

# Minimum Mean Square Error channel estimates for DS-SSS in the presence of ISI

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**Résumé** – Cet article propose une amélioration de l'estimation de canal lorsque de petits facteurs d'étalements sont utilisés dans les systèmes à étalement de spectre par séquences directes. Nous tenons compte de la structure de l'interférence entre symboles pour construire une estimée optimale selon le critère de minimisation de l'erreur quadratique moyenne. Une amélioration significative des performances est ainsi obtenue pour le récepteur en râteau et l'égaliseur linéaire.

**Abstract** – In this paper, we propose to improve channel estimation quality for DS-SSS when small spreading factors are used. We take into account the intersymbol interference structure to construct an optimal estimate according to the minimum mean square error criterion. We show that significant performance improvement is obtained for both the Rake receiver and the linear chip equalizer.

## 1 INTRODUCTION

The Rake receiver is usually used as the conventional matched filter in Direct Sequence Spread Spectrum Systems (DS-SSS) [1]. Since the performance of such a receiver degrades for small spreading factors [2], the use of a linear chip equalizer followed by a simple correlator was proposed [3-4]. This approach has the advantage of reducing both the intersymbol and the multiuser interference in the downlink of the UMTS FDD mode [5]. In fact, the multiuser interference is due to the multipath channel since the orthogonality of the user signatures.

The InterSymbol Interference (ISI) degrades the performance of the conventional channel estimates which are obtained by correlating and averaging over pilot symbols. In this paper, we propose to take into account the ISI structure to construct a best estimate according to Minimum Mean Square Error (MMSE) criterion. Performance improvement is measured for both the conventional Rake and the chip equalizer receivers.

The outline of this paper is as follows. The next section presents the system model. Section 3 recalls the principle of the Rake and the linear equalizer receivers. Section 4 describes the conventional and the MMSE channel estimators. Section 5 gives some simulation results. Finally, section 6 draws some conclusions.

## 2 SYSTEM MODEL

In the presence of multipath propagation, the received signal at time  $t$  can be written as

$$r(t) = \sum_i d(i) \sum_{l=0}^{L-1} \lambda_l(t) g(t - iT_c - \tau_l(t)) + w(t), \quad (1)$$

where  $d(i) = c(i)s(\lfloor \frac{i}{N} \rfloor)$  is the  $i$ -th chip symbol,  $s(i)$  is the  $i$ -th QPSK data symbol,  $c(i)$  is the spreading code which is assumed to be a Walsh Hadamard one multiplied by the scrambling sequence,  $N$  is the spreading factor,  $T_c$  is the chip period,  $g(t)$  is a square root raised cosine filter with roll-off 0.22,  $w(t)$  is a white gaussian noise with one sided power spectral density  $N_0$ ,  $L$  is the number of paths,  $\lambda_l(t)$  and  $\tau_l(t)$  are respectively the complex amplitude and the delay of the  $l$ -th path.

The received signal can also be written as

$$r(t) = \sum_i d(i) h^i(t - iT_c), \quad (2)$$

where

$$h^i(t) = \sum_{l=0}^{L-1} \lambda_l(t + iT_c) g(t - \tau_l(t + iT_c))$$

In agreement with the Nyquist theorem, the received signal is sampled at twice the chip rate. We put in a vector all the received samples where  $d(i)$  appears:

$$\mathbf{r}(\mathbf{i}) = \begin{pmatrix} r((i - M_1)T_c) \\ r((i - M_1)T_c + T_c/2) \\ \vdots \\ r((i + M_2)T_c) \end{pmatrix} = \mathbf{H}(\mathbf{i})\mathbf{d}(\mathbf{i}) + \mathbf{w}(\mathbf{i}), \quad (3)$$

where  $M_1$  and  $M_2$  define the length of  $h^i(t)$  in multiple of  $T_c$ ,

$$\mathbf{H}(\mathbf{i}) = [\mathbf{h}^{M_1+M_2}(\mathbf{i}), \dots, \mathbf{h}^0(\mathbf{i}), \mathbf{h}_1(\mathbf{i}), \dots, \mathbf{h}_{M_1+M_2}(\mathbf{i})],$$

$$\mathbf{h}^j(\mathbf{i}) = (h^i((j - M_1)T_c), h^i((j - M_1)T_c + T_c/2), \dots \\ \dots, h^i(M_2T_c), \mathbf{0}_{1,2j})^T, \quad 0 \leq j \leq M_1 + M_2,$$

$$\mathbf{h}_j(\mathbf{i}) = (\mathbf{0}_{1,2j}, h^i(-M_1T_c), h^i(-M_1T_c + T_c/2), \dots \\ \dots, h^i((M_2 - j)T_c))^T, \quad 0 \leq j \leq M_1 + M_2,$$

and

$$\mathbf{d}(\mathbf{i}) = (d(i - M_1 - M_2), \dots, d(i), \dots, d(i + M_1 + M_2))^T.$$

### 3 Receivers

The conventional Rake receiver and the linear chip equalizer are described in this section. The Rake receiver estimates the  $i$ -th chip symbol by maximal ratio combining over all paths

$$\hat{d}(i) = \mathbf{h}^0(\mathbf{i})^H \mathbf{r}(\mathbf{i}). \quad (4)$$

In order to reduce the ISI effects, the use of a Linear Minimum Mean Square Error (LMMSE) chip equalizer was suggested [3]. The proposed approach estimates the  $i$ -th chip symbol by using the following decision variable [4]

$$\hat{d}(i) = \mathbf{h}^0(\mathbf{i})^H \left( \mathbf{H}(\mathbf{i})\mathbf{H}(\mathbf{i})^H + \frac{N_0}{2\sigma_d^2} \mathbf{I}_{2(M_1+M_2)+1} \right)^{-1} \mathbf{r}(\mathbf{i}), \quad (5)$$

where  $\sigma_d^2$  is the variance of  $d(i)$ .

### 4 MMSE channel estimates

The performance of the conventional channel estimator which obtains its estimates by correlating and averaging

over pilot symbols, degrades at small spreading factors because of ISI. In this section, we propose to improve channel estimation quality by using the knowledge of the ISI structure. If we suppose that path delays are separated by multiples of the chip period, the conventional channel estimates are given by

$$\hat{\boldsymbol{\lambda}} = (\hat{\lambda}_0, \dots, \hat{\lambda}_{L-1})^T = \mathbf{M}\boldsymbol{\lambda} + \mathbf{n}, \quad (6)$$

where

$$\mathbf{M} = [M_{ij}]_{0 \leq i, j \leq L-1},$$

$$M_{ii} = 1, \quad 0 \leq i \leq L-1,$$

$$M_{ij} = \sum_{p=0}^{P-1} \frac{s_p^*}{|s_p|^2} \left[ s_p \sum_{n=pN}^{(p+1)N-1-\tau_{ij}} c_n^* c_{n+\tau_{ij}} \right. \\ \left. + s_{p+1} \sum_{n=(p+1)N-\tau_{ij}}^{(p+1)N-1} c_n^* c_{n+\tau_{ij}} \right], \quad \text{if } \tau_i > \tau_j$$

$$M_{ij} = \sum_{p=0}^{P-1} \frac{s_p^*}{|s_p|^2} \left[ s_p \sum_{n=pN-\tau_{ij}}^{(p+1)N-1} c_n^* c_{n+\tau_{ij}} \right. \\ \left. + s_{p-1} \sum_{n=pN}^{pN-1-\tau_{ij}} c_n^* c_{n+\tau_{ij}} \right], \quad \text{if } \tau_i < \tau_j$$

$P$  is the number of pilot symbols,  $\boldsymbol{\lambda} = (\lambda_0, \dots, \lambda_{L-1})^T$ ,  $\tau_{ij} = (\tau_i - \tau_j)/T_c$ ,  $\mathbf{n}$  is channel estimation noise which is assumed to have a variance equal to  $N_0/E_{pilot}$  and  $E_{pilot}$  is the energy of the pilot symbols.

MMSE channel estimates are given by

$$\hat{\boldsymbol{\lambda}}^{\text{MMSE}} = \mathbf{L}^H \hat{\boldsymbol{\lambda}}, \quad (7)$$

where

$$\mathbf{L} = \text{argmin} \left\| \hat{\boldsymbol{\lambda}}^{\text{MMSE}} - \boldsymbol{\lambda} \right\|.$$

By using (6), we deduce

$$\hat{\boldsymbol{\lambda}}^{\text{MMSE}} = \mathbf{M}^H \left( \mathbf{M}\mathbf{M}^H + \frac{N_0}{2E_{pilot}} \mathbf{I}_L \right)^{-1} \hat{\boldsymbol{\lambda}}. \quad (8)$$

## 5 Simulation results

Figures 1 and 2 show simulation results of the conventional Rake receiver and the LMMSE chip equalizer. The simulated channel is a Rayleigh fading one with four paths of equal average power and delays separated by the chip period. The spreading factor is equal to four and channel estimation is obtained from four pilot symbols. We see that, at high signal to noise ratios, the use of MMSE channel estimates gives respectively 1.5 dB and 2.5 dB performance improvement for the Rake receiver and the LMMSE chip equalizer. Moreover, we notice that the latter receiver is more sensitive to channel estimation quality than the former one.

## 6 Conclusion

In this paper, we have proposed an improvement of channel estimation quality for direct sequence spread spectrum systems when small spreading factors are used. MMSE channel estimates are derived by using the knowledge of the intersymbol interference structure. Significant performance improvement for the Rake receiver and the linear chip equalizer was obtained when this improved channel estimation is used.

## Références

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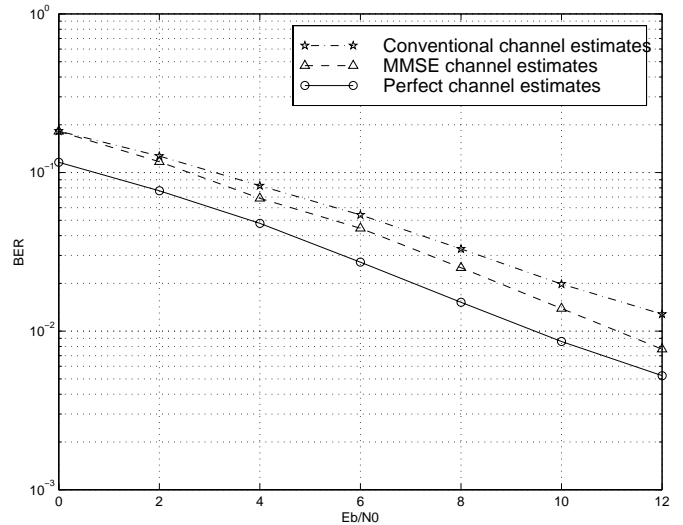


FIG. 1: Rake receiver performance.

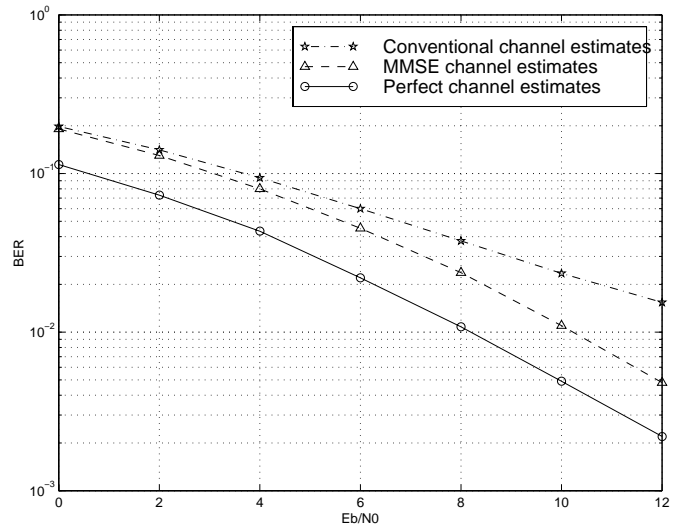


FIG. 2: LMMSE chip equalizer performance.