}$ = set of nodes with degree $> k$; $N_{>k} \equiv |S_{>k}|$; $E_{>k} \equiv$ # edges in $S_{>k}$

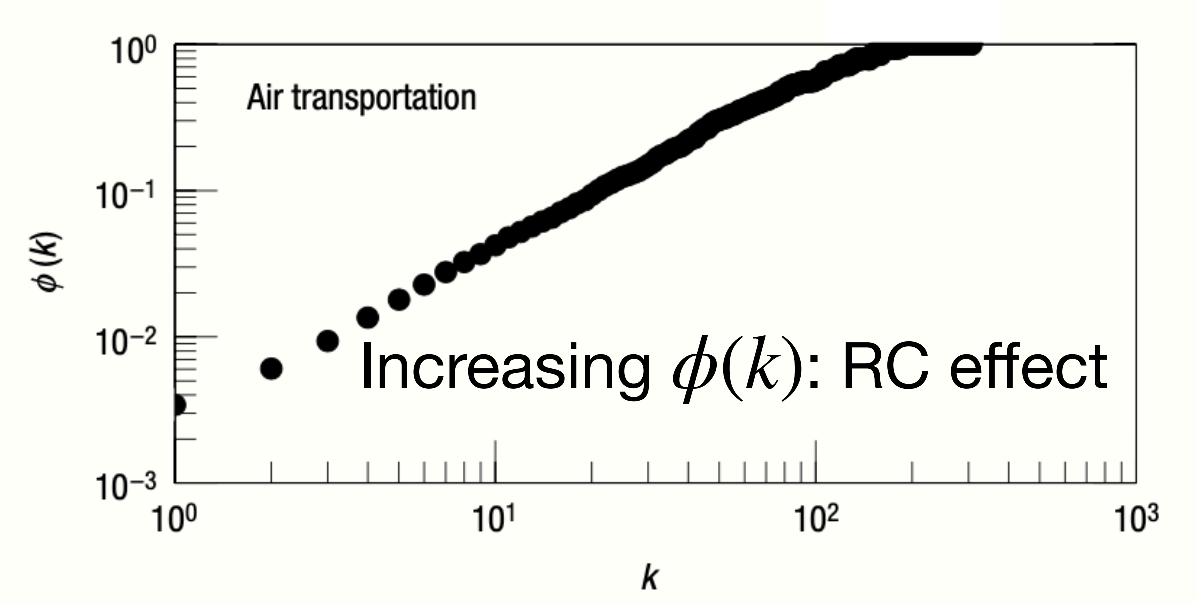
Rich Club coefficient:

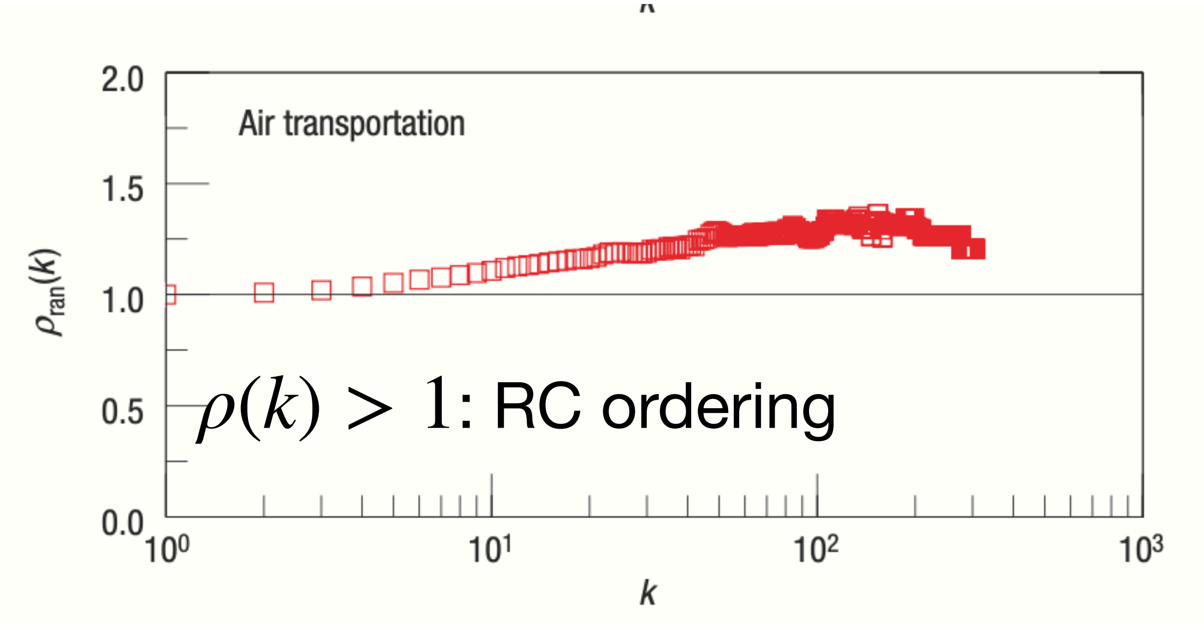
$$\phi(k) \equiv \frac{2E_{>k}}{N_{>k}(N_{>k}-1)}$$

comparison with null model →

Rich Club ordering:

$$\rho(k) \equiv \frac{\phi(k)}{\phi_{rand}(k)}$$





Zhou & Mondragon 2004, Colizza et al. 2006

Case of temporal networks

Does a rich club effect/ordering in a static aggregated network correspond to

- connections at unrelated, possibly casual times?

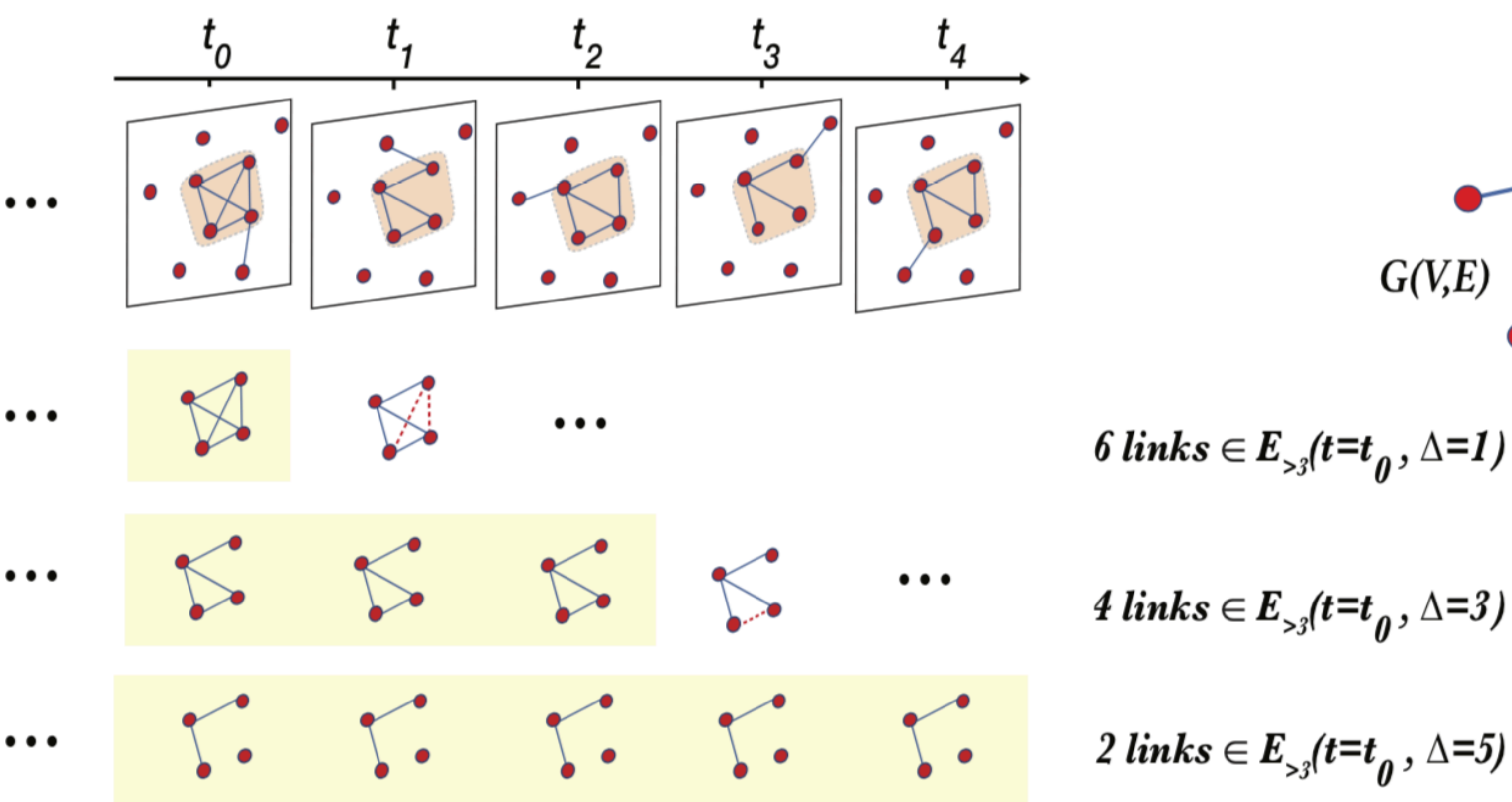
or to

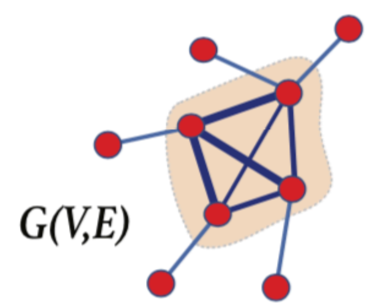
- actual simultaneous connections between the high degree nodes?
    - when?
    - how cohesive?
    - how stable?

Case of temporal networks

Are “rich” nodes (large degree) more inter connected ***in a simultaneous and stable way***, i.e., forming a “***temporal rich club***”?

$S_{>k}$ = set of nodes with degree $> k$ in the aggregated network; $N_{>k} \equiv |S_{>k}|$
$E_{>k}(t, \Delta) \equiv$ # edges in $S_{>k}$ that remain stable on $[t, t+\Delta[$





Cohesion $\epsilon_{>k}(t, \Delta)$: density of links among $S_{>k}$ **stable** during $[t, t+\Delta[$

Temporal rich club coefficient = maximal cohesion

$$M(k, \Delta) = \max_t \epsilon_{>k}(t, \Delta)$$

$M_{rand}(k, \Delta)$:
for randomised data with same
- activity timeline
- aggregated network

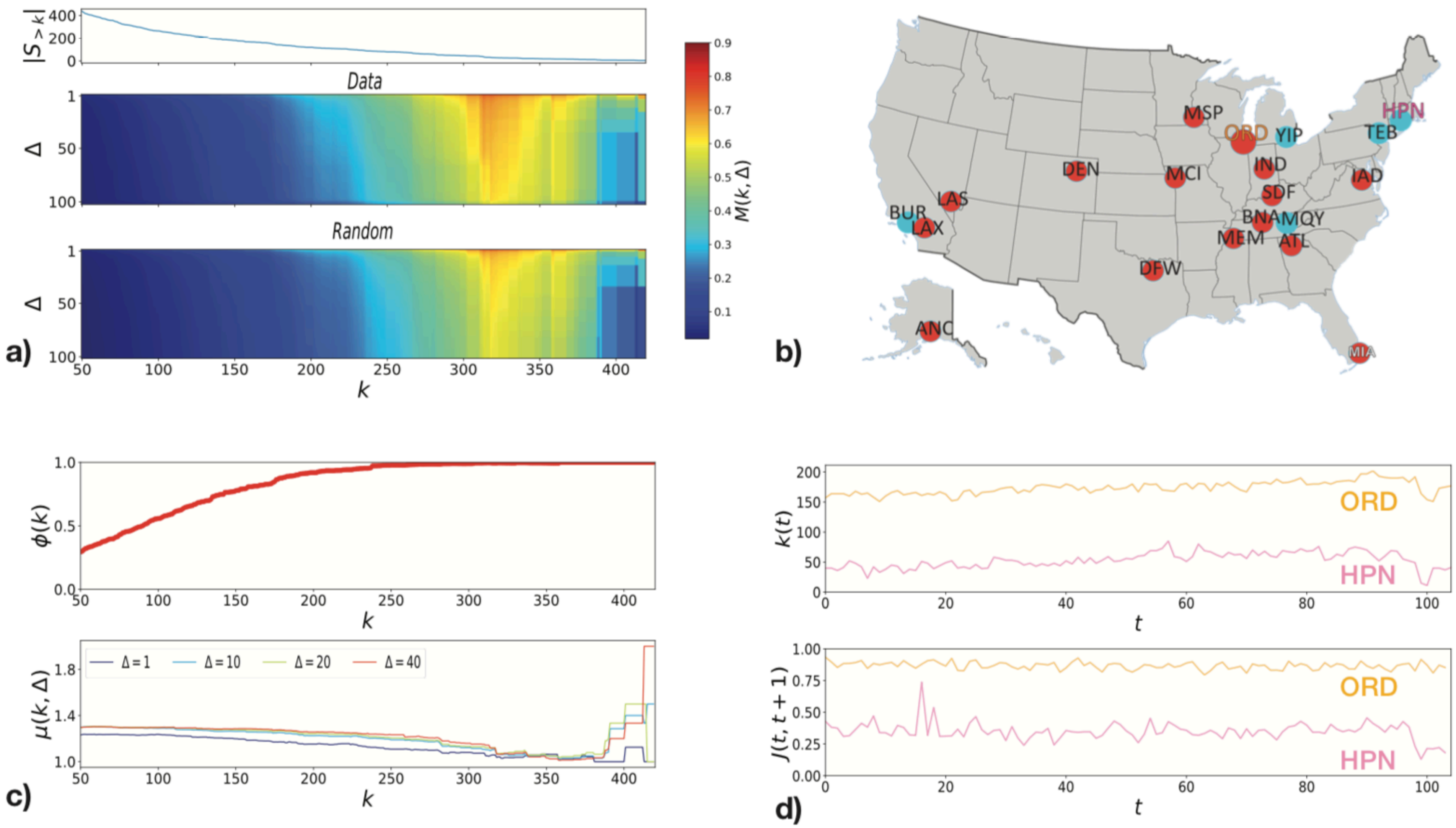
Example 1: US air transportation network (monthly resolution, 2012-2020)

Temporal rich club coefficient = maximal cohesion

$$M(k, \Delta) = \max_t \epsilon_{>k}(t, \Delta)$$

$$\mu(k, \Delta) = M(k, \Delta)/M_{ran}(k, \Delta)$$

Comparison with null model conserving
- activity timeline
- aggregated static structure



$\mu > 1$: more stable cohesion than expected by chance

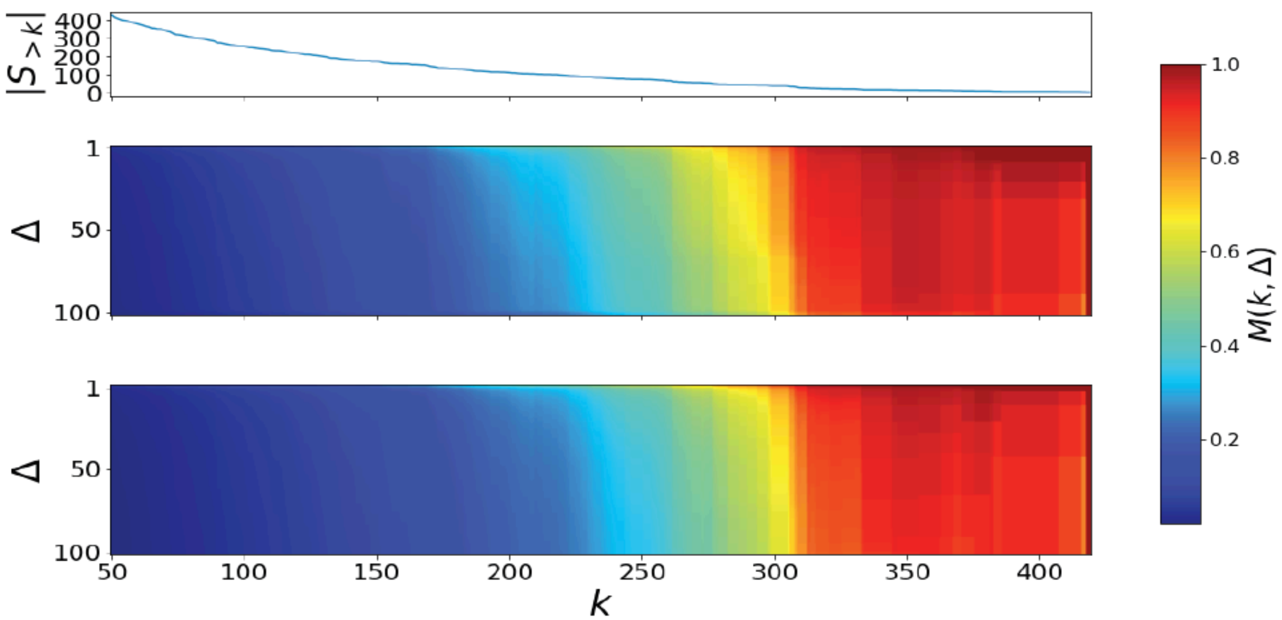
Hubs: stable neighbourhoods
Relievers: fluctuating neighbourhoods
Stable degrees for both

M first increases with k but decreases at very large k:

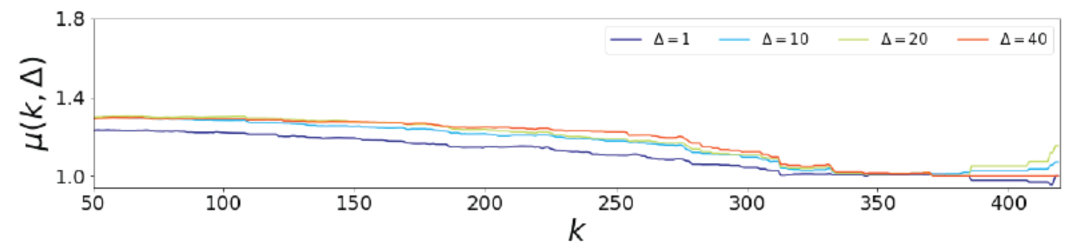
Hubs vs
reliever airports

Very large k: mainly relievers, hence less stable structures

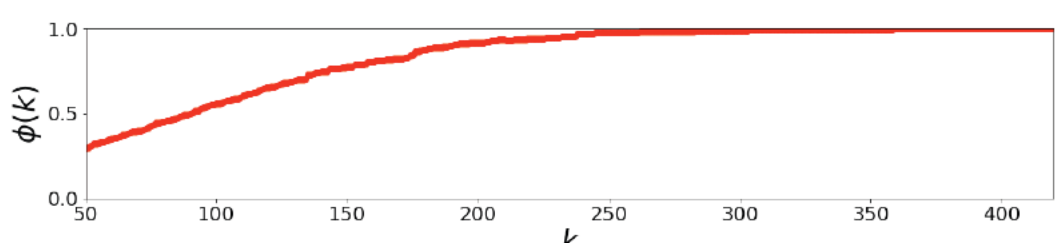
Example 1: US air transportation network (monthly resolution, 2012-2020) with hubs and reliever airports merged



Temporal RC effect + ordering



NB: similar static RC as original data

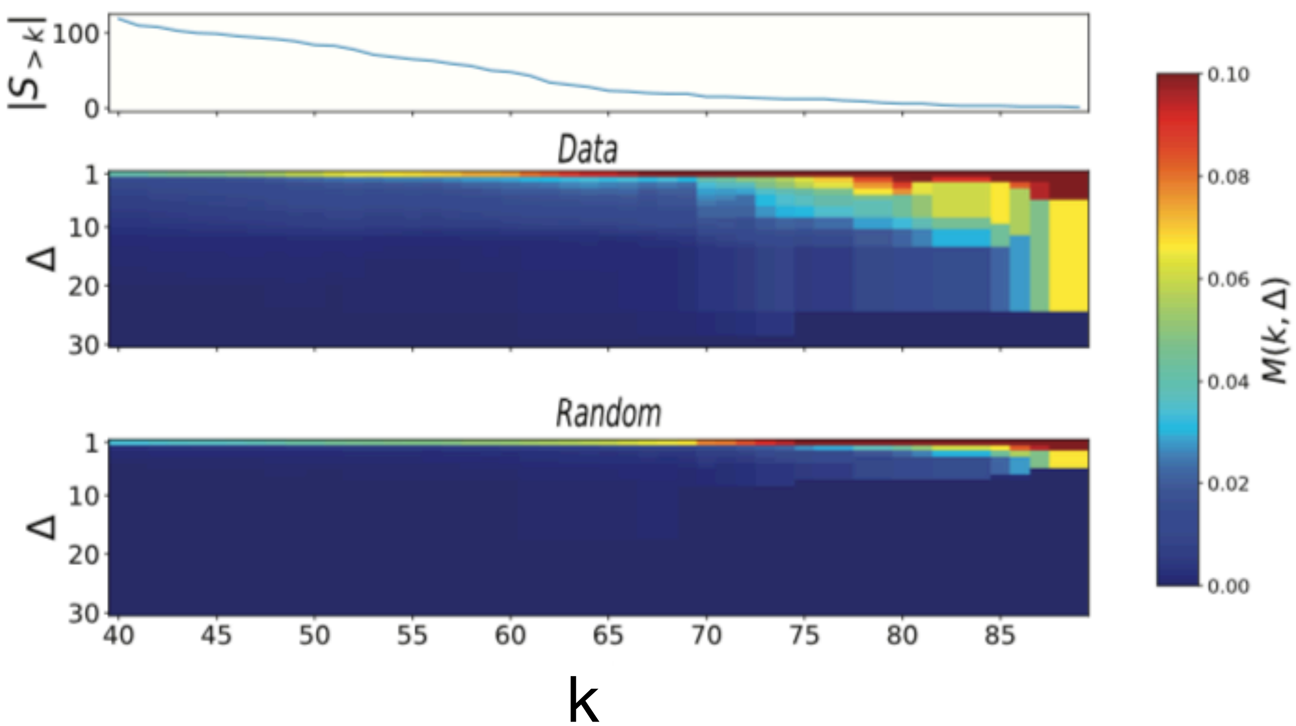


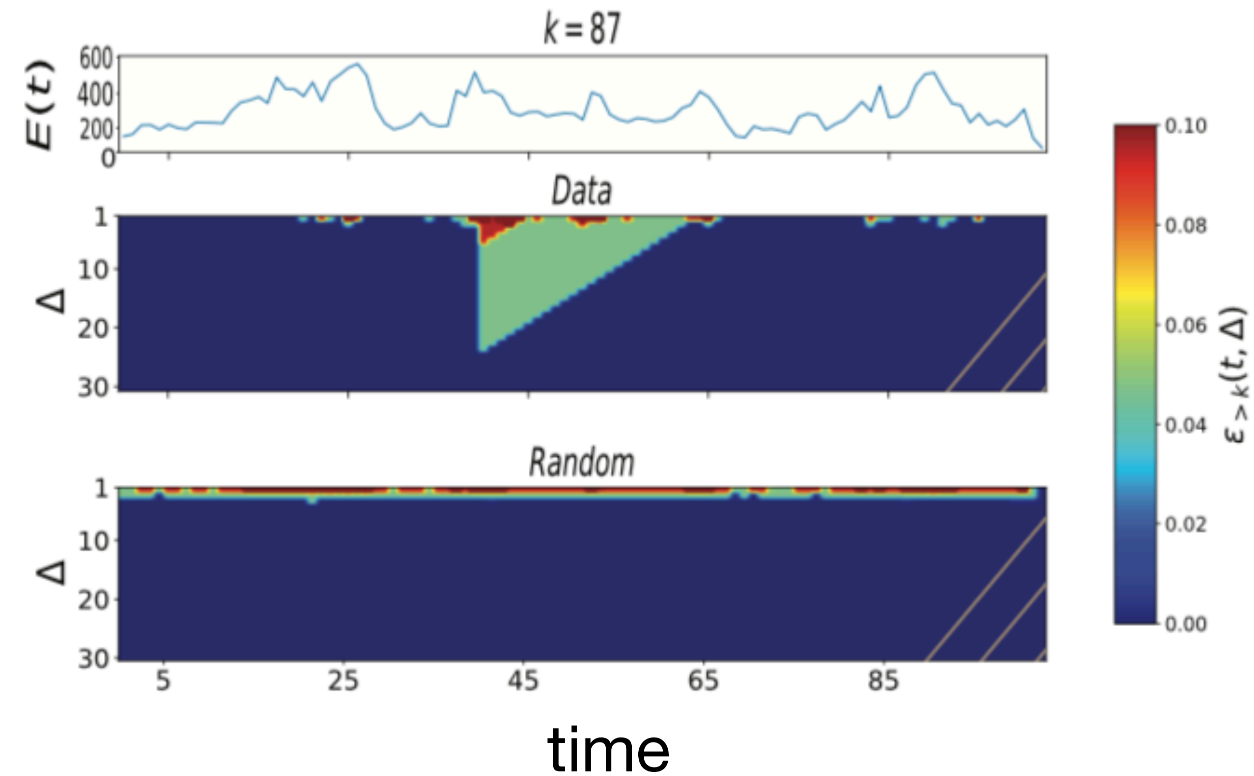
SocioPatterns

Example 2: primary school contact network

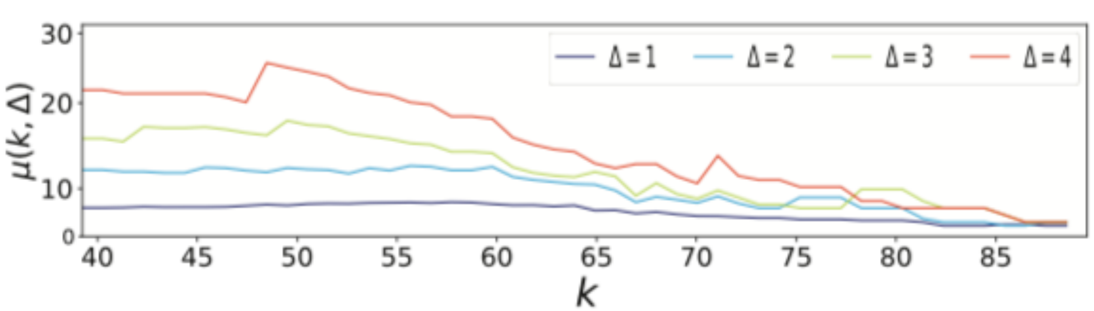
Temporal rich club coefficient = maximal cohesion

$$M(k, \Delta) = \max_t \epsilon_{>k}(t, \Delta)$$

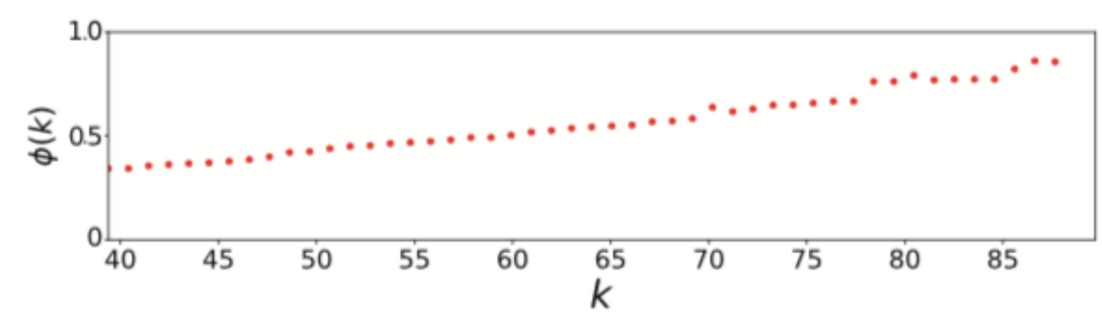




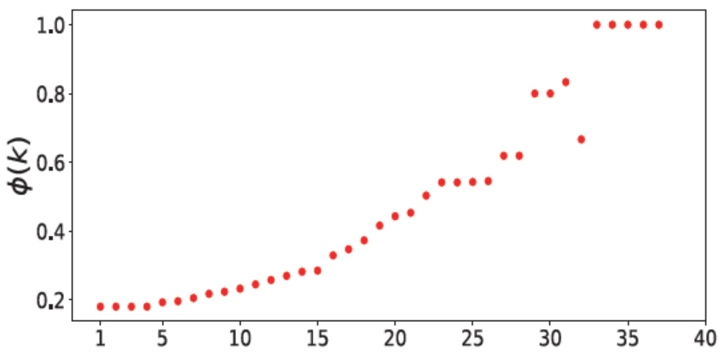
Temporal RC effect + ordering



Much smaller densities than in static network

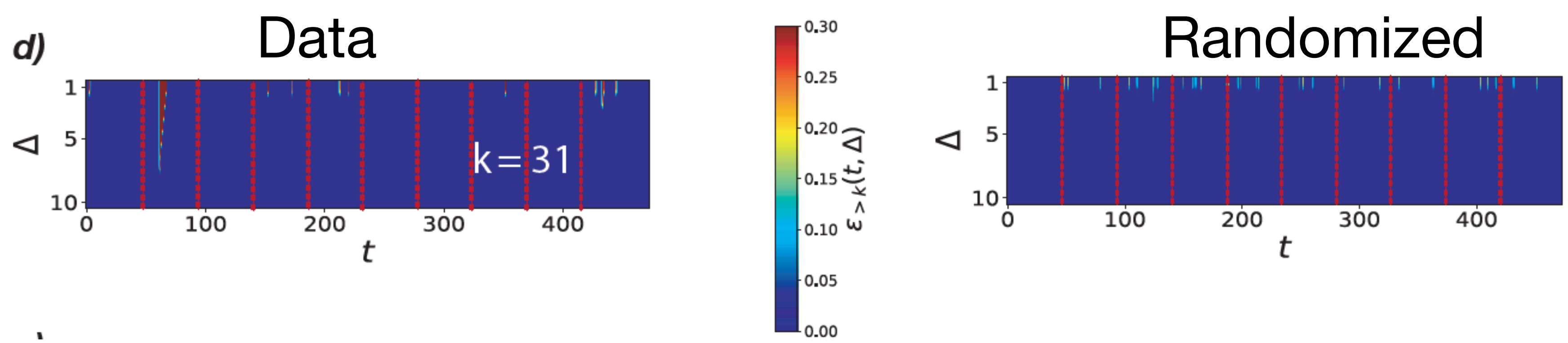


Example 3: workplace contact network

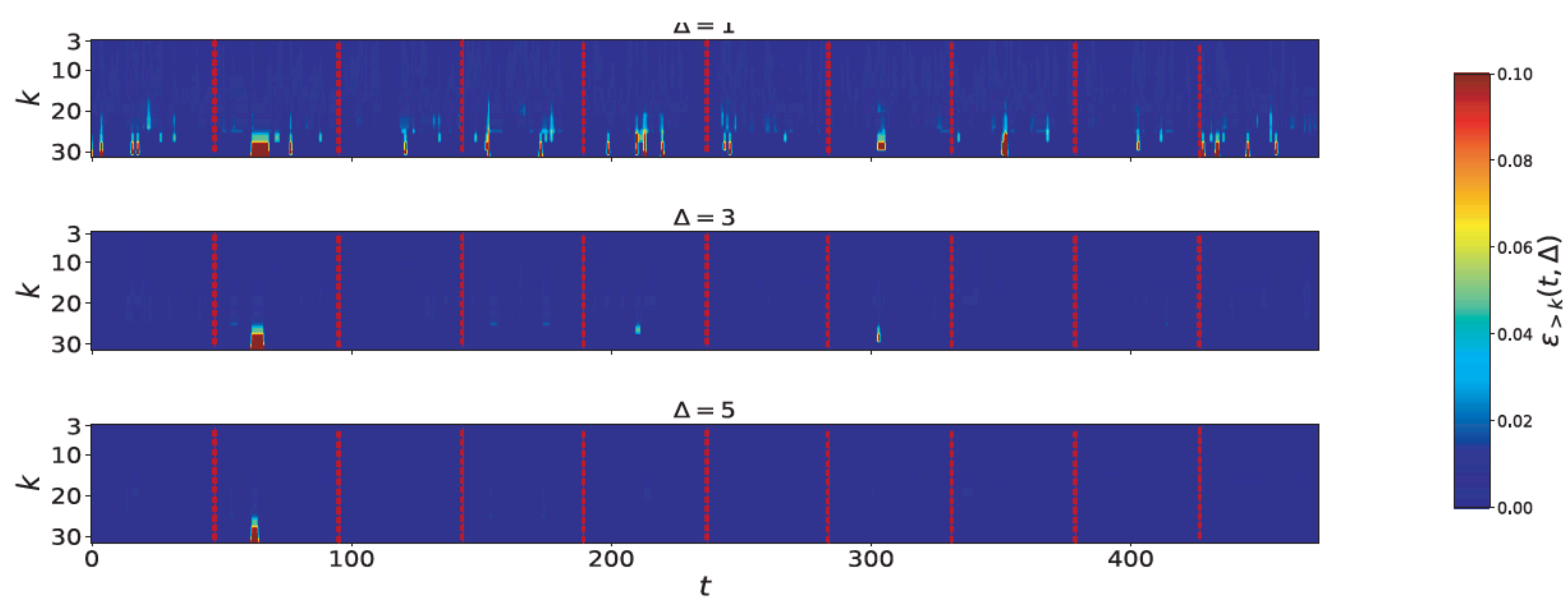


Static rich club effect

No temporal rich club

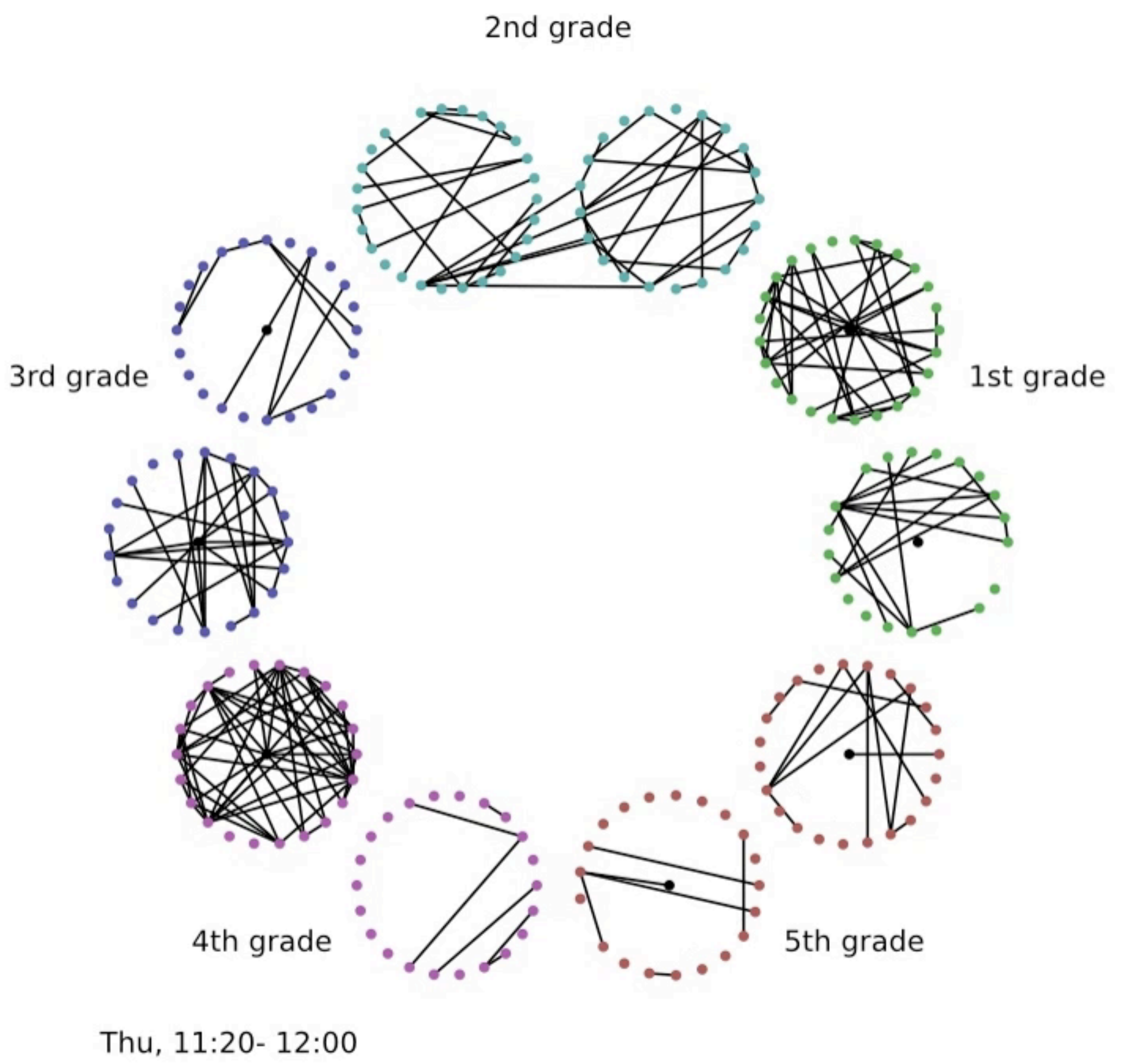


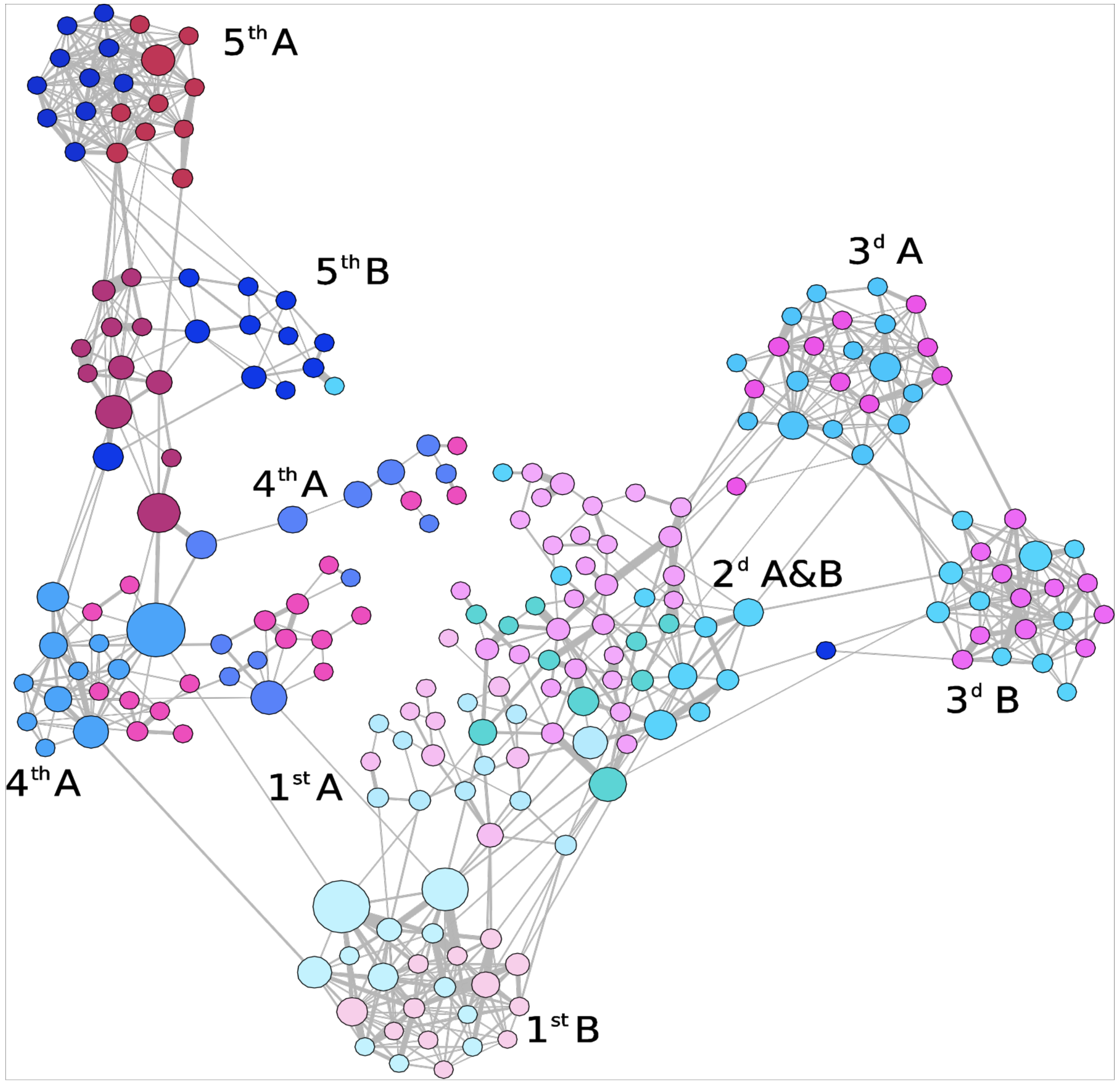
Data



>Timescales and states

Example: contacts in a primary school,





J. Stehlé et al. PLoS ONE 6(8):e23176 (2011)

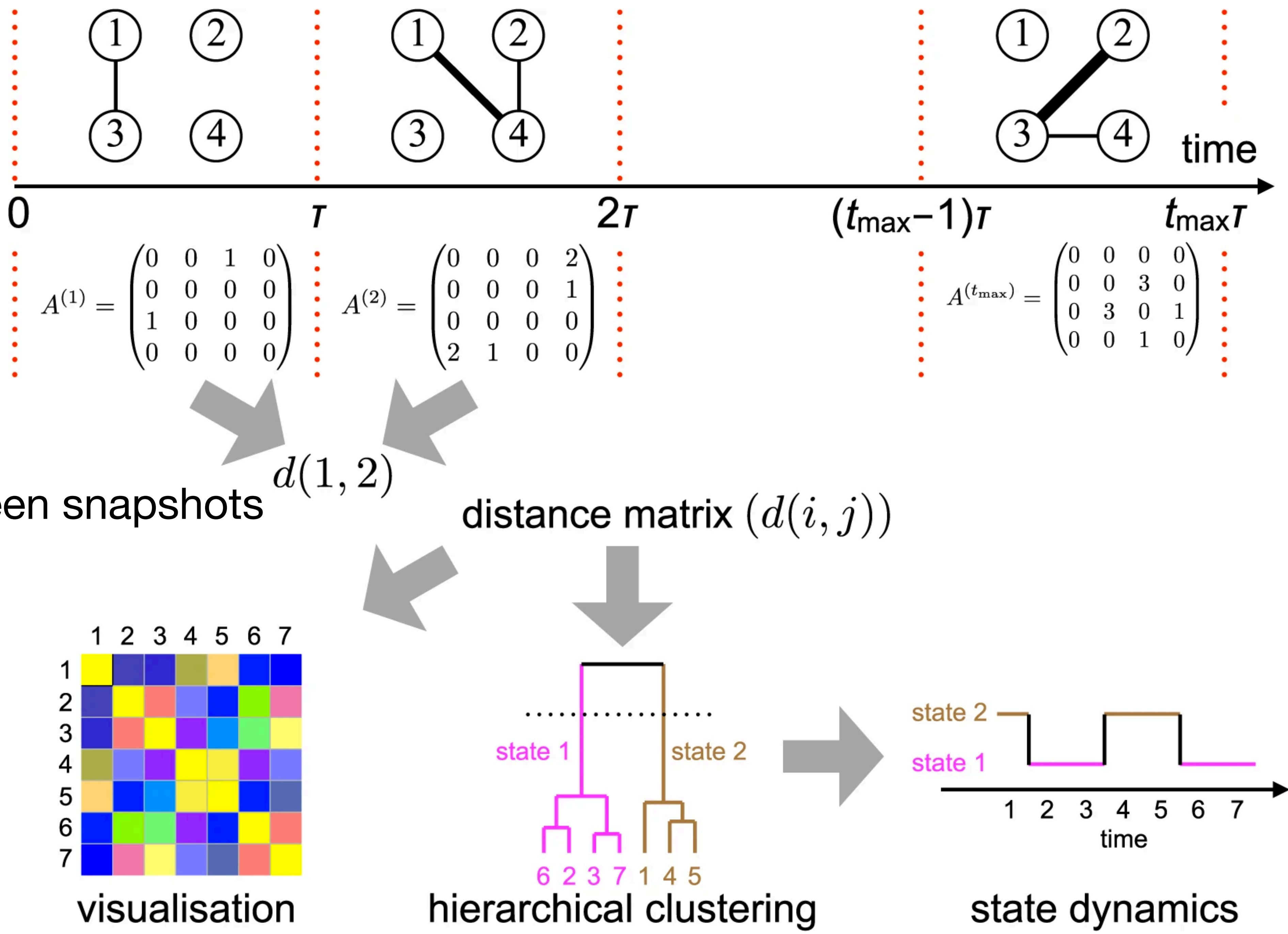
SCIENTIFIC REPORTS

Article | Open Access | Published: 28 January 2019

Detecting sequences of system states in temporal networks

Naoki Masuda & Petter Holme

Scientific Reports **9**, Article number: 795 (2019)



1. matrix of distance between snapshots
2. hierarchical clustering

➡ states

NB:
various possible distances
various clustering algorithms

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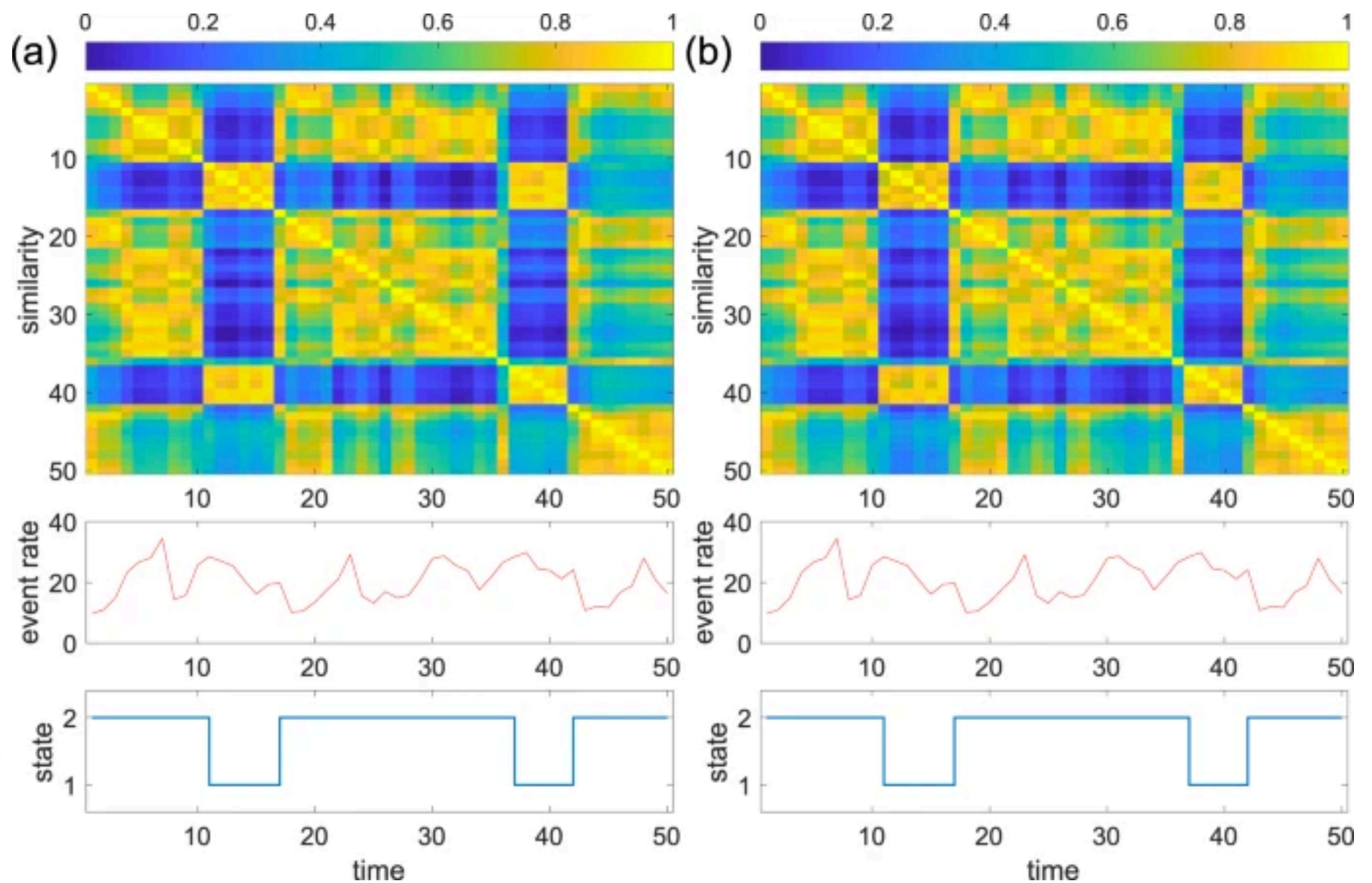
Article | Open Access | Published: 28 January 2019

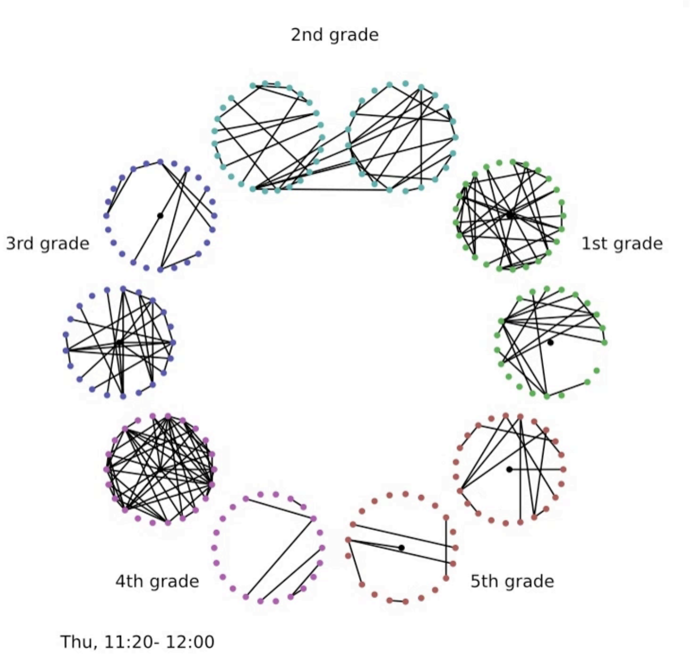
Detecting sequences of system states in temporal networks

Naoki Masuda & Petter Holme

Scientific Reports **9**, Article number: 795 (2019) | Cite this article

Matrices of similarities between snapshots at times t,t'





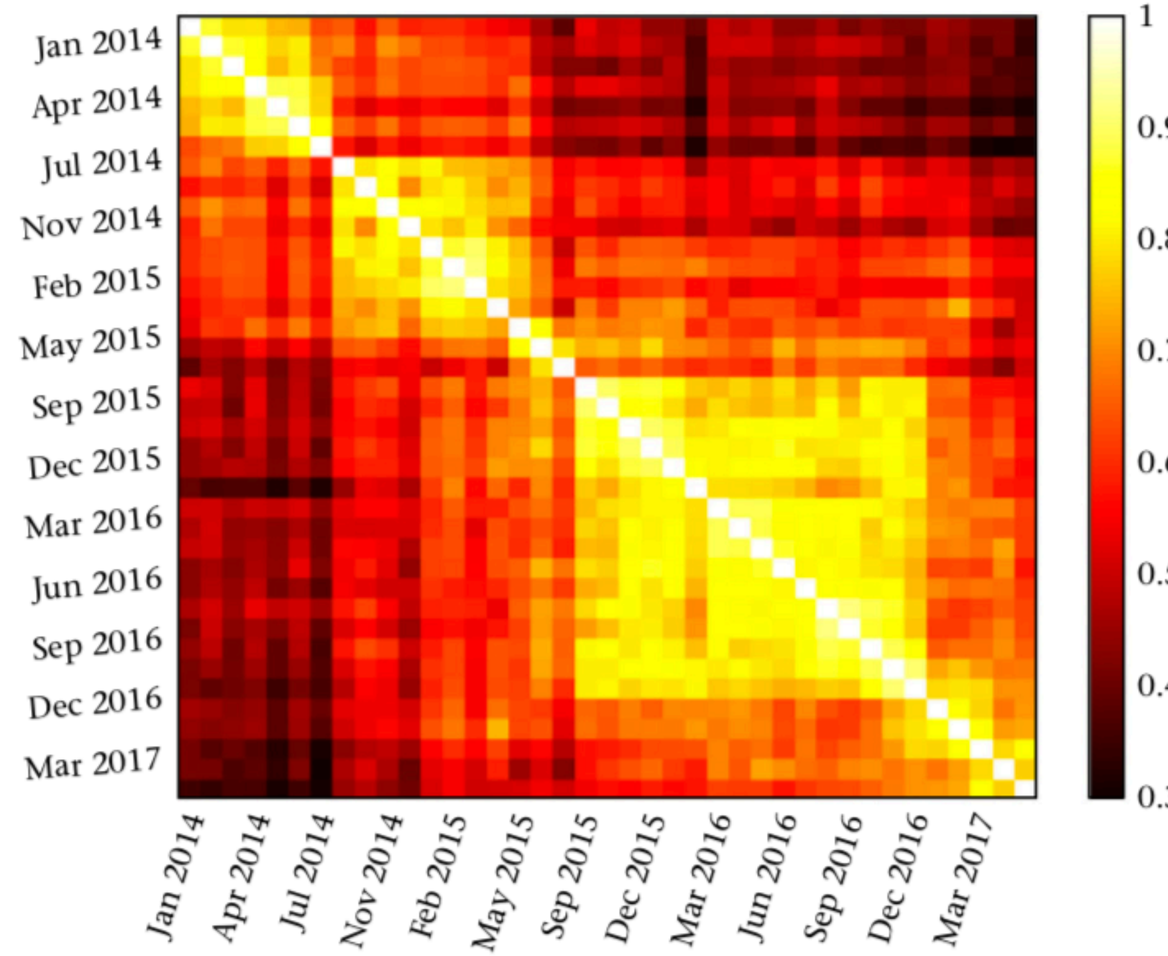
Face-to-face contacts in a primary school

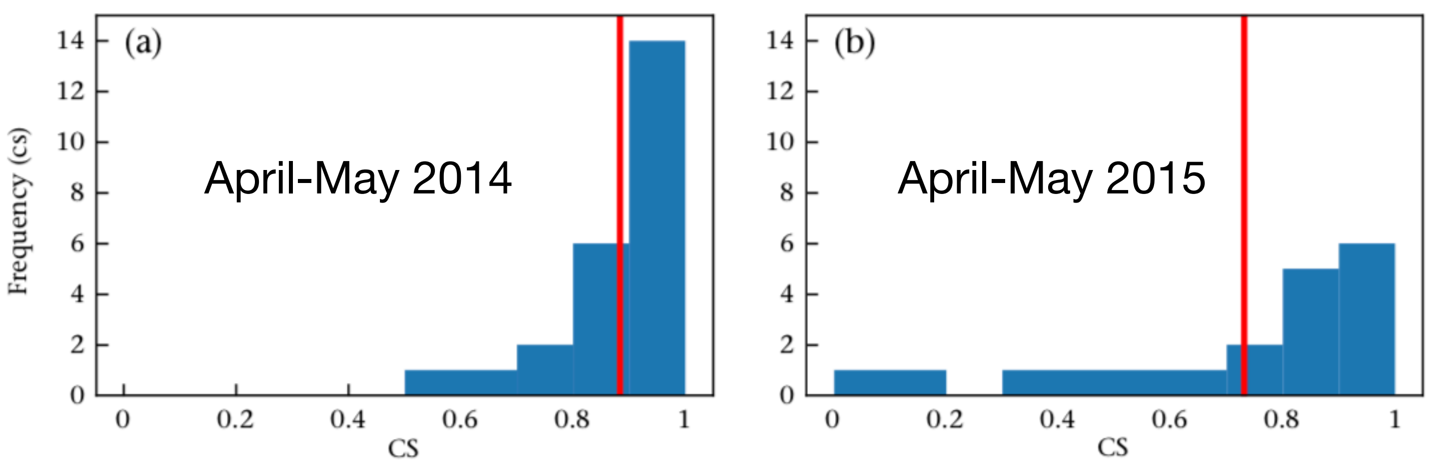
Animal Behaviour
Volume 157, November 2019, Pages 239-254

Detecting social (in)stability in primates from their temporal co-presence network

Valeria Gelardi [a], Joël Fagot [b], Alain Barrat [a, c], Nicolas Claidière [b]

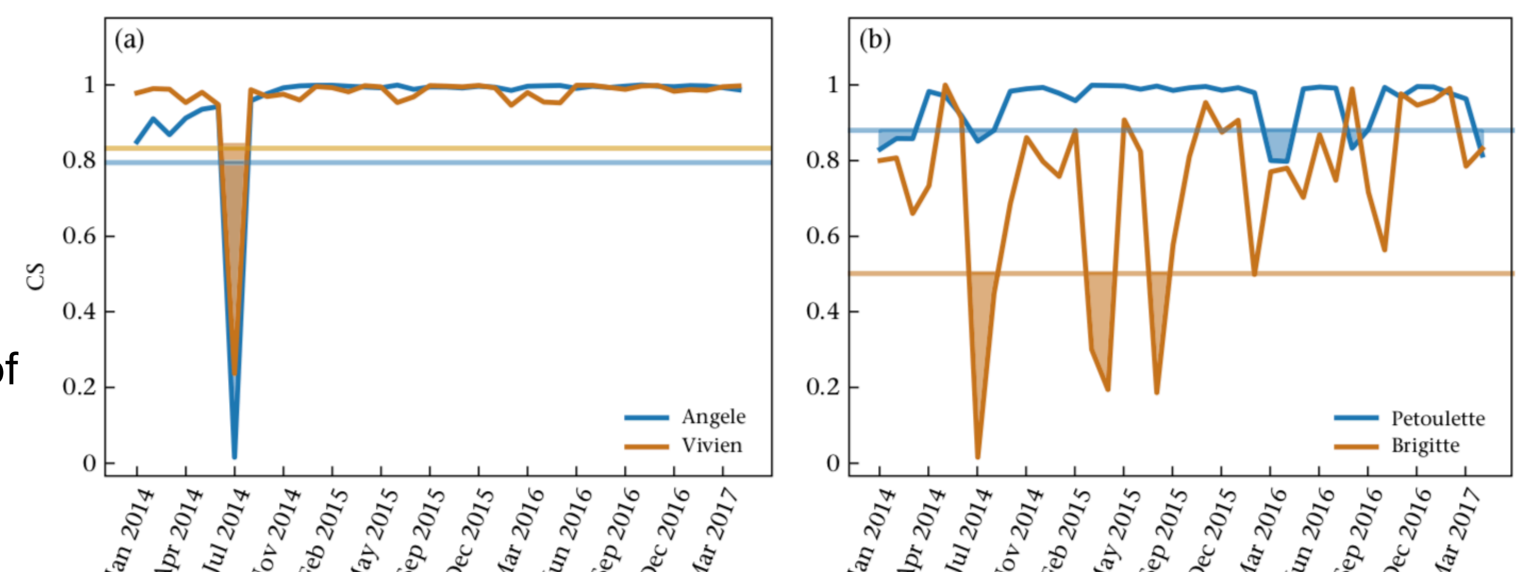
Average cosine similarities between monthly networks





Distributions of similarities of ego-networks (analysis beyond the average):

Comparison between successive ego networks of an individual, CS(t,t+1):

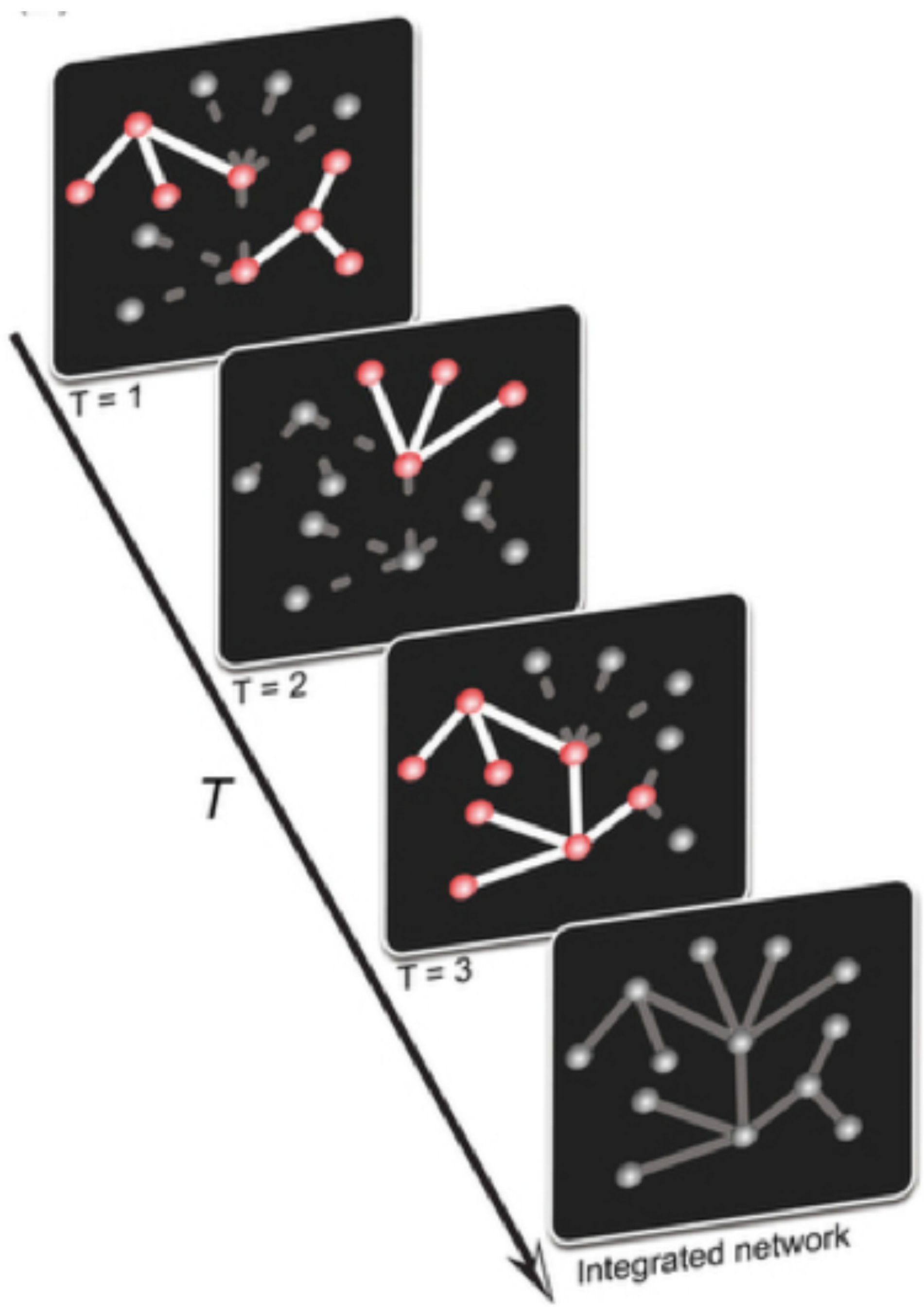


>Models of temporal networks

> activity-driven model

Activity-driven network

Model: N nodes, each with an "activity" a, taken from a distribution F(a)



At each time step:

- node i active with probability a(i)
- each active node generate m links to other randomly chosen nodes
- iterate with no memory

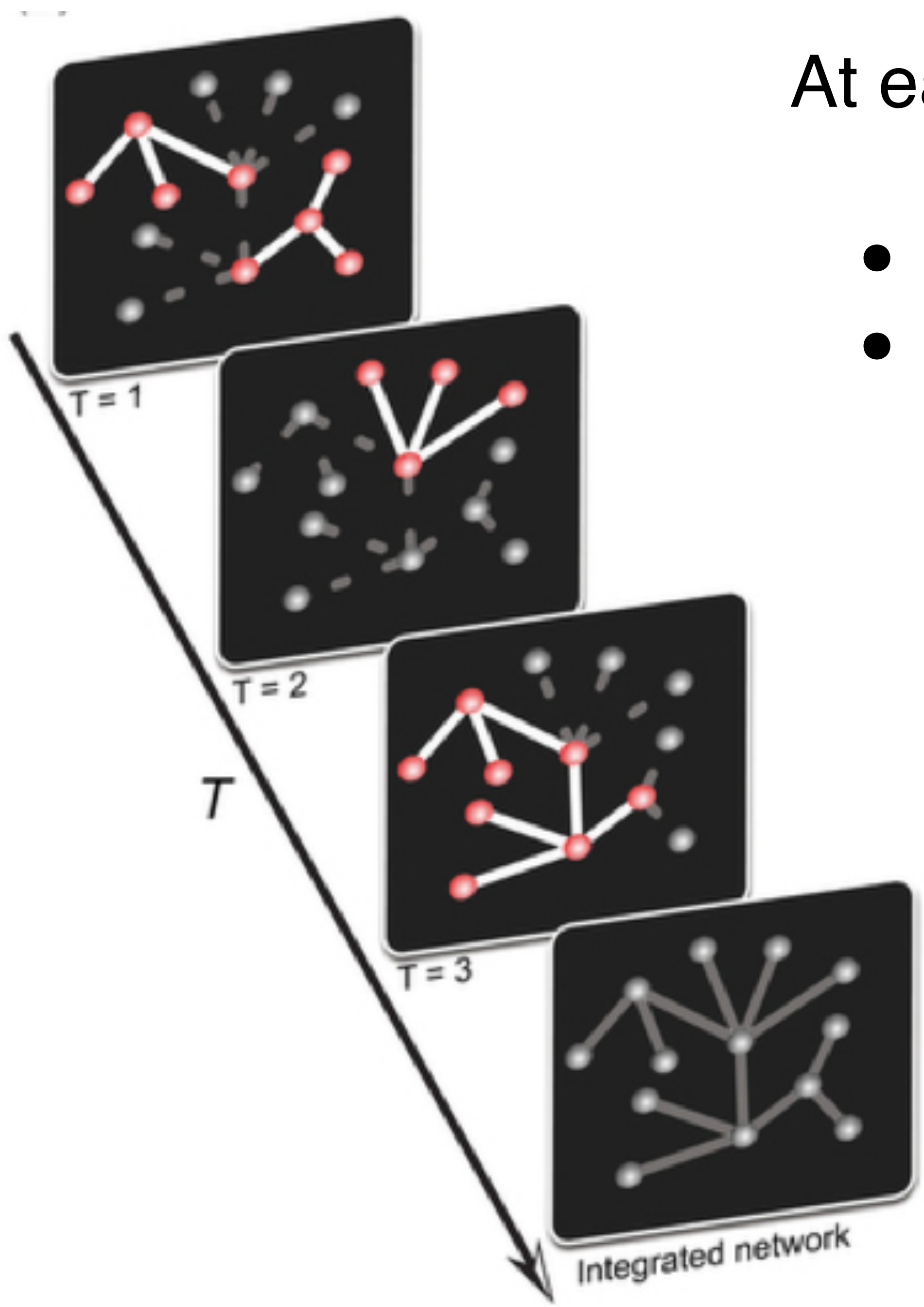
Aggregate degree distribution ~ F

No memory, no correlations…
"Toy" model allowing for analytical computations

Perra et al., Sci. Rep. (2012)

Activity-driven network **with memory**

Model: N nodes, each with an “activity” a, taken from a distribution F(a)



At each time step:

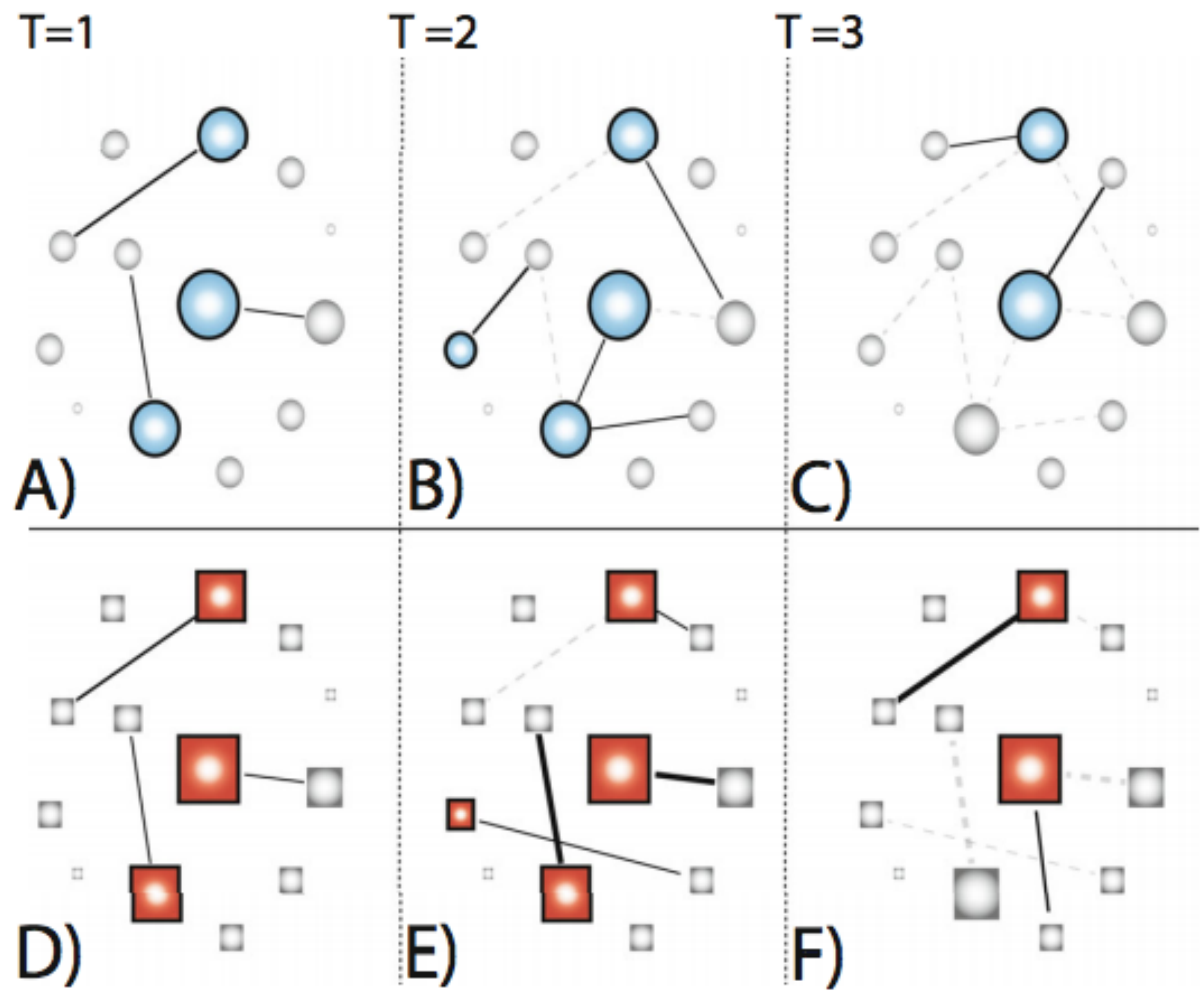
- each node i becomes active with probability a(i)
- each active node generate m links to other nodes; if it has already been in contact with n distinct nodes, the new link is
  - towards a new node with proba p(n)=c/(c+n)
  - towards an already contacted node with proba 1-p(n)

(reinforcement mechanism seen in data)

Karsai et al., Sci. Rep. (2014)

Memory-less

With Memory



Sun et al., EPJB (2015)

>Generative models from data

>Surrogate temporal networks from known empirical statistics

- Generate static underlying structure
- Generate timelines of events
  - known $P(\Delta t)$: generate successive events with inter-event times taken from $P(\Delta t)$
  - known $P(\Delta t)$ and $P(n)$: assign a number of events to each link from $P(n)$, and inter-event times from $P(\Delta t)$
  - known $P(\Delta t)$, $P(n)$, $P(\tau)$
  - known intervals of activity or overall activity timeline
  - …

NB: no temporal correlations, some can be introduced (through correlated intervals of activities)

L. Gauvin et al., Sci Rep 3:3099 (2013)
M. Génois et al., Nat Comm 6:8860 (2015)

>Null Models of temporal networks

What are null models?

- ensemble of instances of randomly built systems
- that preserve some properties of the studied systems

Aim:

• understand which properties of the studied system are simply random, and which ones denote an underlying mechanism or organizational principle

• compare measures with the known values of a random case

Random times

for each link event: pick time at random

keeps:
- link structure
- number of events per link
- corresponding static correlations

destroys:
- global time ordering
- activity timeline
- burstiness
- all temporal correlations

Time shuffling

shuffle times of events

ID1	ID2	time
1	4	2
2	3	8
1	5	12
3	4	15
…..		

ID1	ID2	time
1	4	8
2	3	15
1	5	2
3	4	12
…..		

keeps:

- link structure
- number of events per link
- corresponding static correlations
- global time ordering and activity timeline

destroys:

- interevent times
- all temporal correlations

Interval shuffling

for each link: randomize sequence of inter-event durations

for each link: randomize sequence of contacts

keeps:

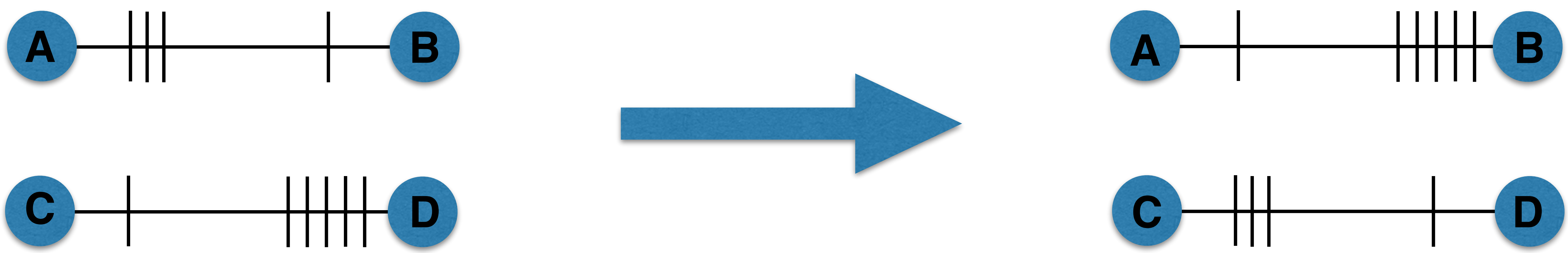
- link structure
- number of events per link
- corresponding static correlations
- link burstiness

destroys:

- all temporal correlations
- activity timeline

Link-sequence shuffling

Randomly exchange sequence of events of different links



keeps:

- link structure
- distribution of link weights
- global activity timeline
- link burstiness

destroys:

- number of events per link & corresponding correlations
- correlations between structure and activity
- all temporal correlations

NB: weight-conserving link-sequence shuffling

Home → SIAM Review → Vol. 64, Iss. 4 (2022) → 10.1137/19M1242252

Randomized Reference Models for Temporal Networks

Authors: Laetitia Gauvin, Mathieu Génois, Márton Karsai, Mikko Kivelä, Taro Takaguchi, Eugenio Valdano, and Christian L. Vestergaard | AUTHORS INFO & AFFILIATIONS

https://doi.org/10.1137/19M1242252

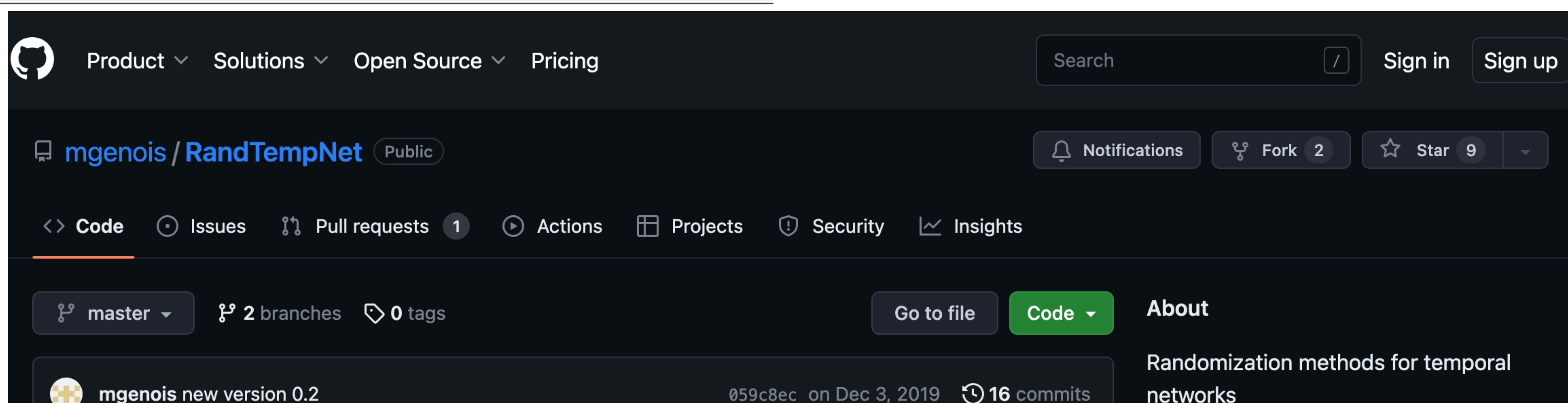
[Submitted on 11 Jun 2018 (v1), last revised 15 Dec 2022 (this version, v4)]

Randomized reference models for temporal networks

Laetitia Gauvin, Mathieu Génois, Márton Karsai, Mikko Kivelä, Taro Takaguchi, Eugenio Valdano, Christian L. Vestergaard

TABLE IV.2: Effects of MRRMs on features of temporal networks. See Table IV.1 for definitions of features. Colored symbols show to what extent each feature is conserved. Informal definitions are found in the tablenotes (detailed definitions are found in Supplementary Table S1).

Canonical name	Common name	topological G^{stat}	k_i	L	weighted $a_i{}^\dagger$	s_i	$n_{(i,j)}{}^\dagger$	$w_{(i,j)}$	temp. A^t	node $\alpha_i^{m\dagger}$	$\Delta\alpha_i^m$	d_i^t	link $\tau^m_{(i,j)}{}^\dagger$	$\Delta\tau^m_{(i,j)}$	$t^1_{(i,j)}$	$t^w_{(i,j)}$
P[E]	*Instant-event shuffling*	–	–	–	–	μ	–	–	μ	–	–	μ	–	–	–	–
P[$p(\boldsymbol{\tau})$]	*Event shuffling*	–	–	–	–	μ	–	–	μ	–	–	μ	p	–	–	–
Link shufflings (LS):																
P[$p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*LS*	–	μ	x	μ	μ	p	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
P[$\chi_\lambda, p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Connected LS*	χ_λ	μ	x	μ	μ	p	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
P[$\mathbf{k}, p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Degree-constrained LS*	–	x	x	μ	μ	p	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
P[$\mathbf{k}, \chi_\lambda, p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Connected, degree-constr. LS*	χ_λ	x	x	μ	μ	p	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
Timeline shufflings (TS):																
P[$\mathcal{L}, E$]	*TS*	x	x	x	–	μ	–	μ	μ	–	–	μ	–	–	–	–
P[$\mathbf{w}$]	*Weight-constrained TS*	x	x	x	–	x	–	x	μ	–	–	μ	–	–	–	–
P[$\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\Delta\tau}),\mathbf{t}^1$]	*Inter-event shuffling*	x	x	x	x	x	x	x	μ	–	–	μ	$\boldsymbol{\mu}_{\mathcal{L}}$	$\boldsymbol{\pi}_{\mathcal{L}}$	x	x
P[$\mathcal{L},p(\boldsymbol{\tau})$]	*TS*	x	x	x	μ	μ	μ	μ	μ	–	–	μ	p	–	–	–
P[$\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\tau})$]	*Local TS*	x	x	x	x	x	x	x	μ	–	–	μ	$\boldsymbol{\pi}_{\mathcal{L}}$	–	–	–
P[$\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\tau}),\mathbf{t}^1,\mathbf{t}^w$]	*Activity-constrained TS*	x	x	x	x	x	x	x	μ	–	–	μ	$\boldsymbol{\pi}_{\mathcal{L}}$	$\boldsymbol{\mu}_{\mathcal{L}}$	x	x
P[$\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\tau}),\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\Delta\tau})$]	*Interval shuffling*	x	x	x	x	x	x	x	μ	–	–	μ	$\boldsymbol{\pi}_{\mathcal{L}}$	$\boldsymbol{\pi}_{\mathcal{L}}$	–	–
P[$\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\tau}),\boldsymbol{\pi}_{\mathcal{L}}(\boldsymbol{\Delta\tau}),\mathbf{t}^1$]	*Inter-event shuffling*	x	x	x	x	x	x	x	μ	–	–	μ	$\boldsymbol{\pi}_{\mathcal{L}}$	$\boldsymbol{\pi}_{\mathcal{L}}$	x	x
P[$\mathbf{per}(\boldsymbol{\Theta})$]	*Timeline shifting*	x	x	x	x	x	x	x	μ	–	–	μ	x	x	–	–
Sequence shufflings (SeqS):																
P[$p_{\mathcal{T}}(\boldsymbol{\Gamma})$]	*SeqS*	x	x	x	–	x	–	x	p	–	–	$p_{\mathcal{T}}$	–	–	–	–
P[$p_{\mathcal{T}}(\boldsymbol{\Gamma}),\boldsymbol{\chi}_{\mathbb{N}^+}(\mathbf{A})$]	*Activity-constrained SeqS*	x	x	x	–	x	–	x	p, H	–	–	$p_{\mathcal{T}}$	–	–	–	–
Snapshot shufflings (SnapS):																
P[$\mathbf{t}$]	*SnapS*	–	–	–	–	μ	–	–	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	–	–	–	–
P[$\mathbf{d}$]	*Degree-constrained SnapS*	–	–	–	–	μ	–	–	x	x	x	x	–	–	–	–
P[$\mathbf{iso}(\boldsymbol{\Gamma})$]	*Isomorphic SnapS*	–	–	–	–	μ	–	–	x	–	–	$\boldsymbol{\pi}_{\mathcal{T}}$	–	–	–	–
P[$p(\mathbf{t},\boldsymbol{\tau})$]	*SnapS*	–	–	–	–	μ	–	–	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$\boldsymbol{\pi}_{\mathcal{T}}$	–	–	–
Link-timeline intersections:																
P[$\mathcal{L},p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Topology-constrained LS*	x	x	x	μ	μ	p	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
P[$\mathbf{w},p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Weight-constrained LS*	x	x	x	μ	x	p	x	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
P[$\mathbf{n},p_{\mathcal{L}}(\boldsymbol{\Theta})$]	*Weight-constrained LS*	x	x	x	x	μ	x	p	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$p_{\mathcal{L}}$	$p_{\mathcal{L}}$	p	p
Link-snapshot intersections:																
P[$\mathbf{w},\mathbf{t}$]	*Timestamp shuffling*	x	x	x	–	x	–	x	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	–	–	–	–
P[$\mathcal{L},p(\mathbf{t},\boldsymbol{\tau})$]	*Topology-constrained SnapS*	x	x	x	μ	μ	μ	μ	x	–	–	$\boldsymbol{\mu}_{\mathcal{T}}$	$\boldsymbol{\pi}_{\mathcal{T}}$	–	–	–



Temporal networks: Still very open field!

Data collection and analysis
Structures in data
Incompleteness of data
Timescales
Models
Processes on temporal networks
…

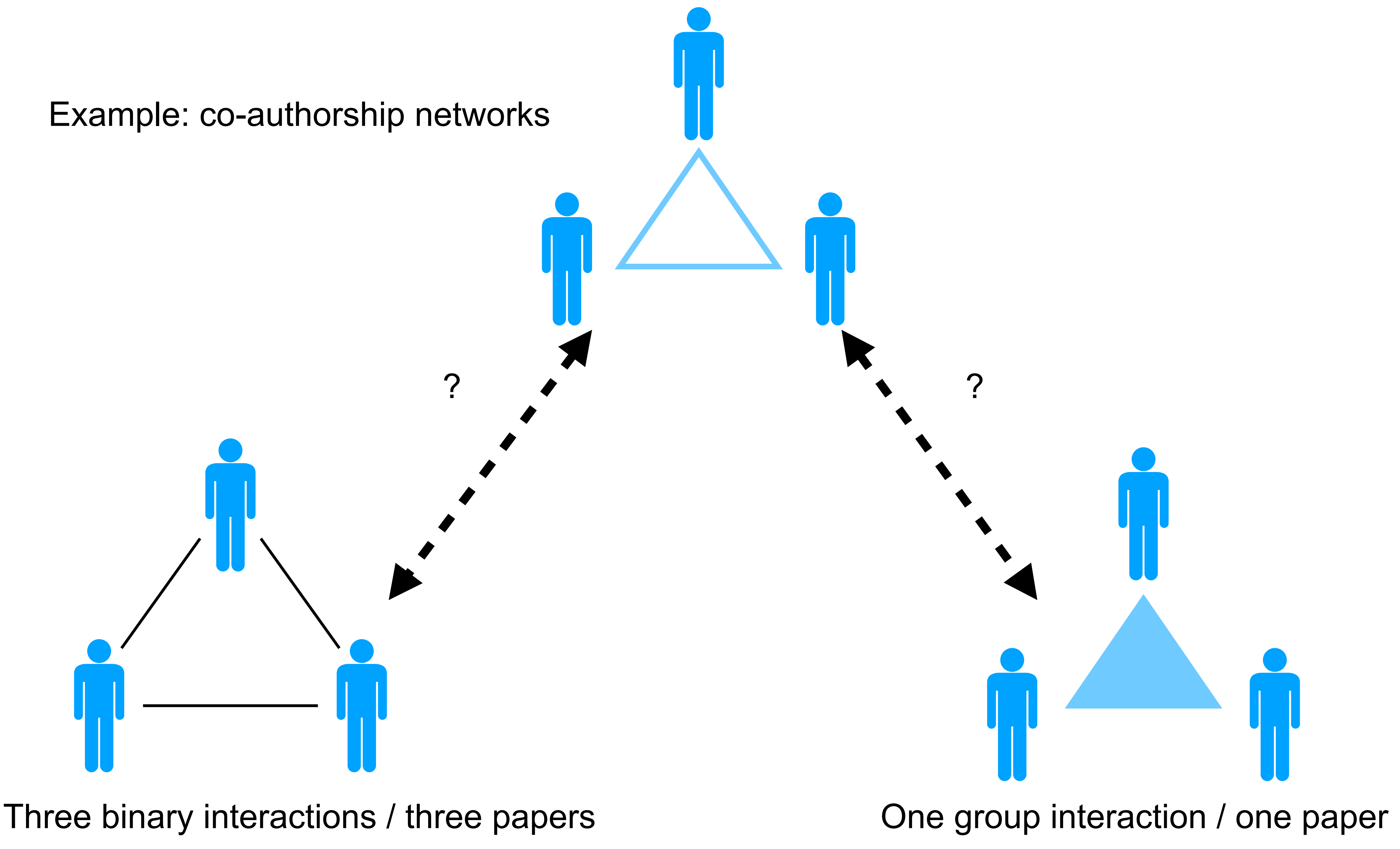
Datasets:

http://www.sociopatterns.org/datasets
http://networkrepository.com/
https://snap.stanford.edu/data/index.html
https://networks.skewed.de/

Further: beyond networks!

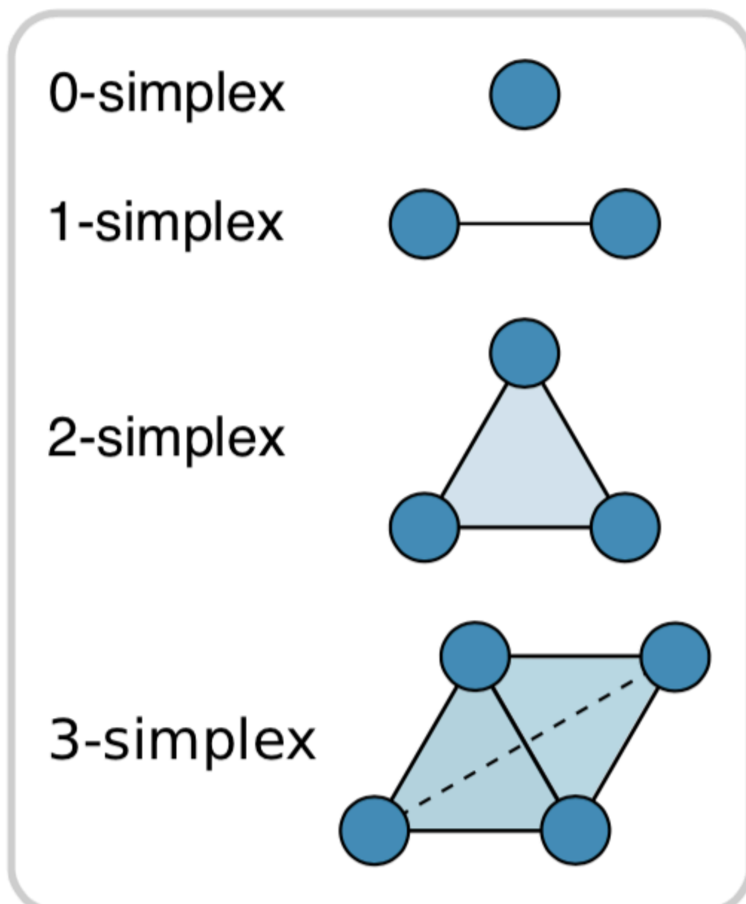
Beyond networks:
Higher-order interactions

Example: co-authorship networks

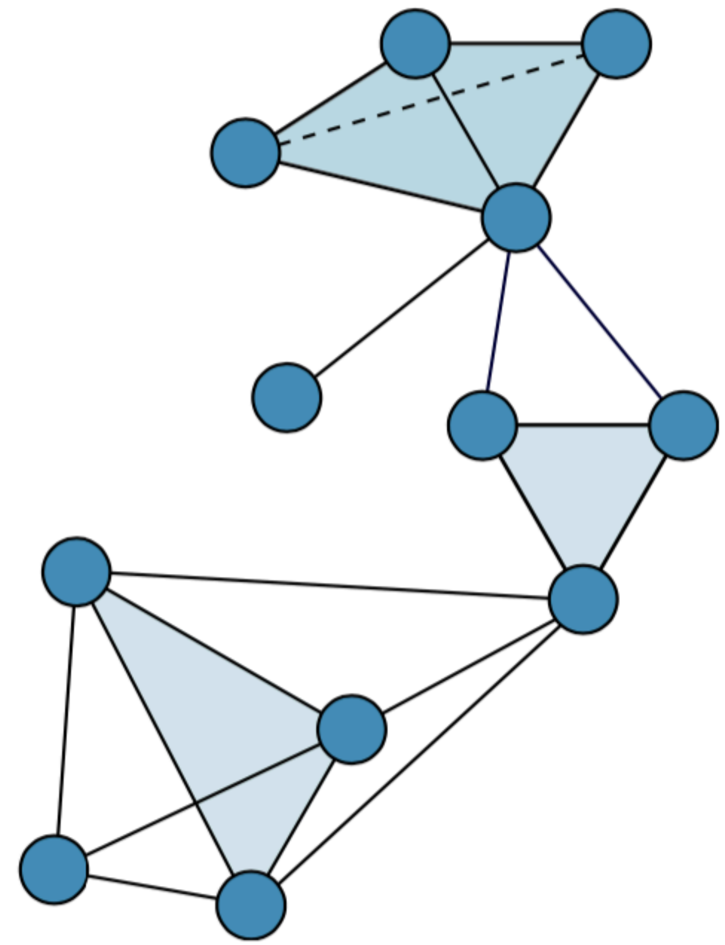


Beyond networks:
Higher-order interactions, hypergraphs

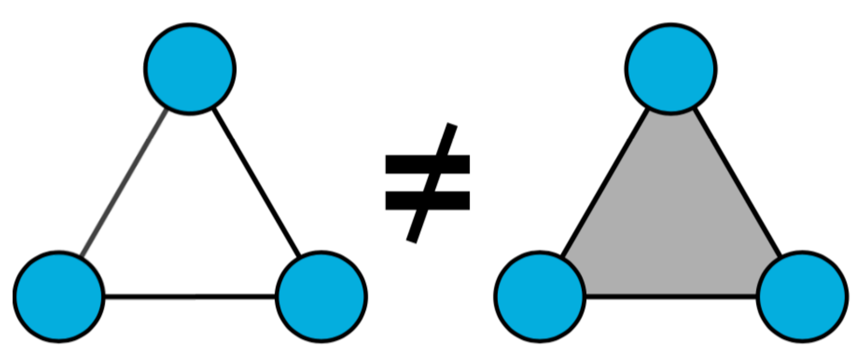
d-dimensional
group interactions



Social structure:
simplicial complex



clique in a network ≠ simplex/hyperedge



ELSEVIER

Physics Reports
Volume 874, 25 August 2020, Pages 1-92

Networks beyond pairwise interactions: Structure and dynamics

Federico Battiston [a], Giulia Cencetti [b], Iacopo Iacopini [c d], Vito Latora [c e f g], Maxime Lucas [h i j], Alice Patania [k], Jean-Gabriel Young [l], Giovanni Petri [m n]

Perspective | Published: 04 October 2021

The physics of higher-order interactions in complex systems

Federico Battiston, Enrico Amico, Alain Barrat, Ginestra Bianconi, Guilherme Ferraz de Arruda, Benedetta Franceschiello, Iacopo Iacopini, Sonia Kéfi, Vito Latora, Yamir Moreno, Micah M. Murray, Tiago P. Peixoto, Francesco Vaccarino & Giovanni Petri

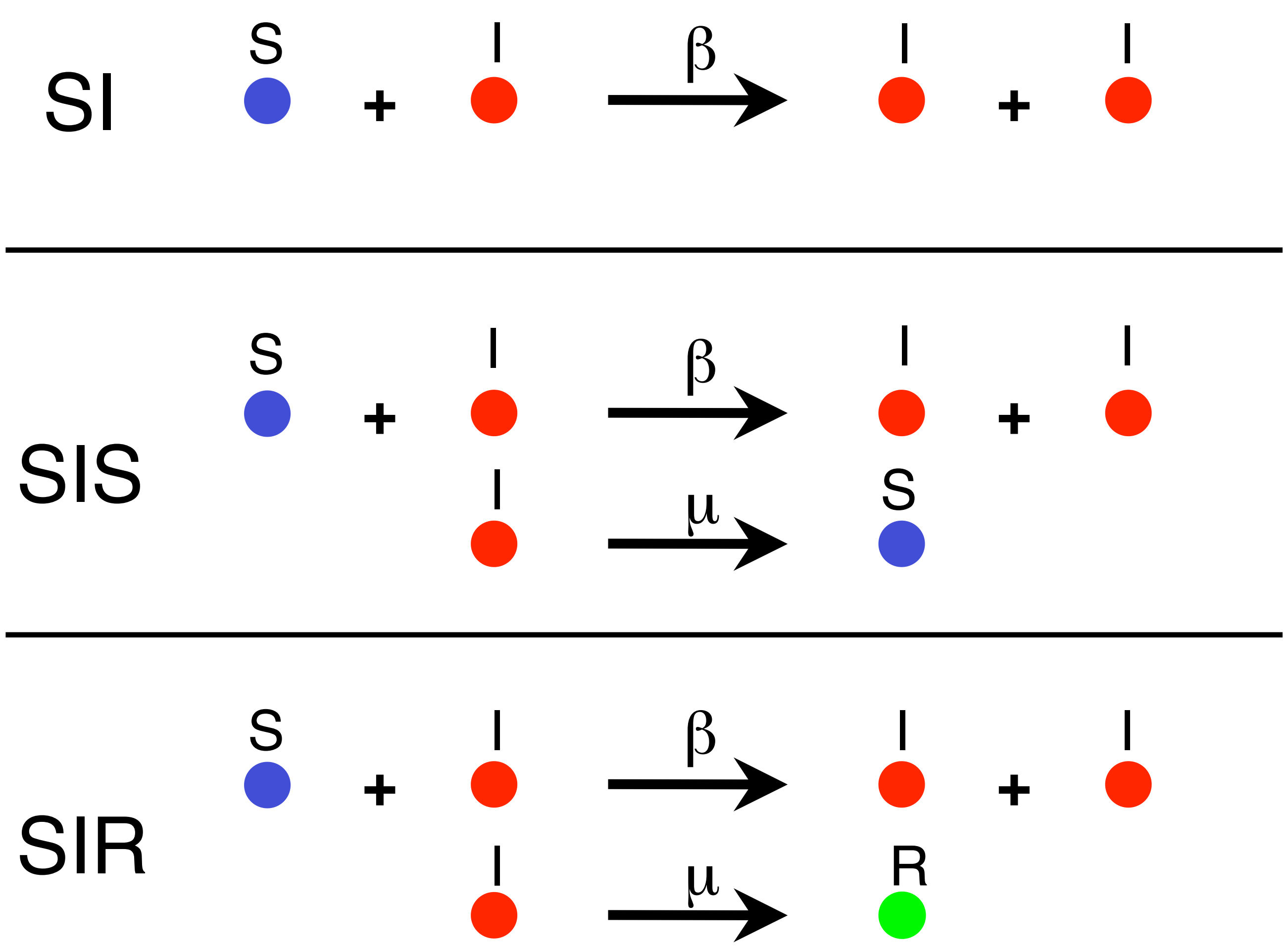
Nature Physics **17**, 1093–1098 (2021) | Cite this article

- **New tools for structure characterisation**
- **New models**
- **Processes on hypergraphs**

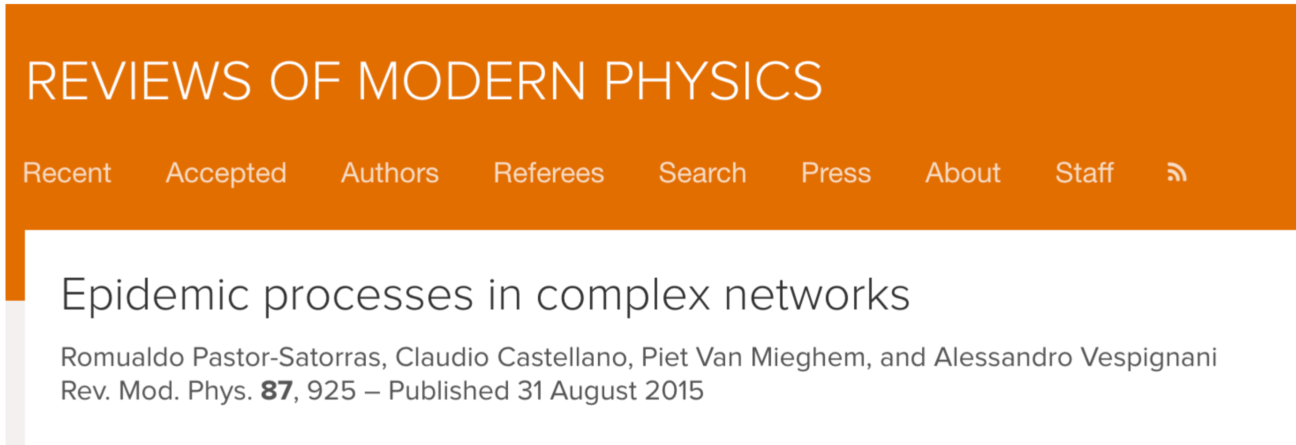
Modeling spreading processes

Propagation models

“Simple contagion”, Epidemic-like: SI, SIS, SIR



PAIRWISE PROCESSES => NETWORKS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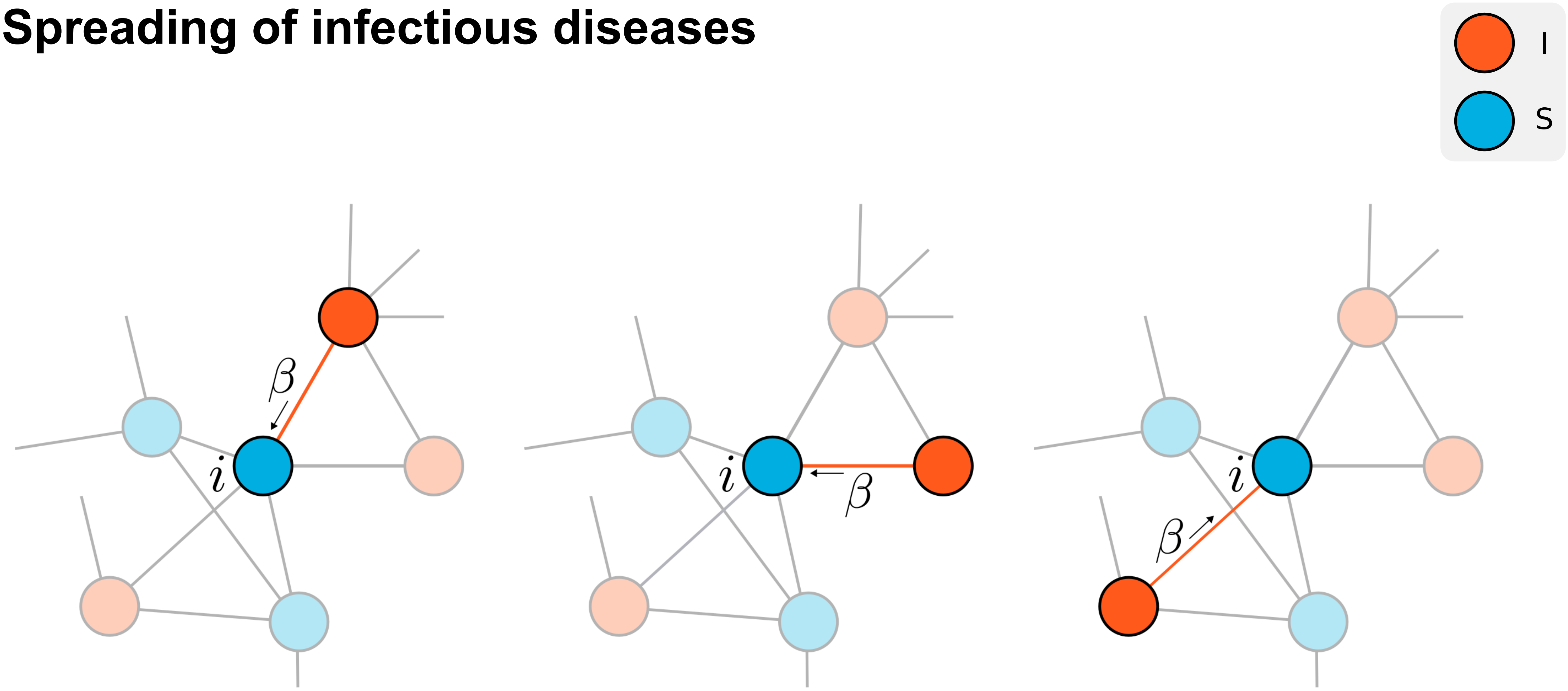
Epidemic processes in complex networks

Romualdo Pastor-Satorras, Claudio Castellano, Piet Van Mieghem, and Alessandro Vespignani

Rev. Mod. Phys. **87**, 925 – Published 31 August 2015

Simple Contagion

Spreading of infectious diseases



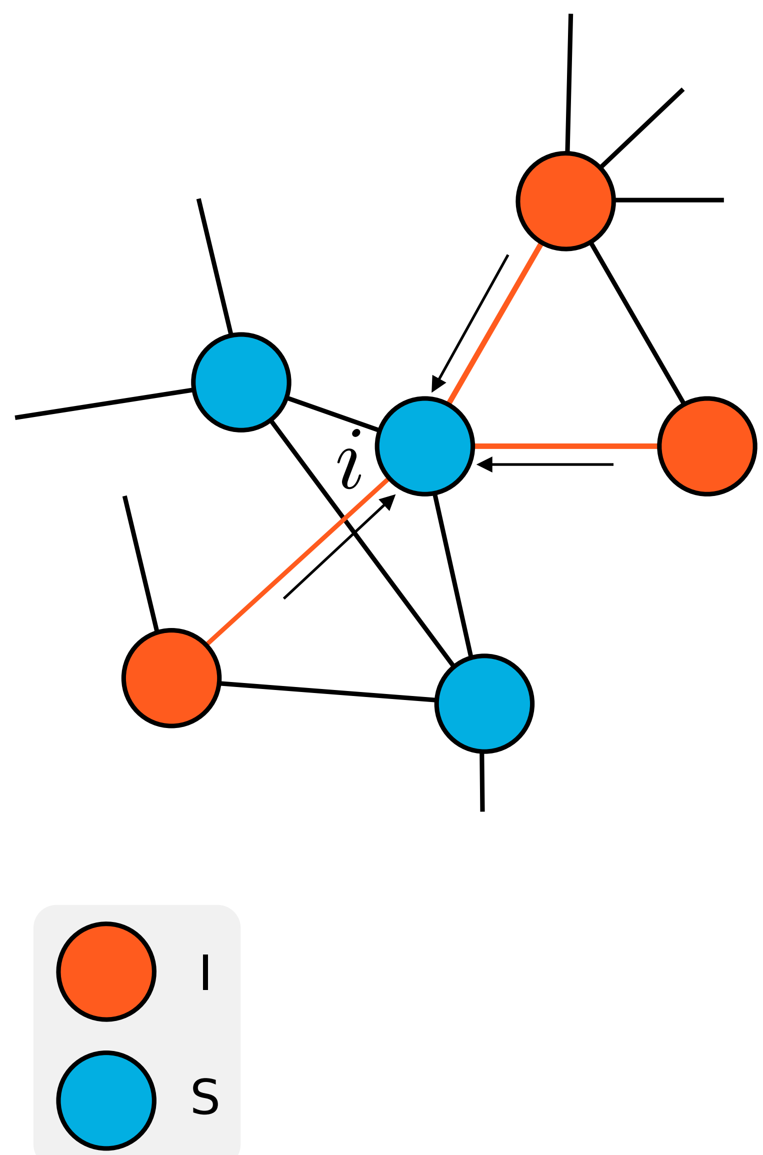
Independent sources

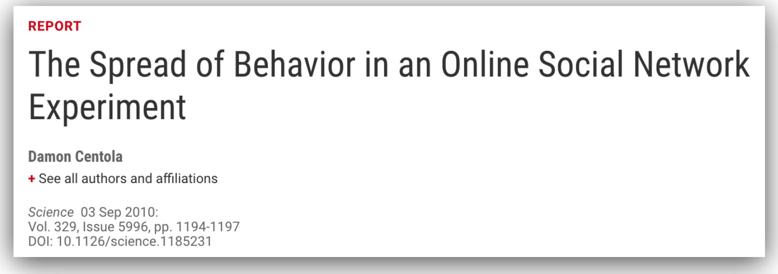
$$I+S \longrightarrow 2I$$

β: probability of infection

Social contagion

Multiple sources of activation are required for a transmission





REPORT

The Spread of Behavior in an Online Social Network Experiment

Damon Centola

+ See all authors and affiliations

Science 03 Sep 2010:
Vol. 329, Issue 5996, pp. 1194-1197
DOI: 10.1126/science.1185231

JOURNAL OF THE ROYAL SOCIETY Interface

Complex contagion process in spreading of online innovation

Márton Karsai[1,2,3,4], Gerardo Iñiguez[2], Kimmo Kaski[2,5] and János Kertész[2,6,7]

Structural diversity in social contagion

Johan Ugander, Lars Backstrom, Cameron Marlow, and Jon Kleinberg

PNAS April 17, 2012 109 (16) 5962-5966; https://doi.org/10.1073/pnas.1116502109

Edited by Ronald L. Graham, University of California at San Diego, La Jolla, CA, and approved February 21, 2012

PLOS | ONE

RESEARCH ARTICLE

Evidence of complex contagion of information in social media: An experiment using Twitter bots

Bjarke Mønsted[1], Piotr Sapieżyński[1], Emilio Ferrara[2,3], Sune Lehmann[1]*

PRL **115,** 218702 (2015) PHYSICAL REVIEW LETTERS week ending 20 NOVEMBER 2015

Kinetics of Social Contagion

Zhongyuan Ruan,[1,2] Gerardo Iñiguez,[3,4] Márton Karsai,[5] and János Kertész[1,2,4,*]
[1]*Center for Network Science, Central European University, H-1051 Budapest, Hungary*
[2]*Institute of Physics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary*
[3]*Centro de Investigación y Docencia Económicas, Consejo Nacional de Ciencia y Tecnología, 01210 México D.F., Mexico*
[4]*Department of Computer Science, Aalto University School of Science, FI-00076 AALTO, Finland*
[5]*Laboratoire de l'Informatique du Parallélisme, INRIA-UMR 5668, IXXI, ENS de Lyon, 69364 Lyon, France*

D. Centola & M. Macy, Americ. J. Sociol. (2007); N. O. Todas & K. Lerman, Sci. Rep. (2014); D. Guilbeault et al., Springer (2018)

Complex contagion

"Complex" contagion: multiple sources needed for a transmission

"*a contagion is complex if its transmission requires an individual to have contact with two or more sources of activation*", i.e. if a "*contact with a single active neighbor is not enough to trigger adoption*"
(Centola & Macy, Am. J. Socio. 2007)

REPORT

The Spread of Behavior in an Online Social Network Experiment

Damon Centola
+ See all authors and affiliations

Science 03 Sep 2010:
Vol. 329, Issue 5996, pp. 1194-1197
DOI: 10.1126/science.1185231

"Individual adoption was much more likely when participants received social reinforcement from multiple neighbors in the social network."

PLOS | ONE   PUBLISH   ABOUT

OPEN ACCESS   PEER-REVIEWED
RESEARCH ARTICLE

Evidence of complex contagion of information in social media: An experiment using Twitter bots

Bjarke Mønsted, Piotr Sapieżyński, Emilio Ferrara, Sune Lehmann

Published: September 22, 2017 • https://doi.org/10.1371/journal.pone.0184148

"We provide experimental evidence that the complex contagion model describes the observed information diffusion behavior more accurately than simple contagion."

Structural diversity in social contagion

Johan Ugander, Lars Backstrom, Cameron Marlow, and Jon Kleinberg

PNAS April 17, 2012 109 (16) 5962-5966; https://doi.org/10.1073/pnas.1116502109

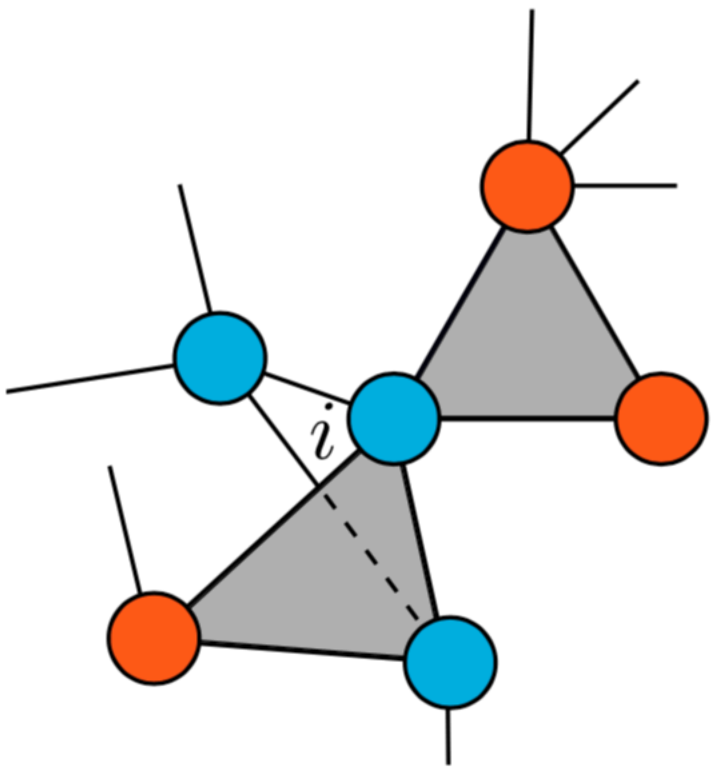
Edited by Ronald L. Graham, University of California at San Diego, La Jolla, CA, and approved February 21, 2012

"We find that the probability of contagion is tightly controlled by the number of connected components in an individual's contact neighborhood, rather than by the actual size of the neighborhood."

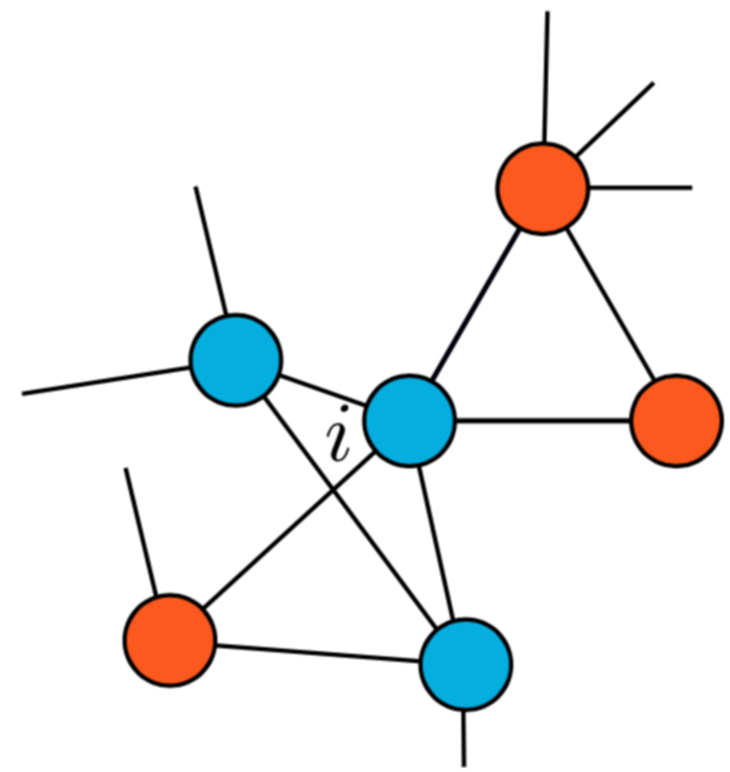
Still: networks, i.e.,
pairwise interactions

What about group interactions?

Social structure



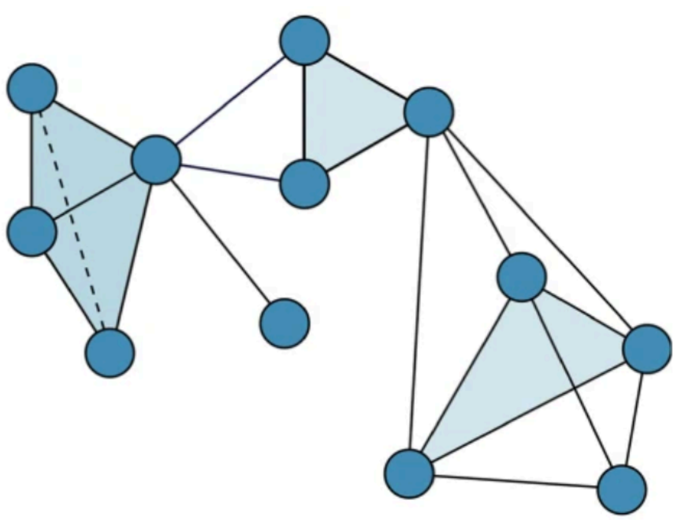
Network representation,
hides group interactions



Mixing simple and complex contagion: epidemic-like models on simplicial complexes

“Simplagion”

(Simplicial contagion)



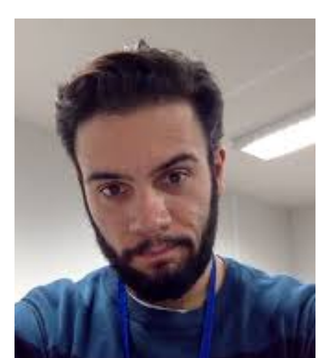
nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-019-10431-6 OPEN

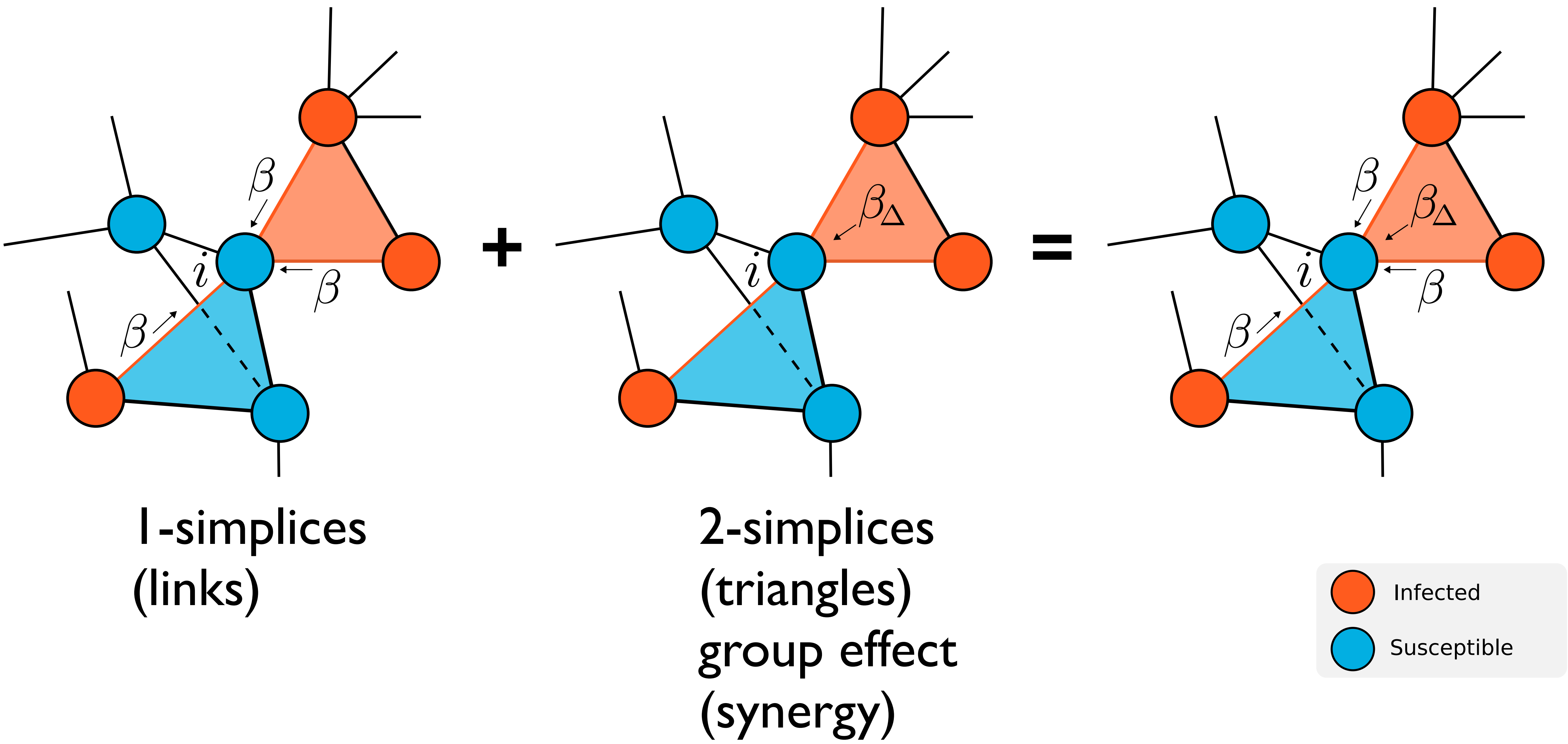
Simplicial models of social contagion

Iacopo Iacopini[1,2], Giovanni Petri[3,4], Alain Barrat[3,5] & Vito Latora[1,2,6,7]

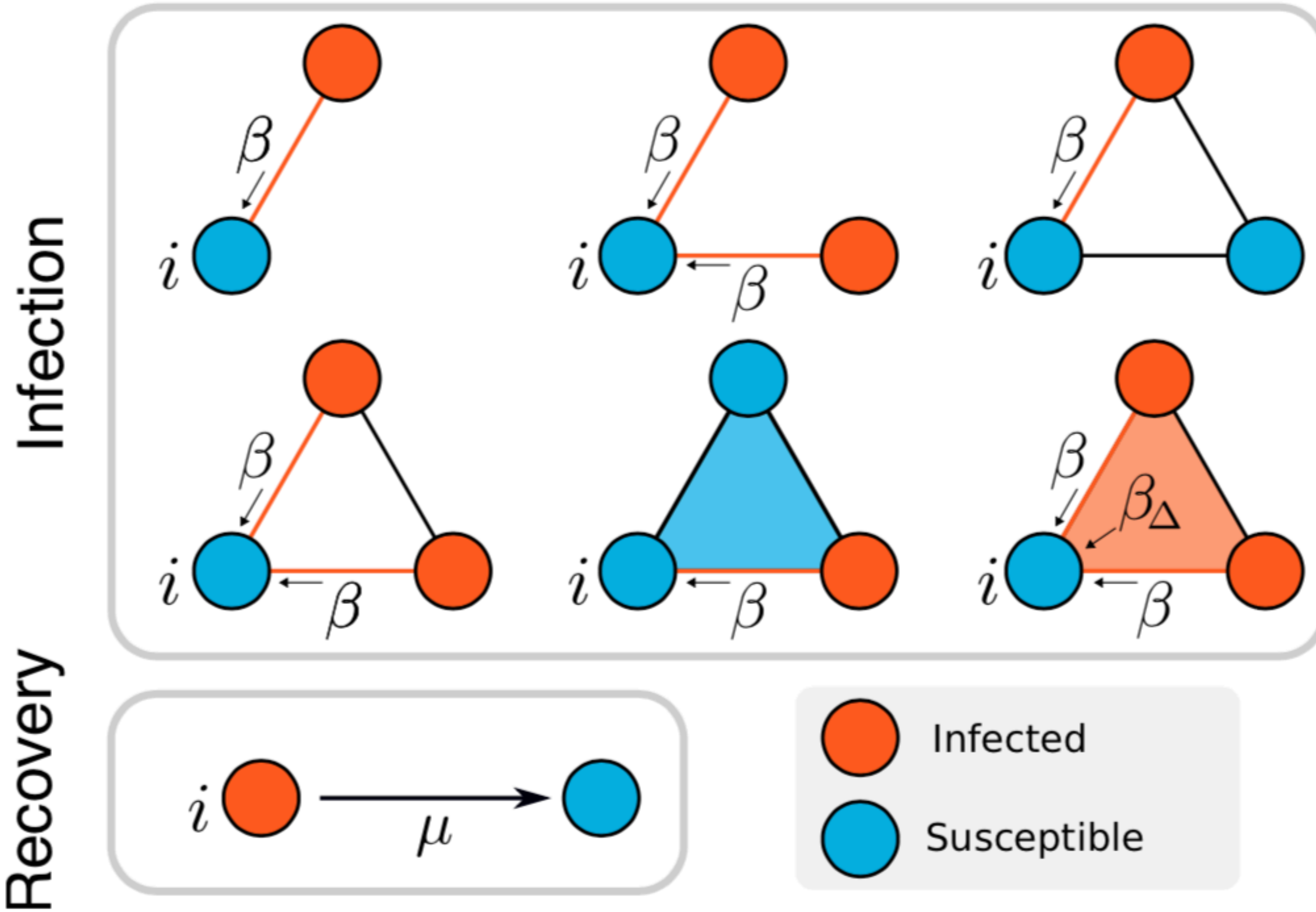


SIMPLicial contAGION

The Model (D=2)



The model (D=2)



control parameters

$$\lambda = \beta \langle k \rangle / \mu$$
$$\lambda_\Delta = \beta_\Delta \langle k_\Delta \rangle / \mu$$

k_w : generalised (simplicial) degree

$$\langle k_1 \rangle = \langle k \rangle$$
$$\langle k_2 \rangle = \langle k_\Delta \rangle$$

Mean-field approach

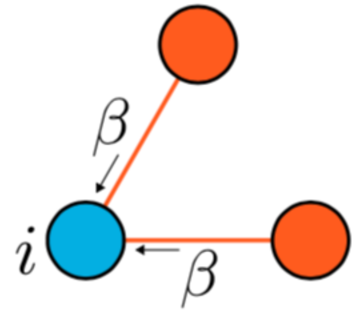
Case D=2

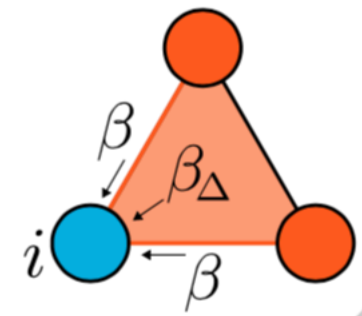
Density of infectious nodes

$$d_t\rho(t) = -\mu\rho(t) + \beta\langle k\rangle\rho(t)\big[1-\rho(t)\big] + \beta_\Delta\langle k_\Delta\rangle\rho^2(t)\big[1-\rho(t)\big]$$

new infections from 1-simplices

new infections from 2-simplices





$$d_t\rho(t) = -\rho(t)\Big(\rho(t) - \rho^*_{2+}\Big)\Big(\rho(t) - \rho^*_{2-}\Big)$$

Steady state: $d_t\rho(t) = 0$ => up to 3 physical solutions

$$\rho^*_1 = 0$$

Absorbing state, all nodes S

$$\rho^*_{2\pm} = \frac{\lambda_\Delta - \lambda \pm \sqrt{(\lambda-\lambda_\Delta)^2 - 4\lambda_\Delta(1-\lambda)}}{2\lambda_\Delta}$$

Potential non-trivial solutions

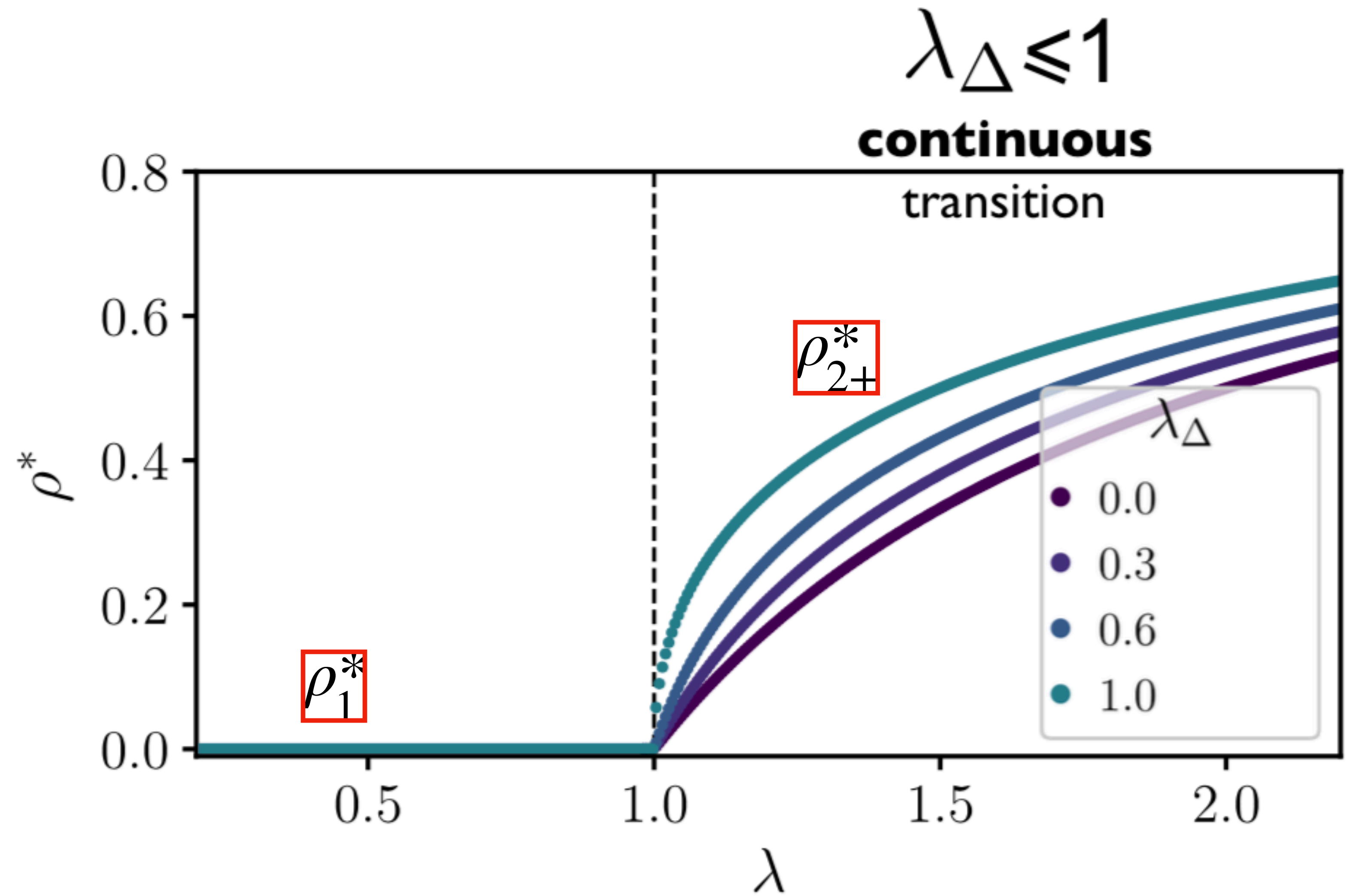
$\lambda_\Delta \leq 1$: continuous SIS-like transition

$\rho^*_{2-} < 0$

$$d_t\rho(t) = -\rho(t)\left(\rho(t) - \rho^*_{2+}\right)\left(\rho(t) - \rho^*_{2-}\right)$$

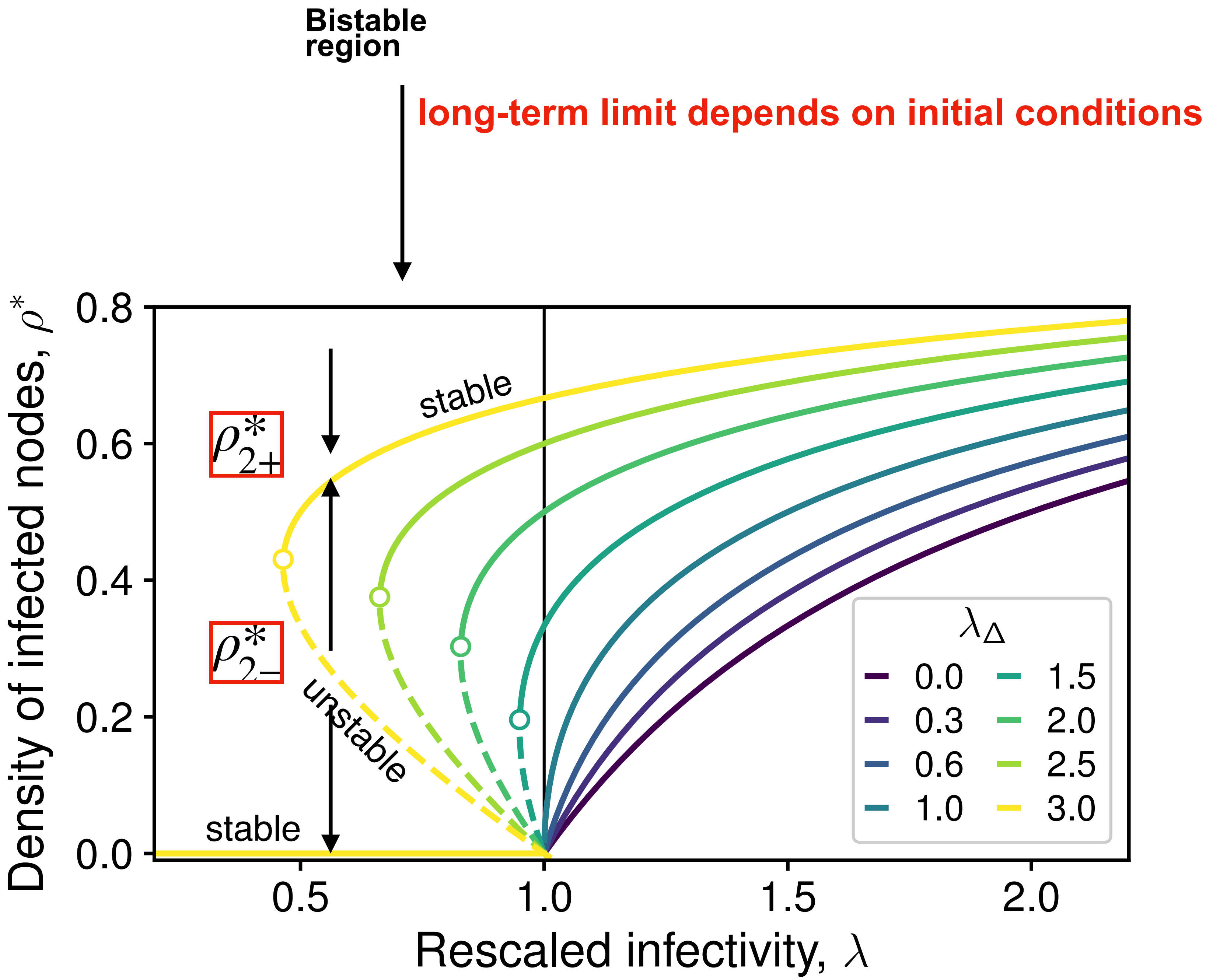
For $\lambda < 1$: $\rho^*_{2+} < 0$ => $\rho^*_1 = 0$ only solution

For $\lambda > 1$: $\rho^*_{2+} > 0$ => $\rho^*_1 = 0$ becomes unstable, ρ^*_{2+} stable solution



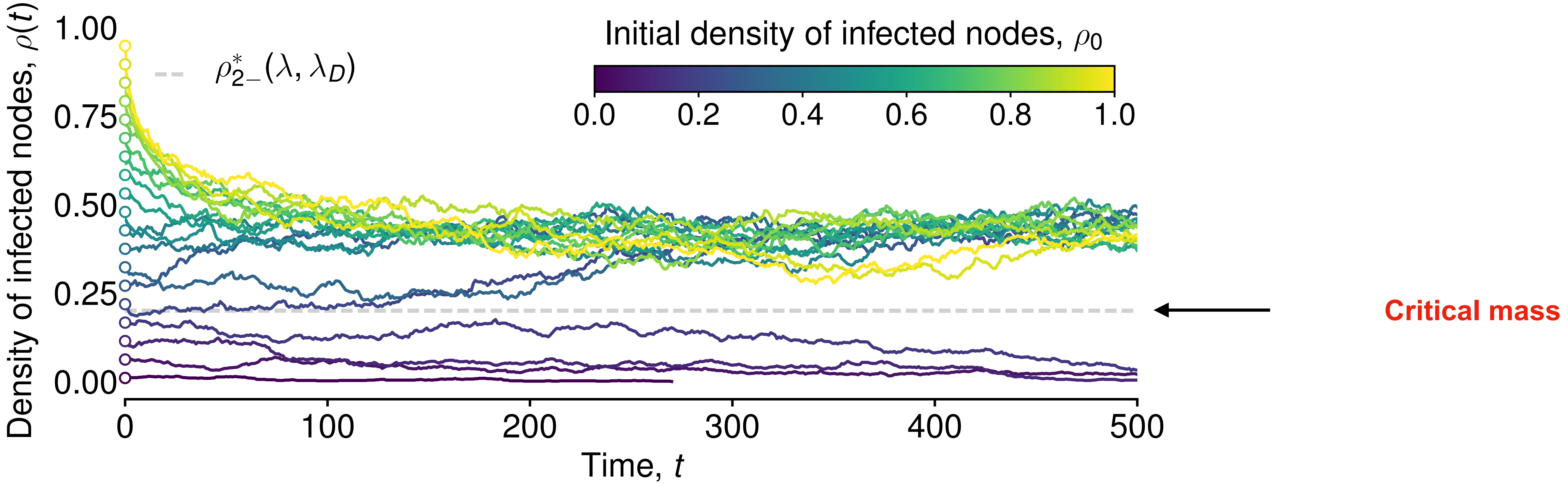
$\lambda_\Delta > 1$: discontinuous transition

$$d_t\rho(t) = -\rho(t)\left(\rho(t) - \rho^*_{2+}\right)\left(\rho(t) - \rho^*_{2-}\right)$$

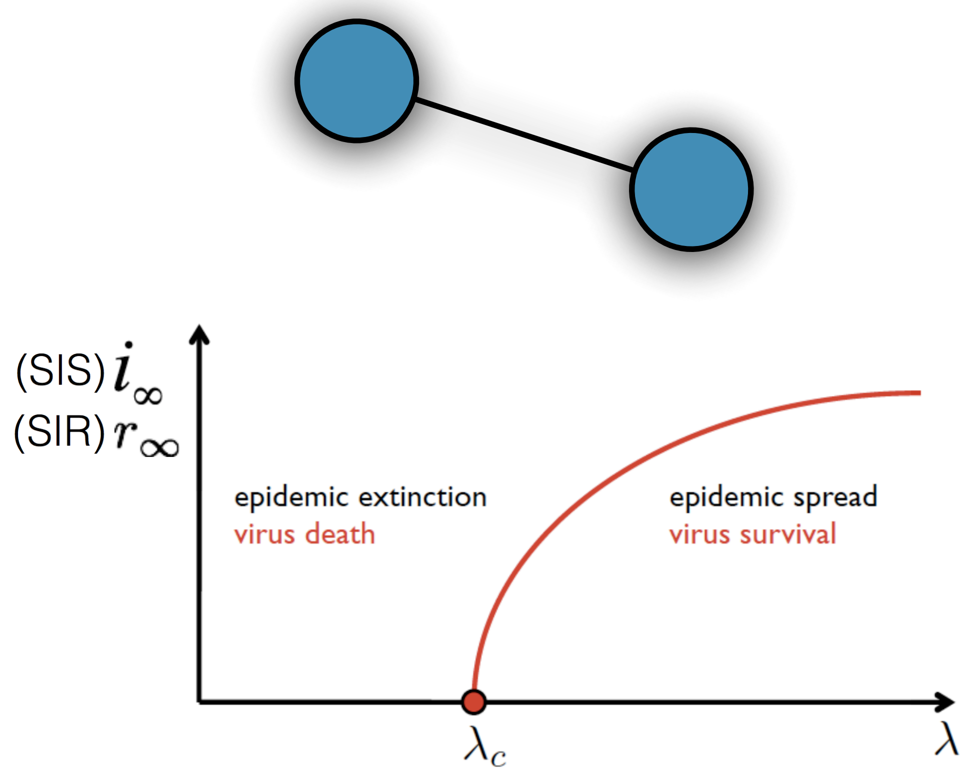


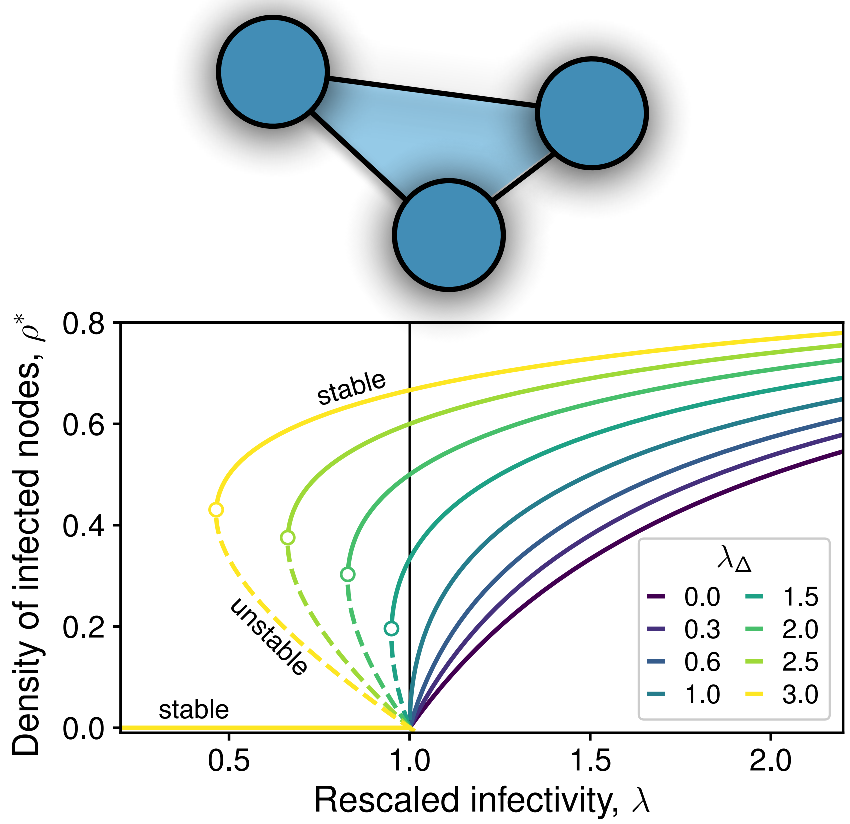
Numerics on random simplicial complexes

Role of initial conditions



Summing up - “SIMPLAGION”





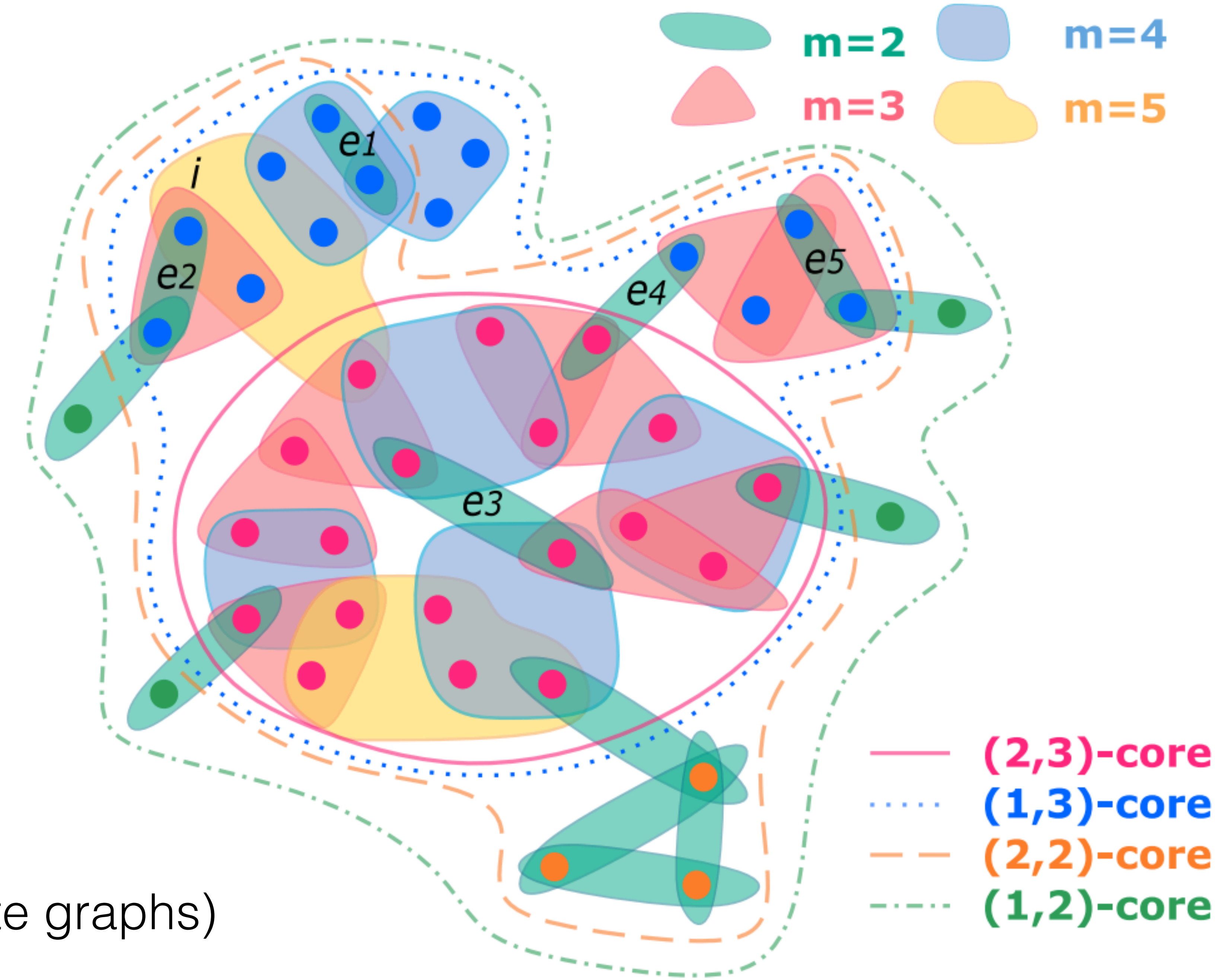
- **Model:**
  - Social structure modelled as a **simplicial complex/hypergraph**
  - Contagion occurs in **group interactions** (with different transmission rates)
- **New phenomenology**
  - Discontinuous transition
  - **Dependence on the size of the seed (critical mass)**
  - Various extensions (hypergraphs, heterogeneous higher order graphs, etc)

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019).
Simplicial models of social contagion. Nature communications, 10(1), 2485.

Beyond networks: Higher-order interactions

Development of new tools:

- characterization
- finding structures: hyper cores
- comparison
- temporal evolution
- …



(k,m) hyper-core:
maximal subhypergraph where all nodes have at least k hyper edges, each of size at least m

(NB: equivalent concept in bipartite graphs)

arXiv > physics > arXiv:2301.04235

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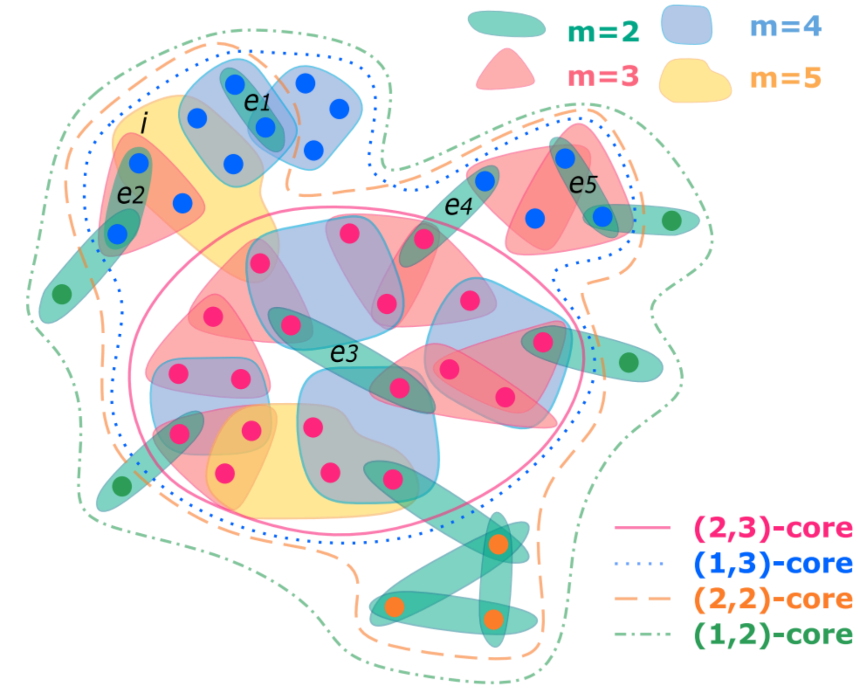
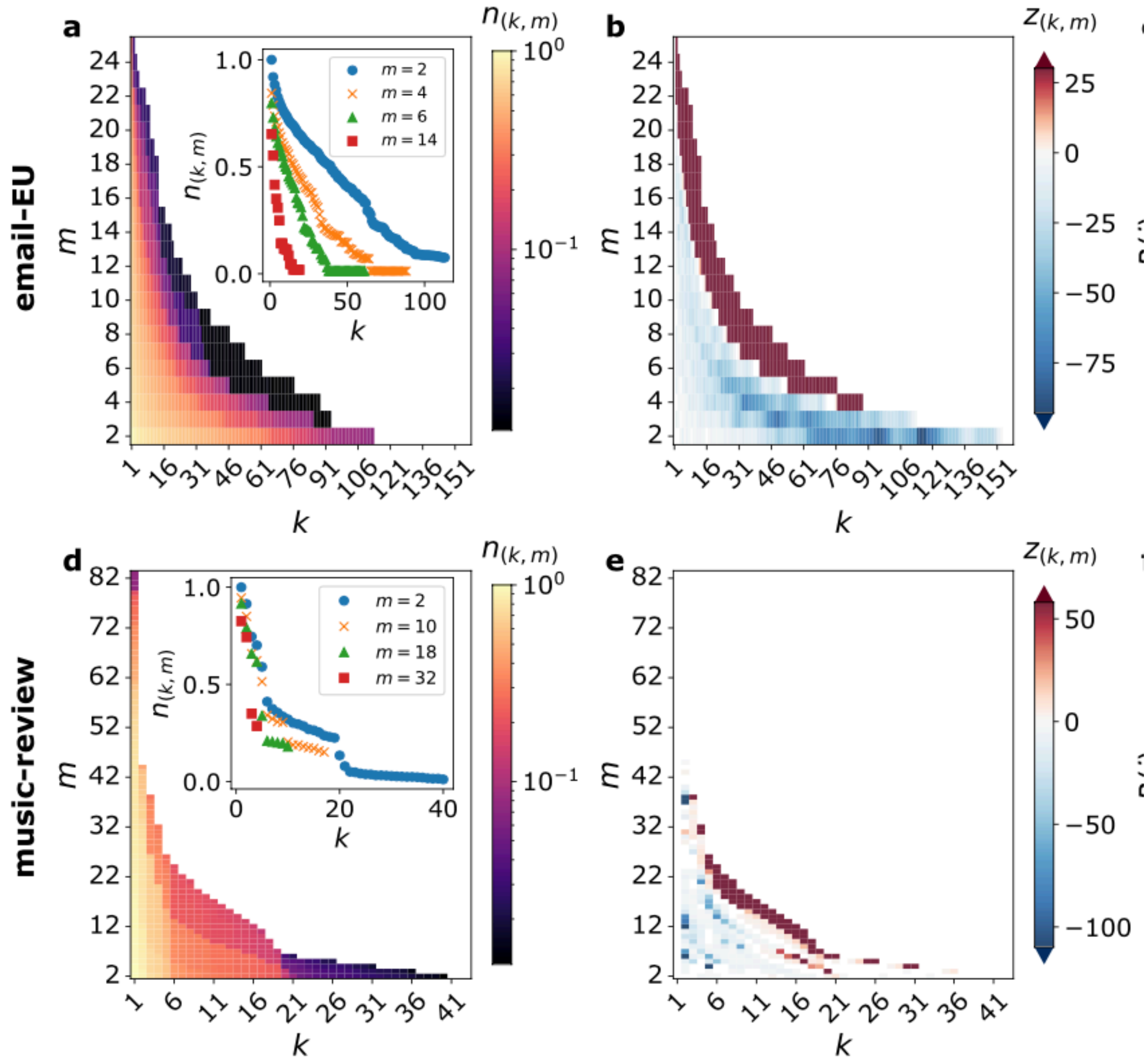
[Submitted on 10 Jan 2023]

Hyper-cores promote localization and efficient seeding in higher-order processes

Marco Mancastroppa, Iacopo Iacopini, Giovanni Petri, Alain Barrat

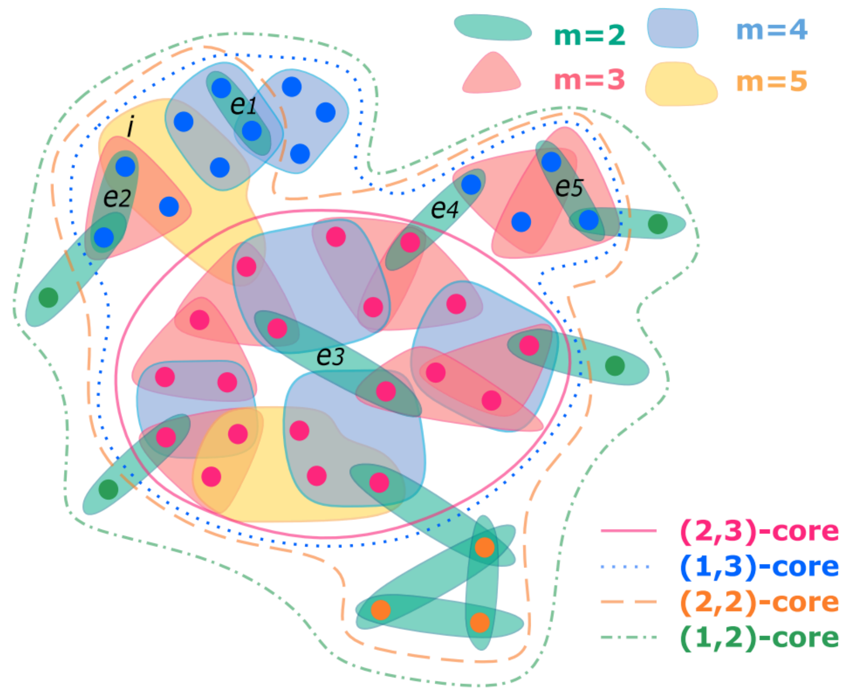
Hyper-cores





Empirical decomposition
+
Comparison with null model

Hyper-cores: impact on processes



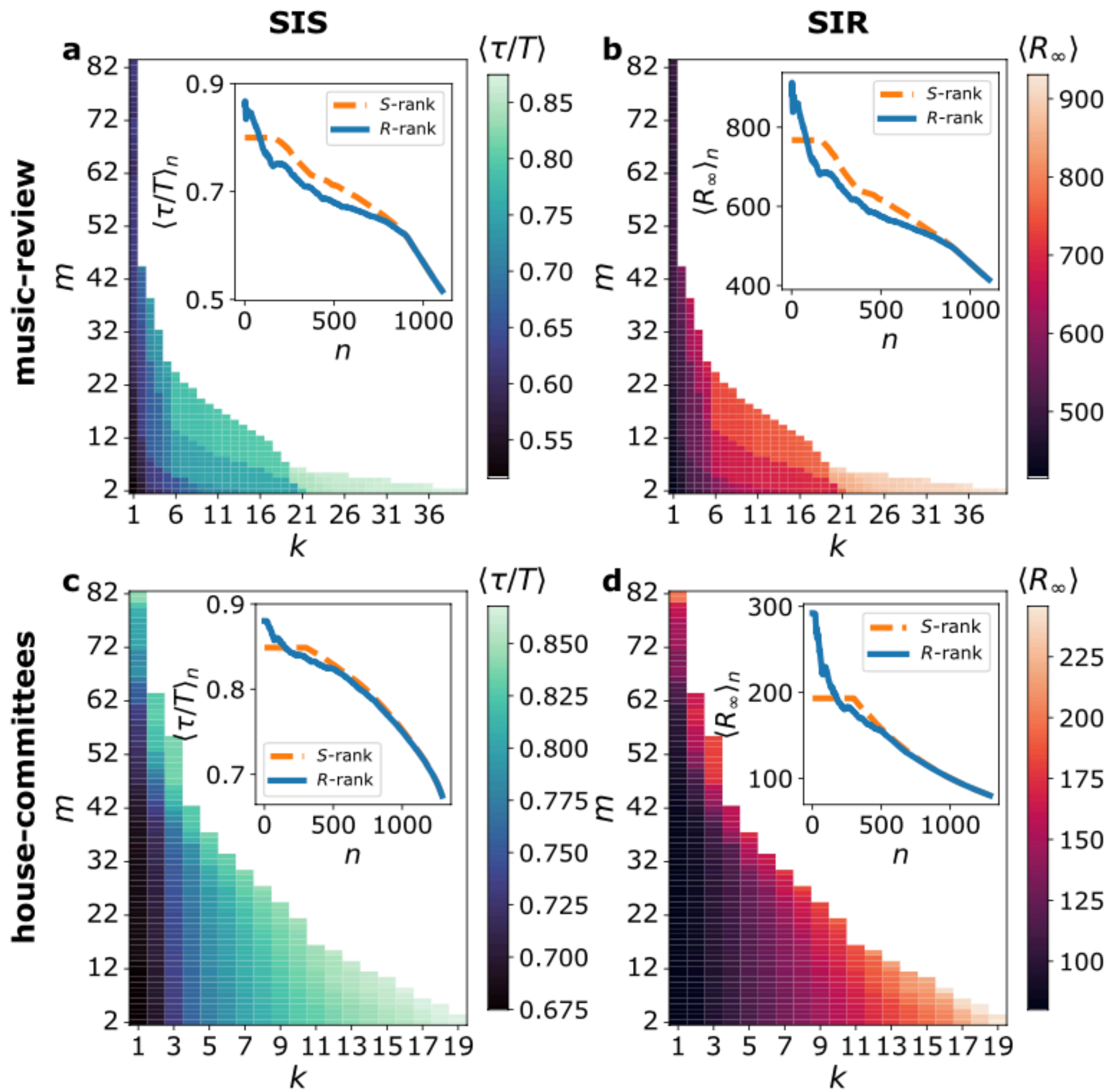
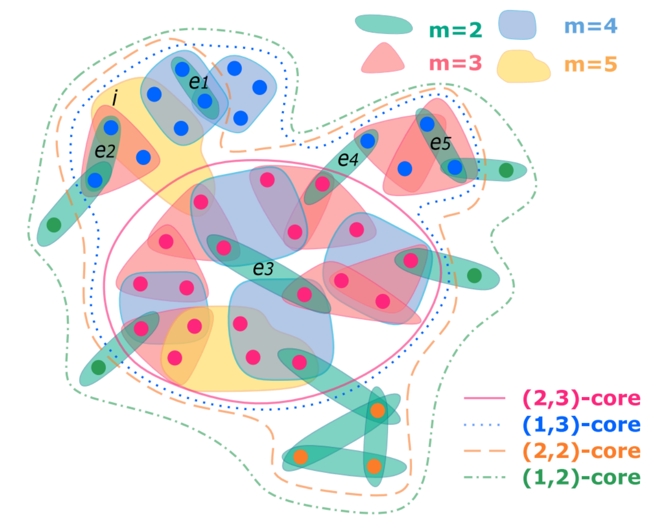


FIG. 3. **Hyper-cores for seeding and localization in higher-order non-linear contagion processes.** For the

Average time spent in state I (SIS model) larger for nodes in more central cores

Final epidemic size (SIR) are larger for processes seeded in the more central cores

Hyper-cores: impact on processes



Committed minorities in the more central nodes convince faster and more easily in the Naming Game

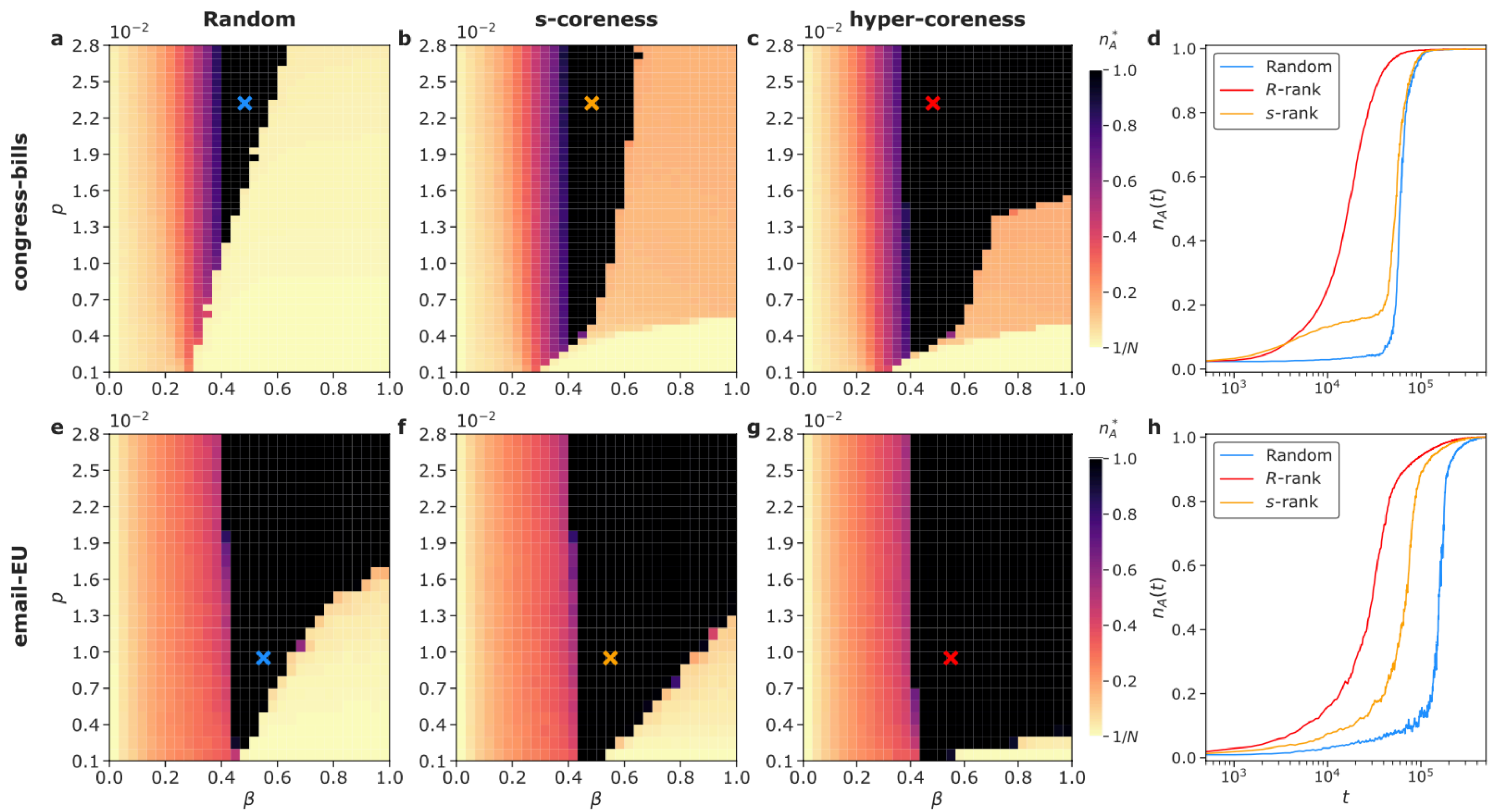


FIG. 4. **Comparison of seeding strategies for committed minorities in a higher-order naming-game process.** In

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Physics Reports

Volume 874, 25 August 2020, Pages 1-92

Networks beyond pairwise interactions: Structure and dynamics

Federico Battiston [a], Giulia Cencetti [b], Iacopo Iacopini [c d], Vito Latora [c e f g], Maxime Lucas [h i j], Alice Patania [k], Jean-Gabriel Young [l], Giovanni Petri [m n]

Perspective | Published: 04 October 2021

The physics of higher-order interactions in complex systems

Federico Battiston, Enrico Amico, Alain Barrat, Ginestra Bianconi, Guilherme Ferraz de Arruda, Benedetta Franceschiello, Iacopo Iacopini, Sonia Kéfi, Vito Latora, Yamir Moreno, Micah M. Murray, Tiago P. Peixoto, Francesco Vaccarino & Giovanni Petri

Nature Physics **17**, 1093–1098 (2021) | Cite this article

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