Capacity of multi-antenna time-dispersive channels subject to fading

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Résumé – L'effet de dispersion temporelle sur la capacité des canaux avec évanouissement est étudiée quand différentes structures multi-antennes sont utilisées. Nous montrons comment l'exploitation de diversité temporelle permet de combattre l'évanouissement du canal. En plus, l'impact de la distribution optimale de puissance à l'émetteur (la solution *water filling*) sur la capacité des canaux sélectifs en fréquence est étudié.

Abstract – The effect of time-delay dispersion on the capacity of fading channels is studied when different multiple antenna structures are employed. It is shown how the exploitation of *time diversity* can help in combating signal fading. Also, the impact of optimal power allotment at transmitter (*water filling* solution) on the capacity of delay-dispersive channels is discussed.

1 Introduction

It is well known that channel dispersion can be regarded as another source of diversity, and can increase the channel capacity under fading conditions. Evidently, to profit from this property, channel estimation/equalization should be performed at receiver.[1] In this paper, we are interested to see the impact of channel dispersion on the channel capacity of multi-antenna systems which are subject to Rayleigh fading.
One interesting case is the Multiple inputs Multiple outputs (MIMO) structure. Previous works have shown that wireless transceivers using antenna arrays at both sides of the radio link can achieve very large capacities, provided that the propagation medium is rich scattering [1]-[4]. MIMO channels have been firstly envisaged to be used in nondispersive media, i.e., media satisfying flat fading conditions, or to be used under OFDM (orthogonal frequency division multiplexing) signaling [2, 3]. That is because the equalization of MIMO channels is a very complicated task, and adds a non-negligible complexity to the system. Recent works have proposed the use of channel shortening filters in order to facilitate the task of channel equalization [5].

In this paper, we consider MIMO and SIMO (single input multiple outputs) structures under frequency selective channel conditions. Another subject to be treated is to study the impact of optimal power distribution at transmitter on the channel capacity. Previous studies have shown that in flat channels the optimal solution is of interest for small number of antennas and under low SNR [6].

[1]Note that assuming perfect equalization, the capacity of flat and time dispersive *deterministic* channels are the same.

The paper is organized as follows. In Section 2, we will state our basic assumptions, and provide a channel model for the general MIMO case. Capacity expressions will be given in Section 3 for this case, which can be used for other simplified structures too. After providing two models for channel dispersion in Section 4, we will study the impact of delay dispersion on capacity in Section 5. In Section 6, the increase in capacity when performing optimal power allotment at the transmitter is studied. Conclusions are given in Section 7.

2 Assumptions and channel model

Consider a general MIMO channel where M_T and M_R antenna elements are used in Tx (transmitter) and Rx (receiver), respectively. No beam forming is considered for the antenna arrays, and antenna elements' patterns are considered as omni-directional with unity gain. Also, the signal attenuation corresponding to all Tx-Rx antenna pairs is considered the same and equal to 1. It is assumed that there are many reflectors in the propagation medium, notably around both Tx and Rx. Channel is considered as quasi-static, that is, almost constant during one or several bursts.
Channel model is given for the MIMO case; deriving the corresponding models for other structures is straight forward. In the case of flat channel, we can define a channel matrix, $\boldsymbol{H}$ ($M_R \times M_T$), with entries h_{ij}, the equivalent baseband channel impulse response from jth transmit to ith receive antennas. h_{ij} are considered as normalized circularly symmetric complex Gaussian random processes.[2]
In the case of frequency selective channel, a space-time channel model is considered.[3] Consider the vector of trans-

[2]This channel model is usually referred to as the i.i.d. channel.
[3]Note that in OFDM signaling, since each subband is narrow enough, the channel can be decomposed to a series of flat channels

mitted symbols as in equation (1), where a block of N transmit symbols is considered for each transmit antenna.

$$\boldsymbol{x} = [x_1(1) \dots x_1(N) \quad \cdots \quad x_{M_T}(1) \dots x_{M_T}(N)]^T \qquad (1)$$

where $.^T$ denotes the vector transpose. Equation (2) describes channel input-output relationship.

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{y} + \boldsymbol{n} \qquad (2)$$

where $\boldsymbol{n}$ is the equivalent baseband receiver noise considered as zero-mean white Gaussian with variance of σ^2.

Keeping in mind the channel model for the flat case, each h_{ij} in this model should be replaced by a Toeplitz matrix $\boldsymbol{H}_{ij}$ of dimension $(N + \upsilon_c) \times N$, as shown in (3), with υ_c the channel dispersion length ($N \gg \upsilon_c$). The same υ_c is considered for all Tx/Rx antenna pairs.
The total spatio-temporal channel matrix $\boldsymbol{H}$ will be of dimension $M_R(N + \upsilon_c) \times M_T N$. As explained later in Section 4, $h_{ij}(k)$ are assumed to be independent.

$$\boldsymbol{H}_{ij} = \begin{bmatrix} h_{ij}(0) & 0 & 0 & \cdots & 0 \\ h_{ij}(1) & h_{ij}(0) & 0 & \cdots & \\ \vdots & \ddots & & & \\ h_{ij}(\upsilon_c) & \cdots & h_{ij}(0) & 0 & \vdots \\ 0 & h_{ij}(\upsilon_c) & & 0 & \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & h_{ij}(\upsilon_c) & \dots & h_{ij}(0) \\ \vdots & & & \ddots & \vdots \\ 0 & & \cdots & 0 & h_{ij}(\upsilon_c) \end{bmatrix} \qquad (3)$$

3 Capacity expressions

It is assumed that the channel is known at Rx. The capacity is defined under the constraint that the total transmit power over N sample time is equal to NP_T. Note that the capacity of a discrete-time Gaussian channel using a per-block average energy input constraint is equal to the capacity when using the per-symbol average energy constraint [8]. We give the capacity expressions when the channel state information (CSI) is provided or not at Tx.

A. No CSI available at transmitter
When the transmitter does not know the channel, the logical way for power allotment is to distribute the available power equally over space and time. In this case, the average received power from each of transmit antennas at the receiver array at each sample time is $\frac{P_T}{M_T}$, which is considered the same for all receive antennas. The capacity is given by

$$C = \frac{1}{N} \sum_{i=1}^{NM} \log_2 \left(1 + \frac{P_T}{M_T \sigma^2} \, \lambda_{H,i}^2 \right) \; bps/Hz \qquad (4)$$

where $\lambda_{H,i}$ are the singular values of $\boldsymbol{H}$ and

$$M = \min(M_T, M_R) \; . \qquad (5)$$

B. CSI is provided at transmitter
In this case the transmitter can allot the available power (over time and the antenna elements) with an optimal manner, in order to achieve maximum capacity. This optimal power allotment is usually referred to as *water filling* (WF). The WF capacity is given by

$$C = \frac{1}{N} \sum_{i=1}^{NM} \log_2 \left(1 + \frac{\lambda_{X,i}}{\sigma^2} \lambda_{H,i}^2 \right) \; bps/Hz \qquad (6)$$

$$\lambda_{X,i} = \left(\varphi - \frac{\sigma^2}{\lambda_{H,i}^2} \right)^+ \qquad (7)$$

where, $(s)^+ = s$ if $s > 0$ and 0 otherwise. $\lambda_{X,i}$ are the eigenvalues of the autocorrelation matrix of $\boldsymbol{x}$, $\boldsymbol{R}_X$ ($NM_T \times NM_T$). φ is chosen so as to satisfy the per-block transmit power constraint,

$$\sum_{i=1}^{MN} \lambda_{X,i} = NP_T \qquad (8)$$

The optimum $\boldsymbol{R}_X$ to achieve the capacity of equation (6) is

$$\boldsymbol{R}_X = \boldsymbol{V}_H \boldsymbol{P}_X \boldsymbol{V}_H^\dagger \qquad (9)$$

where † denotes the transpose-conjugate transformation and $\boldsymbol{V}_H$ ($NM_T \times NM_T$) is the unitary matrix resulting from the SVD (Singular Value Decomposition) of $\boldsymbol{H}$,

$$\boldsymbol{H} = \boldsymbol{U}_H \boldsymbol{\Lambda}_H \boldsymbol{V}_H^\dagger \; . \qquad (10)$$

$\boldsymbol{P}_X$ is a diagonal matrix with the diagonal entries of $\lambda_{X,i}$ in descending order. $\boldsymbol{R}_X$ determines the appropriate (source) coding of the transmitted data which permits achieving the capacity bound of (6).

4 Channel dispersion modeling

Let $h(\tau)$ be a SISO (single input single output) channel[4] impulse response with the width of τ_c. Assuming *uncorrelated scattering* conditions, the ensemble of $h(\tau)$ are characterized by a *delay power spectrum* $P_h(\tau)$, defined as follows [9].

$$P_h(\tau) = \frac{1}{2} E\{h(\tau)h^*(\tau)\} \; ; \; \int P_h(\tau) d\tau = 1 \qquad (11)$$

for which the capacity expressions can be calculated [7]. Meanwhile, each subband can profit from channel time dispersion due to the particular signaling scheme. This is not the case in our analysis, however; we assume that signal transmission is performed on the entire frequency band and no frequency division is performed. Our results can somehow regarded as an upper bound for the OFDM case.

[4] Corresponding to each Tx-Rx antenna pair in MIMO channel

where $E\{.\}$ denotes the expected value. Regarding our discrete-time model, we define equivalently

$$P_h(n) = A\ E\{h(n)h^*(n)\}\ ;\ \sum_n P_h(n) = 1 \qquad (12)$$

A is determined so as to satisfy the normalization constraint. We will consider for $P_h(n)$ two cases of one-sided exponential and the equi-power multi-spikes, defined according to (13) and (14), respectively.

$$P_{h1}(n) = A\ \exp\left(-\frac{n}{n_0}\right)\ ;\ n = 0, ..., v_c \qquad (13)$$

$$P_{h2}(n) = \frac{1}{v_c + 1}\sum_{m=0}^{v_c} \delta(n-m) \qquad (14)$$

In equation (13), $n_0 = \frac{\tau_0}{T}$ with τ_0 the rms delay spread of channel.[5] For a given τ_0, v_c the channel dispersion length, is determined such that $P_{h1}(v_c+1)$ be a negligible value, say less than 0.01 (as we will consider).

Equation (13) approximates spectra that have been measured in urban environment [10], while equation (14) considers a simple equi-power multi-Dirac dispersion.
Notice that the capacity given for a multi-ray channel does not depend on the location of rays; in other words, the capacity is the same for multi-ray channels with rays spaced apart with any distance. However, regarding channel equalization, the receiver complexity may not be the same. For example, a double ray channel with rays separated with one time symbol is easier to equalize than when the rays are separated by several time symbols. However, since we consider the capacity bounds (which correspond to ideally equalized channel performances), these two cases result in the same capacity.

We assume that $P_h(n)$ is constant and does not change with time. Also, $P_h(n)$ is considered the same for all Tx/Rx antenna pairs. For a given $P_h(n)$, considering the discrete-time channel impulse response as $[h(0)\ ...\ h(v_c)]$, the variance of real and imaginary parts of $h(i)$ equals to $P_h(i)/2$, for $i = 0, ..., v_c$. We will assume independent $h(i)$, which is a reasonable assumption because for different i, $h(i)$ corresponds to different clusters of scatterers over which the scattering can be assumed to take place independently.

5 Impact of dispersion on the unknown CSI capacity

All results in this paper are given for the *outage probability* of $P_{out} = 0.01$ and SNR (signal to noise ratio) is defined as P_T/σ^2. Fig.1 contrasts capacities of SIMO (with $M_R = 2, 4$) and MIMO ($M_T = M_R = 2$) systems for two cases of exponential and multi-Dirac dispersions, in terms of the normalized rms delay spread τ_0/T and channel dispersion length v_c, respectively. $v_c = 0$ and $\tau_0/T = 0$

[5]The square root of the second central moment of $P_h(\tau)$.

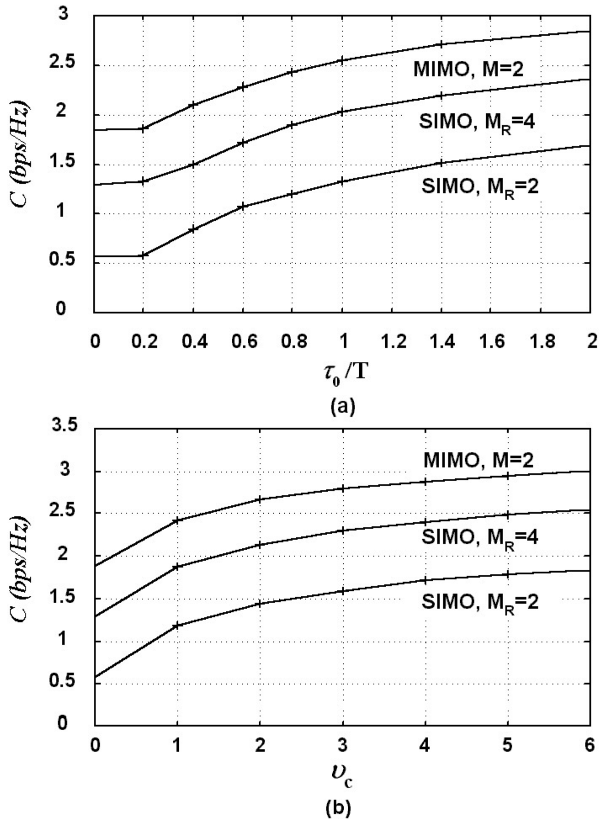

FIG. 1: Capacity of exponential (a) and multi-Dirac (b) dispersive channels in terms of normalized rms delay spread and dispersion length, respectively; $M_T = M_R = M$ for MIMO; $P_{out} = 0.01$, SNR=5 dB

correspond to the flat channel case. It is seen that the capacity increases with dispersion length, and this increase is of most importance for a double-Dirac channel compared to a flat channel. Notice that the total received power at Rx is the same for every dispersion length, because of the normalization made on $P_h(n)$. For Fig.1 as well as other simulation results obtained for dispersive channels, $N = 10\ v_c$ is taken.

As shown for instance in Fig.2, for the case of a double-Dirac channel, the gain in capacity increases with increase in SNR. Our results also correspond with those of Clark *et al.* [11] who has studied the effect of channel dispersion on decreasing the error probability in a QPSK signaling system, considering matched filter bounds, i.e., assuming ideal equalization.

6 Increase in capacity using water filling

Fig.3 shows curves of WF-gain and the normalized WF-gain (to no-WF capacity) versus SNR for a double-Dirac dispersive channel with $M_T = M_R = 2, 4$. Similar to the flat channel case [6], WF gain decreases with increase in SNR. Also, it is less considerable for large M, since the MIMO capacity is actually large for large number of antennas. Comparing Fig.3 with the results of [6], WF gain

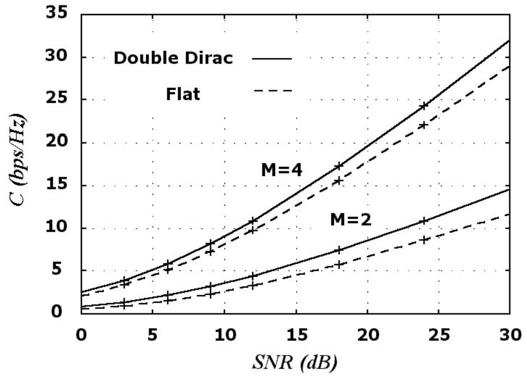

FIG. 2: Capacity of a double-Dirac dispersive channel compared to that of a flat channel, $M_T = M_R = M$, $P_{out} = 0.01$

is less important in the dispersive channel case. It was expected, because by exploiting time diversity we can reduce the effect of channel fading on signal detection, and hence, less WF-gain is achieved for a dispersive channel than for a flat channel.
Notice that to do WF, the CSI must be provided for the transmitter. So, a reverse link should be established from the receiver (which estimates and tracks the channel variations) to the transmitter. Therefore, there seems little interest to perform this optimal solution in the case of dispersive channel for high SNR and large number of antennas, regarding the complexity added to the system and the non-considerable gain in capacity.

7 Conclusion

After introducing a delay dispersive MIMO channel model, capacity expressions were presented when CSI is or not provided at transmitter.
It was shown that channel dispersion can be regarded as another source of diversity and can be exploited –by performing channel equalization– to combat fading, and hence to increase the capacity.
On the other hand, WF solution in delay dispersive channels was considered; it was shown that regarding the complexity of the realization of this solution, it would not be of interest, especially for large number of antennas and high SNR.

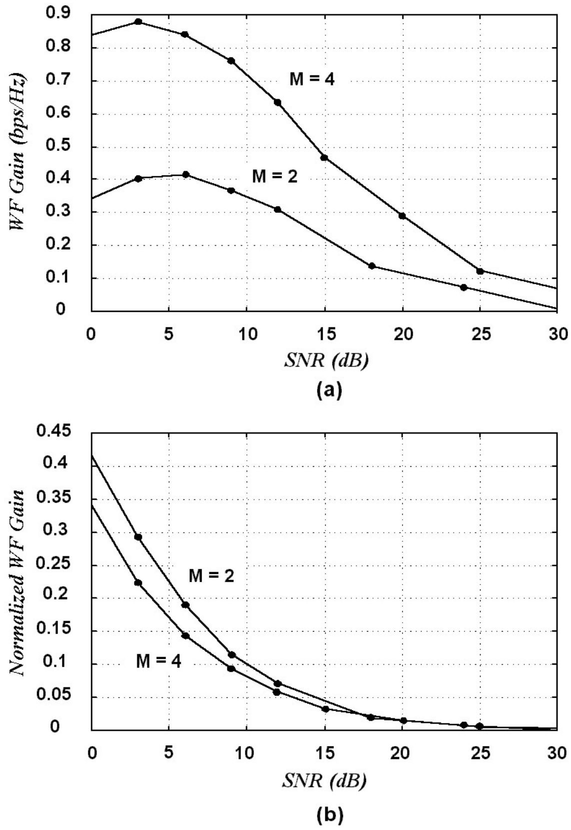

FIG. 3: WF-gain for a double-Dirac dispersive Rayleigh MIMO channel with $M_T = M_R = M$; $P_{out} = 0.01$

References

[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol.6, 1998, pp.311-335

[2] G.G. Raleigh and J.M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Transactions on Communications*, vol. COM-46, No.3, Mar. 1998, pp.357-366

[3] G.G. Raleigh and V.K. Jones, "Multivariate modulation and coding for wireless communication," *IEEE Journal on Selected Areas in Communications*, vol. SAC-17, No.5, May 1999, pp.851-866

[4] M.A. Khalighi, J.M. Brossier, K. Raoof, and G. Jourdain, "On Capacity of wireless communication systems employing antenna arrays," submitted to the Journal of *Wireless Personal Communications*, Oct. 2000

[5] N. Al-Dhahir, "FIR Channel-Shortening Equalizers for MIMO ISI Channels," *IEEE Transactions on Communications*, vol.49, No.2, Feb. 2001 ,pp.213-218

[6] M.A. Khalighi, J.M. Brossier, G. Jourdain, and K. Raoof, "Water Filling Capacity of Rayleigh MIMO channels," to be appeared in the Proceedings of PIMRC 2001, 30 Sept. - 3 Oct. 2001, San Diego, CA, USA

[7] H.Bölcskei, D. Gesbert, and A.J. Paularj, "On the capacity of OFDM-based multi-antenna systems," *In proceedings of ICASSP 2000*, Istanbul, Turkey, June 2000, pp.2569-2572

[8] W. Hirt and J.L. Massey, "Capacity of the discrete-time Gaussian channel with intersymbol interference," *IEEE Transactions on Information Theory*, vol. IT-34, No.3, May 1988, pp.380-388

[9] G.J. Proakis, Digital Communications, McGraw Hill, second edition, 1989

[10] B. Glance and L.J. Greenstein, "Frequency selective fading effects in digital mobile radio with diversity combining," *IEEE Transactions on Communications*, vol. COM-31, No.5, Sept. 1983, pp.1085-1094

[11] M.V. Clark, L.J. Greenstein, W.K. Kennedy, and M. Shafi, "Matched filter performance bounds for diversity combining receivers in digital mobile radio," *IEEE Transactions on Vehicular Technology*, vol. VT-41, No.4, Nov.1992, pp.356-362