Angular Estimation with Leaky Wave Antennas: Toward a Low-Tech Radar?

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Résumé – Les antennes à onde de fuite permettent de balayer un domaine angulaire en paramétrant la fréquence du signal émis. Elles pourraient donc permettre la réalisation de radars avec une faible complexité où une seule voie d'émission-réception suffirait à couvrir un plan. Cet article établit la borne de Cramer-Rao pour l'estimation de la direction d'une cible avec un tel radar.

Abstract – Leaky-wave antennas enable to scan an angular domain with a frequency sweep. They could therefore facilitate the development of low complexity radars since a single transmit and receive channel becomes sufficient to sense a plane. This paper computes the Cramer-Rao Bound for estimating the direction of a target with such a radar.

1 Introduction

Leaky-Wave Antennas (LWA) is a particular topology of antenna which belongs to the class of traveling wave antennas. Unlike resonant antennas, such as patches or dipoles, LWAs radiate electromagnetic energy gradually along the path of wave propagation. Thus, a LWA can be seen as a leaky Transmission Line (TL), where the leakages are caused by radiation. The antenna can be excited by a single port and the phase shifts between the radiating elements can be adapted by adjusting the TL phase constant. As a result, a passive beamforming capability can be obtained with a single channel and controlled by the signal frequency [9]. Additionally, LWAs are known to be efficient (i.e. the antenna efficiency being the ratio between the antenna gain and the directivity in a certain direction). These aforementioned properties allow the development of low-cost systems including radars [10] able to scan a plane with a single channel, resulting in a simpler hardware compared to the classic phased array technology.

However, such systems are in practice limited by the necessary bandwidth to scan a large angular domain in accordance with radio regulations [8]. For that purpose, research is focusing on increasing the scanning rate. For example, [13] proposes the use of metasurfaces in a slotted waveguide, while [16] and [4] employ corrugations or periodical pins in a leaky waveguide. The latest reference reaches an average scanning rate of $50.8°$ per GHz with a fabricated prototype.

Another barrier to the development of LWA-based radar is the lack of theoretical studies on the precision of target parameter estimation. More specifically, some theoretical performances for direction finding have already been studied but do not apply to radar applications. For example, [11] establishes the Cramer-Rao Bound (CRB) for the estimation of the direction of a single tone signal impinging a LWA. Unfortunately, the single tone model does not match with most radars which use pulse compression techniques to improve the range resolution. Similarly, traditional direction finding algorithms have been adapted for signals received by a LWA, such as the well-known Maximum Likelihood Estimator (MLE) [1] or the super-resolution methods MUSIC [11] or PUMA [20]. But again, the signal model does not match a LWA-based radar as the antenna transfer function should also be considered for the transmission and not only for the reception of signals.

This paper addresses the second barrier, which is the study of parameter estimation through the CRB. The paper is organized as follows. First the model describing the transmitted and received radar signal with a LWA is presented. Secondly, the CRB for the estimation of a target direction is computed. Following that, the expression of the MLE is established such that the CRB can be validated in a numerical application which also compares the performance of a LWA-based radar with a phased array radar before concluding.

For the sake of clarity, a regular font style is used for scalars (e.g. x) while a bold font style is used for vectors or matrices (e.g. $\boldsymbol{x}$). In addition, we introduce the Fourier transform as $\mathcal{F}$.

2 Signal Model Incorporating the LWA Transfer Function

The transfer function of a slotted guide LWA describes the relation between a signal at the antenna port, $x(t)$ whose Fourier transform is $X(f)$, and the radiated signal, $y(t, u)$ whose Fourier transform is $Y(f, u)$, namely

$$Y(f,u) \; = \; \boldsymbol{W}^{\top}(f)\, \boldsymbol{A}(f,u)\, X(f). \tag{1}$$

In other words, the LWA transfer function can be interpreted as a filter parameterized by the direction of observation or the direction of the incident signal, namely the direction cosine u. In this model, $\boldsymbol{W}(f)$ describes the propagation of the signal inside the waveguide, while $\boldsymbol{A}(f, u)$ is the slotted array response in the direction cosine u. When the slots are identical (uniform LWA), they can be expressed as [2, 7] [1]

1. The antenna radiation efficiency and the gain of each slot in the direction cosine u may be incorporated into the expressions of $\boldsymbol{W}(f)$ and $\boldsymbol{A}(f, u)$. However, they have not been included in this article to make it easier to read.

$$\boldsymbol{W}(f) = \frac{e^{-\alpha \boldsymbol{p} - j\beta(f)\boldsymbol{p}}}{\|e^{-\alpha \boldsymbol{p}}\|}, \tag{2}$$

and

$$\boldsymbol{A}(f,u) = e^{jk_0(f)u\boldsymbol{p}}, \tag{3}$$

where α represents the attenuation in the waveguide which can be considered as a constant value when the transmitted bandwidth is narrow enough [6], while β is the phase shift induced by the TL, which can be obtained by solving the Maxwell's equations [12]. Introducing ε_r as the permittivity of the dielectric material filling up the waveguide (or the equivalent permittivity when radiofrequency structures such as corrugations load the waveguide), β can be approximated in the TE01 propagation mode by [6]

$$\beta(f) \approx \sqrt{\left(\frac{2\pi f}{c}\right)^2 \varepsilon_r - \left(\frac{\pi}{h}\right)^2}. \tag{4}$$

Furthermore, $\boldsymbol{p}$ are the positions of the slots along the waveguide shallowness, and h is the height of the waveguide as shown in figure 1. One may note that equation (4) can be adapted to the TE10 propagation mode by replacing h by the width of the waveguide. Finally, k_0 is the wavenumber in the vacuum, known as

$$k_0(f) = \frac{2\pi f}{c}. \tag{5}$$

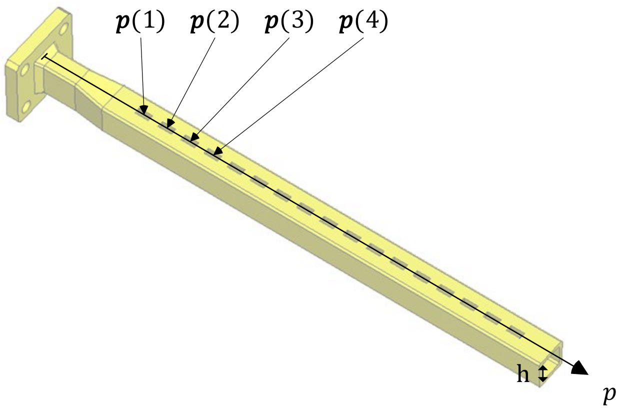

Figure 1 – Description of the LWA.

In this paper, we consider the case of a radar with a unique transmit and receive channel connected to the LWA port. The transmitted signal, $s(t)$, is radiated and then back-scattered from a single target in the direction cosine u. The signal impinging the LWA is combined with a white noise jammer whose signal is $q(t)$ and arriving from the direction cosine v. As a result, the received signal at the LWA port, $r(t)$, is given by

$$r(t) = b\left(s * \mathcal{F}^{-1}\{(\boldsymbol{W}^\top(f)\,\boldsymbol{A}(f,u))^2\}\right)\Big|_{t-\tau} + \left(q * \mathcal{F}^{-1}\{\boldsymbol{W}^\top(f)\,\boldsymbol{A}(f,v)\}\right)\Big|_{t} + \varepsilon(t), \tag{6}$$

where τ is the propagation delay (proportional to the range of the target) while b is an unknown complex amplitude accounting for the link budget and the phase shift during the propagation. Moreover, $\varepsilon(t)$ is the thermal noise added by the receive channel. Both the jamming signal and the thermal noise follow circular Gaussian distributions, namely

$$\begin{aligned} q(t) &\sim \mathcal{CN}(0, \varsigma^2), \\ \varepsilon(t) &\sim \mathcal{CN}(0, \sigma^2). \end{aligned} \tag{7}$$

In the general case of equation (6), the jammer contribution to the received signal is therefore a colored noise. However, in the absence of jammer, the observation model becomes a Conditional Signal Model (CSM) [18] where the received signal is observed amid a circular complex Gaussian noise.

Finally, one should note that the proposed model corresponds to a non-fluctuating target as b is constant. This implies that the necessary bandwidth to scan the field of view is small enough such that the Radar Cross Section (RCS) of the target included in b is not fluctuating. Therefore, the results only apply to LWAs with improved frequency beam scanning velocity, such as LWAs with dielectric or radiofrequency structures filling up the waveguide.

3 Cramer-Rao Bounds for Angular Estimation

The CRB for the estimation of a target direction is now computed in the absence of jamming and for a known delay. In this case, the signal model corresponding to the LWA simplifies to a CSM, where the received signal is corrupted solely by thermal noise. The received signal at the LWA port can therefore be described by the following distribution:

$$\begin{aligned} r(t) &\sim \mathcal{CN}(b\,\mu(t,u),\, \sigma^2), \\ \mu(t,u) &= \left(s * \mathcal{F}^{-1}\{(\boldsymbol{W}^\top(f)\,\boldsymbol{A}(f,u))^2\}\right)\Big|_{t-\tau}. \end{aligned} \tag{8}$$

The CRB of any unbiased estimator of the direction cosine u can be computed by inverting the Fisher Information Matrix (FIM). In the case of a LWA, the well-known Woodward formulae [19], which expresses the FIM as a function of the Hessian of the ambiguity function, cannot be applied. This is because the variations of the radar wavelength significantly impact the formed beam. Therefore, the CRB is obtained here by applying the Slepian-Bangs formulae [17, 3]. As a result,

$$CRB_{\text{LWA}} = \frac{\sigma^2}{2\,|b|^2}\left(\int \left|\frac{\partial\,\mu(t,u)}{\partial\,u}\right|^2 dt\right)^{-1} \tag{9}$$

The derivative of the mean signal value (μ) with respect to the direction cosine (u) of the target is given by

$$\frac{\partial\,\mu(t,u)}{\partial\,u} = s * \mathcal{F}^{-1}\left\{2\,\boldsymbol{W}^\top(f)\,\boldsymbol{A}(f,u)\,\boldsymbol{W}^\top(f)\,\frac{\partial\,\boldsymbol{A}(f,u)}{\partial\,u}\right\}, \tag{10}$$

where [2]

$$\frac{\partial\, \boldsymbol{A}(f,u)}{\partial\, u} \;=\; j\, k_0(f)\, \boldsymbol{p} \cdot \boldsymbol{A}(f,u). \tag{11}$$

4 Maximum Likelihood Estimation

We recall that the Maximum Likelihood Estimator (MLE) is asymptotically efficient in the case of the CSM [14] and its Mean Squared Error (MSE) is therefore expected to converge to the CRB as the Signal-to-Noise Ratio (SNR) increases. Consequently, the MLE can be used to verify the calculation of the CRB. Its expression corresponds to the search of the parameters, namely the direction of the target in our case, which maximize the likelihood function. In more details,

$$\hat{u} \;=\; \underset{u}{\text{argmax}} \Bigg\{ \int \left| r(t) - b\left(s * \mathcal{F}^{-1}\{ (\boldsymbol{W}^\top(f)\, \boldsymbol{A}(f,u))^2 \} \right)\Big|_{t-\tau} \right|^2 dt \Bigg\}. \tag{12}$$

5 Numerical Application

In the numerical application, a LWA operating at X-band is considered. The antenna is a waveguide filled with a dielectric material, such as aluminum oxide, with a relative permittivity $\epsilon_r = 10$. The antenna has a height of $h = 4.8$ mm and features $N = 32$ slots uniformly spaced every $\Delta p = 12$ mm. The leakage rate is taken as $\alpha = 0.015\, k_0(f_0)$ with $f_0 = 10$ GHz.

The radar emits a Linearly Frequency Modulated (LFM) waveform with constant amplitude, also known as a chirp, over a period of time T. The transmitted bandwidth is 300 MHz centered at f_0. As a result, the radar scans an angular domain from 22.8° to 58.4° as illustrated in figure 2.

The Mean Squared Error (MSE) for estimating the direction of a single target located at 45° is then computed using Monte-Carlo simulations (1 000 runs per point). The results are shown in figure 3, where the MSE is compared to the CRB whose expression is given by equation 9. As expected, the MSE corresponds to the variance of a uniform distribution at low SNR. After the well-known threshold effect [15], the MSE asymptotically converges to the CRB as the SNR increases, which validates our calculation.

Additionally, the CRB for estimating the direction is also presented for a radar using a Uniform Linear Array (ULA) [5], to provide a point of comparison with a more conventional antenna technology. For this comparison, the ULA is configured to have the same radiation efficiency as the LWA, and a similar structure but where each slot is connected to a receive channel, resulting in $N = 32$ channels. Consequently, the radiated power by each channel is $1/N$ of the total radiated power by the LWA. Moreover, the radar illuminates the same angular domain with both the ULA or the LWA. Thus, the averaged transmit gain radiated by the ULA in the direction of the target is

2. $\boldsymbol{x} \cdot \boldsymbol{y}$ represents the element-wise product between two matrices $\boldsymbol{x}$ and $\boldsymbol{y}$, also known as the Hadamard product.

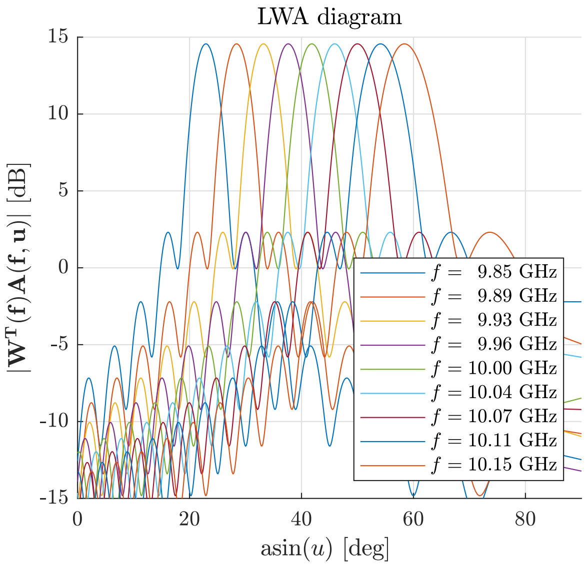

Figure 2 – Antenna diagram as function of the direction and parametrized by the operating frequency.

$$G_t \;\approx\; \frac{1}{T} \int_{\sin 22.8°}^{\sin 58.4°} \boldsymbol{A}^{\mathsf{H}}(f_0, v)\, \boldsymbol{A}(f_0,u)\, dv. \tag{13}$$

As a result, the signal model corresponding to the ULA becomes

$$\boldsymbol{r}(t) \;\sim\; \mathcal{CN}\left(b\, \sqrt{N^{-1}\, G_t\, T}\, \boldsymbol{A}(f_0,u),\, \sigma^2 \boldsymbol{Id} \right), \tag{14}$$

Finally, applying the Slepian-Bangs formulae [17, 3] to the model above yields to the corresponding CRB, which is

$$CRB_{\,\text{ULA}} \;=\; \frac{N\,\sigma^2}{2\, G_t\, T\, |b|^2} \left(\left\| \frac{\partial\, \boldsymbol{A}(f_0,u)}{\partial\, u} \right\|^2 \right)^{-1}. \tag{15}$$

The numerical analysis presented in figure 3 indicates that the performance achieved using the specified LWA is degraded by 0.5 dB compared to that achieved with the specified ULA when estimating the direction of a target. This outcome is not surprising, as the specified LWA parameters allow it to form a beam pattern resembling the cardinal sine function (cf. figure 2), similar to what can be achieved with a ULA.

6 Conclusion

The CRB for the estimation of the direction of a non-fluctuating target has been established for a radar with a single transmit and receive channel feeding a LWA, in the absence of jamming and for a known delay. Comparing this CRB with that obtained for a ULA-based radar where the phase of each radiating element is individually controlled, motivates further exploration into low-tech radar systems using LWAs.

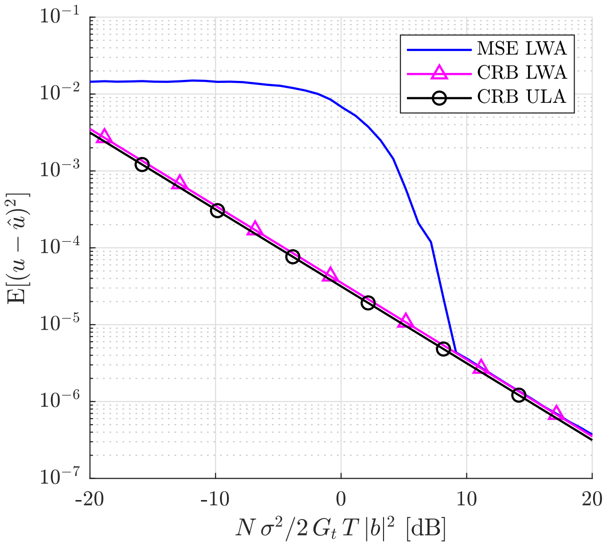

Figure 3 – MSE for the estimation of the direction of a single target with a LWA-based radar or a more conventional ULA-based radar.

To develop such a radar, the CRB for joint delay, Doppler and angular estimation should be studied to predict the asymptotical performance of the resulting radar matched filter. This CRB will also elucidate the price to pay when leveraging the simplicity offered by LWAs. Specifically, using a LWA is expected to involve a compromise between range precision (or resolution) and angular precision (or resolution) as narrowing the formed beam decreases the frequency bandwidth of the radar signal in the direction of a target.

Finally, the performance should also be studied in the presence of a jammer that could color the noise. For example, a white noise jammer would be filtered by the LWA transfer function based on its direction. Consequently, the spectrum of the received noise would not be uniform across all frequencies.

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