An Epigraphical Convex Optimization Approach for Multicomponent Image Restoration using Non-Local Structure Tensor

G. Chierchia[1] N. Pustelnik[2] J.-C. Pesquet[3] B. Pesquet-Popescu[1]

[1]LTCI, Télécom ParisTech/Institut Télécom – CNRS UMR 5141
[2]Laboratoire de Physique, ENS Lyon – CNRS UMR 5672
[3]LIGM, Université Paris-Est – CNRS UMR 8049

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Outline

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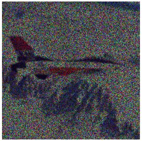
1. Introduction
2. Proposed approach
3. Numerical results
4. Conclusions

Problem definition


Original.


Degraded.

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- *original (multicomponent) image*: $\overline{x} = (\overline{x}_1, \dots, \overline{x}_R) \in (\mathbb{R}^N)^R$
- *linear operator*: $A = (A_{j,i})_{1 \leq j \leq S, 1 \leq i \leq R}$, with $A_{j,i} \in \mathbb{R}^{K \times N}$
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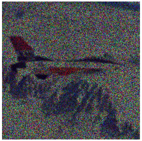

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Objective

How can we recover $\overline{x}$ from the observation z?

Related work

$$\boxed{\underset{x \in (\mathbb{R}^N)^R}{\text{minimize}} \quad \underbrace{\|Ax - z\|_2^2}_{\text{Data fidelity term}} + \lambda \underbrace{g(x)}_{\text{Regularization term}}}$$

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  $\rightarrow$ $\ell_{1,\infty}$ matrix-norm regularization

  [Sapiro 1996] [Goldluecke 2012]

Contributions

$$\boxed{\underset{x \in C}{\text{minimize}} \quad \|Ax - z\|_2^2 \quad \text{subject to} \quad g(x) \leq \eta}$$

where C is a closed convex subset of $(\mathbb{R}^N)^R$

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Problem

How can we efficiently handle ST-NLTV constraints?

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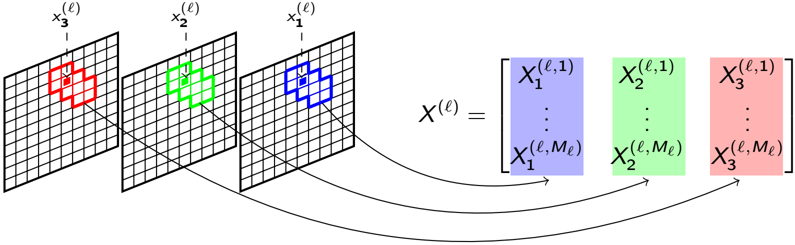
Structure Tensor Non-Local TV

- Non-Local gradient at point $\ell \in \{1, \ldots, N\}$

$$X^{(\ell)} = \left(\omega_{\ell,n}\,(x_i^{(\ell)} - x_i^{(n)})\right)_{n\in\mathcal{N}_\ell,\, 1\le i\le R} \in \mathbb{R}^{M_\ell \times R}$$



$$X^{(\ell)} = \begin{bmatrix} X_1^{(\ell,1)} & X_2^{(\ell,1)} & X_3^{(\ell,1)} \\ \vdots & \vdots & \vdots \\ X_1^{(\ell,M_\ell)} & X_2^{(\ell,M_\ell)} & X_3^{(\ell,M_\ell)} \end{bmatrix}$$

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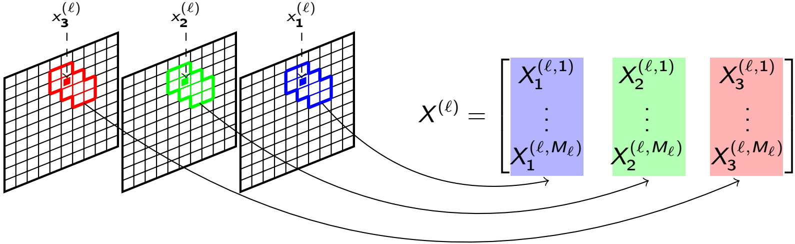

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- *special case*: ST-TV
  - $\mathcal{N}_\ell \rightarrow$ horizontal/vertical neighbours
  - $\omega_{\ell,n} = 1$

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2. matrix $\ell_{1,p}$-norm ball

$$D = \{X \in \mathbb{R}^{M\times R} \mid \sum_{\ell=1}^{N} \|X^{(\ell)}\|_p \leq \eta\}$$

where $M = M_1 + \cdots + M_N$

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  $\rightarrow$ *primal methods*: applicable by reformulating the minimization problem (see [Peyré 2011] [Briceño-Arias 2011])

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$\gamma \in \left]0,\ 1/(2\|A\|^2 + \|F\|)\right[,\quad x^{[0]} \in (\mathbb{R}^N)^R,\quad y^{[0]} \in \mathbb{R}^{M\times R}$
for $t = 0, 1, \ldots$ **do**
$\quad \widetilde{x}^{[t]} = P_C\big(x^{[t]} - \gamma(2A^\top A x^{[t]} - 2A^\top z + F^\top y^{(t)})\big)$
$\quad X^{[t]} = \gamma^{-1} y^{[t]} + F x^{[t]}$
$\quad \widetilde{y}^{[t]} = X^{[t]} - \gamma^{-1} P_D(X^{[t]})$
$\quad y^{[t+1]} = \widetilde{y}^{[t]} + \gamma\, F\,(\widetilde{x}^{[t]} - x^{[t]})$
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Problem

How can we efficiently compute $P_D(X) = \arg\min_{\widehat{X} \in D} \|\widehat{X} - X\|$?

Epigraphical Splitting Technique

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Proposed solution [Chierchia 2012]

Decompose D by adding an auxiliary vector $\zeta = (\zeta^{(\ell)})_{1\leq\ell\leq N} \in \mathbb{R}^N$

$$\sum_{\ell=1}^{N} \|X^{(\ell)}\|_p \leq \eta \quad \Leftrightarrow \quad \begin{cases} \|X^{(\ell)}\|_p \leq \zeta^{(\ell)} & (\forall \ell \in \{1,\ldots,N\}) \\ \sum_{\ell=1}^{N} \zeta^{(\ell)} \leq \eta \end{cases}$$

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$$E = \left\{(X, \zeta) \quad \middle| \quad (X^{(\ell)}, \zeta^{(\ell)}) \in \operatorname{epi} \|\cdot\|_p \quad (\forall \ell \in \{1, \ldots, N\})\right\}$$

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2. closed half-space

$$W = \left\{\zeta \in \mathbb{R}^N \;\middle|\; 1_N^\top \zeta \leq \eta\right\}$$

with $1_N = (1,\dots,1)^\top \in \mathbb{R}^N$

Epigraphical projection for matrix norms

- $P_{\mathrm{epi}\,\|\cdot\|_p}$ exists for vectorial norms with $p \in \{1, 2, +\infty\}$
  [Pang 2003] [Pock 2010] [Ding 2012] [Chierchia 2012]

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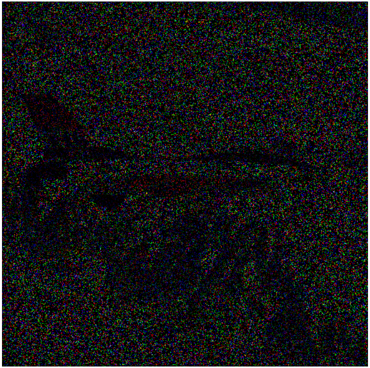
- choice of η based on image characteristics

Qualitatively results (SNR – SSIM)

Original



Degraded

Qualitatively results (SNR – SSIM)

CC-TV-ℓ_2: 22.33 - 0.812

ST-TV-ℓ_2: 23.10 - 0.823

CC-NLTV-ℓ_2: 23.20 - 0.829

ST-NLTV-ℓ_2: 23.69 - 0.836

Qualitatively results (SNR – SSIM)

CC-TV-ℓ_∞: 22.00 - 0.803

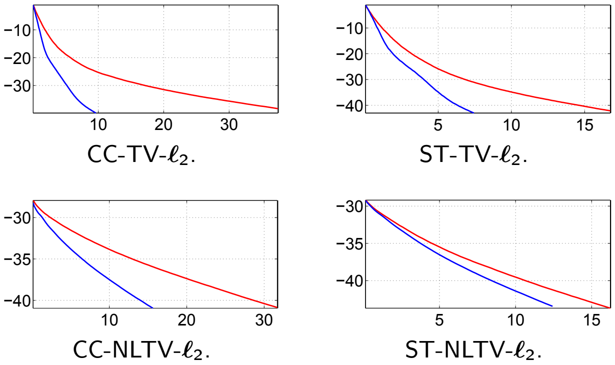
ST-TV-ℓ_∞: 22.68 - 0.817

CC-NLTV-ℓ_∞: 23.28-0.827

ST-NLTV-ℓ_∞: 23.03 - 0.823

Convergence speed ($\|x^{[n]} - x^{[\infty]}\| / \|x^{[\infty]}\|$ vs time)

- *blue line*: algorithm with epigraphical projections
- *red line*: algorithm with direct projections



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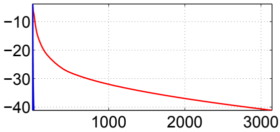
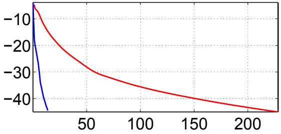
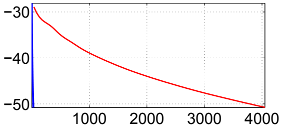


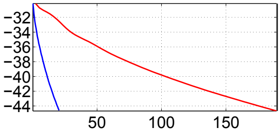
CC-TV-ℓ_∞.



ST-TV-ℓ_∞.



CC-NLTV-ℓ_∞.



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Outline

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Thanks for your attention...