Optimal Constellations for Bit-Interleaved Turbo-Coded Modulations

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Résumé – Les performances en terme de taux d'erreurs binaires des modulations turbo-codées avec entrelacement au niveau du bit dépendent fortement de la constellation à M états employée. Lorsque le paramètre M est de la forme $M = 2^{2p}$ où p désigne un entier, nous mettons en évidence le fait que les modulations d'amplitude en quadrature (MAQ) à constellation rectangulaire sont très attractives pour la conception de puissantes modulations turbo-codées avec entrelacement au niveau du bit. Nous étudions également l'influence de l'entrelacement et du "mapping" sur les performances de ce type de systèmes lorsque des modulations MAQ sont utilisées.

Abstract - The bit error rate performance of bit-interleaved turbo-coded modulation strongly depends on the M-ary signal constellation, which is employed in association with the binary turbo code. When the parameter M is in the form $M = 2^{2p}$, where p is an integer, we show that classical rectangular QAM modulations are particularly attractive for designing power efficient bit-interleaved turbo-coded modulations. We also investigate the influence of the interleaving and the mapping on the performance of BITCM's based on such modulation schemes.

1. Introduction

Introduced in 1993, turbo codes can achieve high coding gains close to the Shannon limit [1]. They are in essence made up of two recursive and systematic convolutional (RSC) codes, also called constituent codes, connected in parallel and separated by a pseudo-random interleaving. To design bandwidth-efficient coding schemes, some successful attempts have been undertaken to combine turbo codes with multilevel modulations. Two different approaches have mainly been suggested.

The first approach consists of using a turbo code in a bit-interleaved coded modulation scheme [2, 3]. The coding scheme thus obtained is termed "bit-interleaved turbo-coded modulation" (BITCM). Basically, BITCM is a serial concatenation of binary turbo coding, bit-by-bit interleaving, and high order modulation [4]. The major characteristic of BITCM is that turbo coding and modulation are optimized separately. Actually, the BITCM approach is so simple and flexible (the same chip can be used to obtain a wide range of spectral efficiencies) that it is sometimes referred as a "pragmatic approach".

In the second approach, the trellis-coded modulation principles [5] are applied to turbo codes [6, 7]. Unlike BITCM, turbo coding and modulation functions are then considered as a single entity, and therefore jointly optimized. It is suggested that this second approach is superior in terms of bit error rate (BER) performance at the decoder output. However, it appears that the actual performance difference between both approaches is quite negligible [3, 8]. Despite their simplicity and flexibility, BITCM's can thus be considered as very power-efficient. For instance, they have been shown to perform very close to the capacity limit for various spectral efficiencies, over both AWGN and Rayleigh channels (see, e.g., [3, 8, 9]). This is why they are so attractive for many band-limited applications.

Until now, most studies concerning BITCM have been done assuming that the modulation is a rectangular M-QAM. However, some signal sets have been reported to be more power-efficient than rectangular constellations. It could therefore be beneficial to employ these signal sets in place of the rectangular ones so as to improve BER performance of BITCM's.

In this paper, we investigate this point, and show that, when the number of states M of the modulation is in the form $M = 4^p$ (p is an integer), rectangular QAM's are, among all known signal sets studied so far, the most attractive ones for designing power-efficient BITCM's. This justifies the excellent error-correction capabilities of BITCM's based on such signal sets. We also investigate the influence of the mapping and the interleaving on performance of BITCM's using rectangular QAM's. Both AWGN and memoryless Rayleigh fading channels with perfect channel state information (CSI) are assumed throughout the paper.

2. Structure of BITCM transmitter and receiver

The block diagram of a BITCM transmitter is depicted in Fig.1. This transmitter is made up of an M-state ($M = 2^m$)

modulator (MOD) combined with a rate R encoder built from a standard rate-1/3 turbo encoder (ENC) by puncturing some redundant bits.

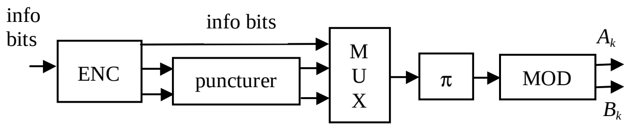

Fig. 1 – Structure of the generic BITCM transmitter.

To obtain uncorrelated samples at the turbo decoder input, an interleaver π is inserted between the puncturer and the modulator. At time k, each signal point of the M-ary constellation is represented by a pair of real-valued symbols $\{A_k, B_k\}$ coded by a set $\{u_{k,i}\}$, $i \in \{1\ldots m\}$, of m bits according to Gray or *quasi*-Gray mapping. It is well-known that, with such mappings, the bits $u_{k,i}$ are not equally protected [2, 3].

In a set $\{u_{k,i}\}$, some bits represent information bits, while the others correspond to redundant bits. Our simulations have indicated that the best BER performance is obtained when the information bits are associated with the most protected bits in the set $\{u_{k,i}\}$. In essence, the suboptimal iterative decoding algorithm (the "turbo" algorithm) relies heavily on the information bits. This is why this decoding algorithm starts being efficient mainly when the BER on the information sequence at the decoder input reaches a certain level. In other words, the signal-to-noise ratio (SNR) at which the decoding algorithm starts performing well depends mainly on the reliability of the information sequence, and is less influenced by that of the redundant sequence. Thus, to ensure that this SNR is as low as possible, we offer a maximum protection to information bits. This will be illustrated later by some simulation results.

Assuming coherent detection, the BITCM receiver acquires, at time k, two samples X_k and Y_k. Over fading channels with perfect CSI, this receiver knows the values taken by the fading samples $\alpha_{k,1}$ and $\alpha_{k,2}$ associated with X_k and Y_k, respectively. AWGN channels can be seen as fading channels for which $\alpha_{k,1} = \alpha_{k,2} = 1$.

The structure of a generic BITCM receiver is depicted in Fig. 2. From samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$, the logarithm of likelihood ratio (LLR) $\Lambda(u_{k,i})$ associated with each bit $u_{k,i}$, $i \in \{1\ldots m\}$, is computed and used as a relevant soft-decision by the turbo decoder. The LLR's $\Lambda(u_{k,i})$ are obtained using the relation, for $i \in \{1\ldots m\}$:

$$\Lambda(u_{k,i}) = K \log \frac{P(u_{k,i} = 1 \mid X_k, Y_k, \alpha_{k,1}, \alpha_{k,2})}{P(u_{k,i} = 0 \mid X_k, Y_k, \alpha_{k,1}, \alpha_{k,2})}, \qquad (1)$$

where K is a constant, and $P(u_{k,i} = j \mid X_k, Y_k, \alpha_{k,1}, \alpha_{k,2})$ denotes the probability that $u_{k,i} = j$ given X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$. At time k, the m LLR's $\Lambda(u_{k,i})$ are correlated since they are obtained from the same samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$. It is thus necessary to use a deinterleaver π^{-1}, associated with the interleaver π used at the transmitter side, which suppresses correlation between samples $\Lambda(u_{k,i})$ and thus ensures an efficient turbo decoding. Actually, we have noticed that the suppression of this interleaving leads only to a slight degradation of the BER at the turbo decoder output. Finally, the depuncturer replaces the punctured bits with zero (neutral) values to provide the standard turbo decoder with three samples at each time k.

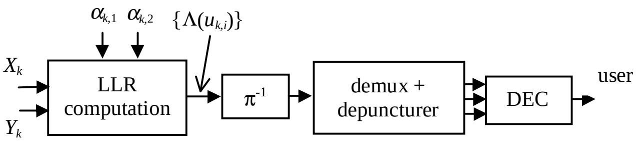

Fig. 2 – BITCM receiver structure.

3. Constellations for designing power-efficient BITCM's

BITCM's based on rectangular QAM signal sets have an excellent error-correcting capability mainly because turbo codes and such modulation schemes are inherently very well matched. It can be justified as follows.

It is well known that turbo codes do not have a large free distance, but they possess a very small average number of nearest neighbor code-words (provided that the size of the pseudo-random interleaving is large enough) [10]. This is why turbo codes achieve excellent BER performance at low SNR. For example, obtaining a BER of 10^{-5} after decoding with the rate-1/2 turbo code proposed in [1] only requires a SNR of 0.7 dB, i.e. a BER of 0.14 at the channel output (BPSK modulation and AWGN channel are assumed). Hence, in order to design a powerful BITCM at moderate BER's, it is necessary to employ a modulation scheme, which is optimized for operation at low SNR.

It can be shown that this optimization mainly consists of minimizing the average number of nearest neighbors of the modulation, and can simply be achieved by using Gray mapping. This shows the profound impact of the mapping on BITCM's performance. The other parameters of the modulation, such as for example the minimum Euclidean distance between signals, play a much less important role in BITCM. It is thus obvious that the most suitable modulation schemes for designing BITCM's are those for which Gray mapping is possible, i.e., in particular rectangular M-QAM modulations.

Some constellations have been reported to perform better than the rectangular ones, for both low and high SNR regions, over both AWGN and fading channels (see, e.g., [11- 13]). They could therefore be of interest for the design of BITCM's. So far, to the best of the authors' knowledge, the comparison between different constellations has always been performed by evaluating their performance in terms of symbol error rate (SER). However, the key parameter in BITCM is the BER of the modulation scheme rather than its SER, since the standard turbo decoder operates from estimates of bits $u_{k,i}$, and not from estimates of symbols A_k

and B_k. As their irregular structure makes Gray mapping impossible, the 'best' constellations do not possess an optimal number of nearest neighbors. This is why their BER performance at low SNR is actually not as good as that of the equivalent rectangular constellations. This makes them unsuitable for designing BITCM schemes.

To illustrate this, consider the example of the 16-state constellation (1, 5, 10) depicted in Fig. 3. We have noticed that this constellation achieves excellent SER performance at low SNR over both AWGN and fading channels [11-13]. It could therefore be beneficial to replace the classical rectangular 16-QAM with the (1, 5, 10) constellation in some BITCM schemes. As Gray mapping is not applicable to (1, 5, 10), we have to use in this case a *quasi*-Gray mapping, i.e., a correspondence between bits $u_{k,i}$ and pairs $\{A_k, B_k\}$ which is as close to Gray mapping as possible. This quasi-Gray mapping leads to a non-optimal average number of nearest neighbors. We indicate in Fig. 3 the best quasi-Gray mapping that we have found. Computer simulation results are given in Fig. 4 for the Rayleigh fading channel.

From Fig. 4, it is seen that the use of (1, 5, 10) in place of 16-QAM improves SER performance. However, at low SNR, the SER performance advantage of (1, 5, 10) over 16-QAM is not sufficient to compensate for the increase in the number of nearest neighbors due to the use of quasi-Gray mapping. Hence, owing to its smaller number of nearest neighbors, rectangular 16-QAM is actually the constellation, which achieves the best BER performance at low SNR, and therefore remains the most attractive for the design of BITCM's. Although not displayed here because of a lack of space, the simulation results obtained on AWGN channels lead exactly to the same conclusion [14]. We mention that one could discover some new constellations, which outperform the rectangular ones in terms of BER performance at low SNR. Further study concerning this point is necessary.

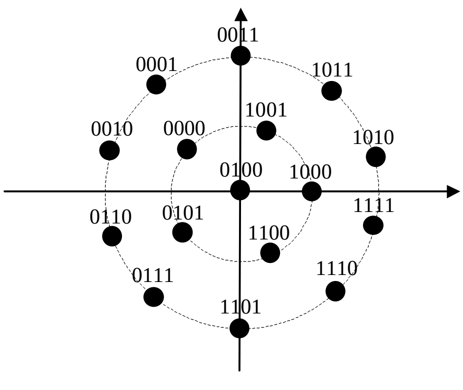

Fig. 3 – Constellation (1, 5, 10). In our simulations, the ring ratio (ratio of the radii of the outer and inner rings) is equal to 2. The indicated mapping is a quasi-gray mapping.

To illustrate this, we have depicted in Fig. 5 the BER versus E_b/N_0 for several 2-bits/s/Hz BITCM's made up of a rate-1/2 turbo code combined with both 16-ary signal sets

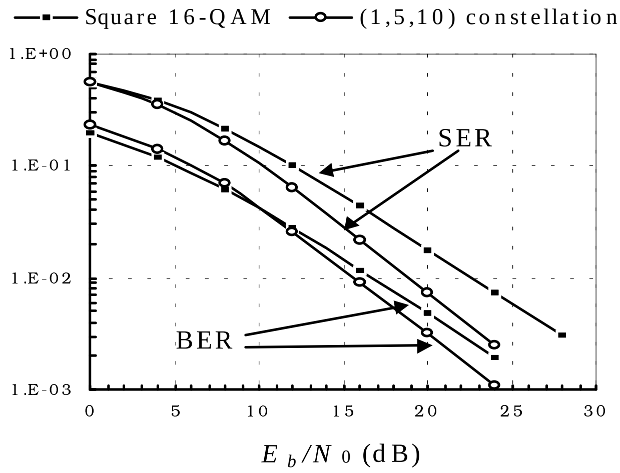

Fig. 4 – Performance comparison between 16-QAM and (1, 5, 10) constellations over the Rayleigh fading channel.

considered previously. Simulations consider a turbo code built from two 16-state RSC codes with polynomials (23, 31) and coding rates 3/5 and 3/4. The pseudo-random interleaving separating these RSC codes has a size of 16384 bits. The MAP algorithm is used for the decoding of each RSC code [1], and turbo decoding is performed in 8 iterations. Note that Gray and quasi-Gray mappings are such that information bits are the most protected bits.

As was expected, Fig. 5 shows that the highest coding gains are obtained when 16-QAM is employed. Over both channels, the SNR advantage of 16-QAM over (1, 5, 10) is approximately equal to 1.25 dB, at a BER of 10^{-5}.

4. Influence of the mapping and the interleaving π/π^{-1} on BER performance of BITCM's

It is also interesting to investigate the actual influence of the mapping and the interleaving function π/π^{-1} on the error-correction capability of BITCM schemes based on QAM signal sets. For this analysis, we consider the 2-bits/s/Hz BITCM made up of 16-QAM and rate-1/2 code. We have checked that all conclusions obtained for this particular configuration are also valid for any other BITCM with different spectral efficiency. In Fig. 6 are depicted the BER curves obtained with this scheme over Rayleigh fading channels when:

- interleaving is used and maximum protection is given to information bits in the Gray mapping ('With int., info bits') ;
- no interleaving is utilized and the information bits are the most protected ones ('Without int., info bits') ;
- interleaving is employed and Gray mapping is such that maximum protection is given to redundant bits ('With int., redundant bits').

These results show that the suppression of the interleaving function π/π^{-1} leads only to a performance degradation of 0.1 dB at a BER of 10^{-5}, which is much less

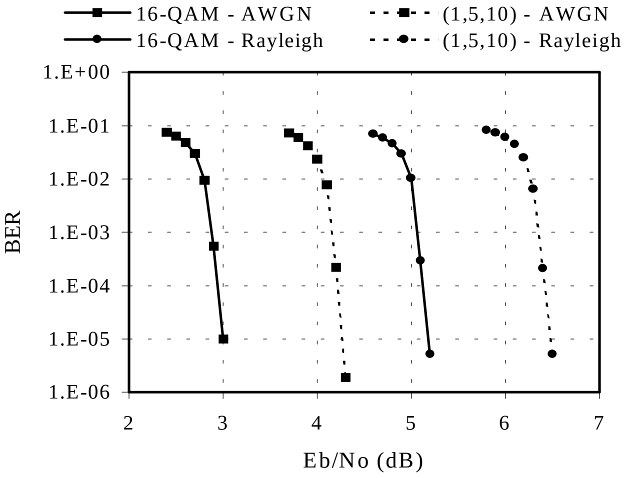

Fig. 5 – BER performance of several 2-bits/s/Hz BITCM's using 16-QAM and (1, 5, 10) constellations, over AWGN and Rayleigh fading channels.

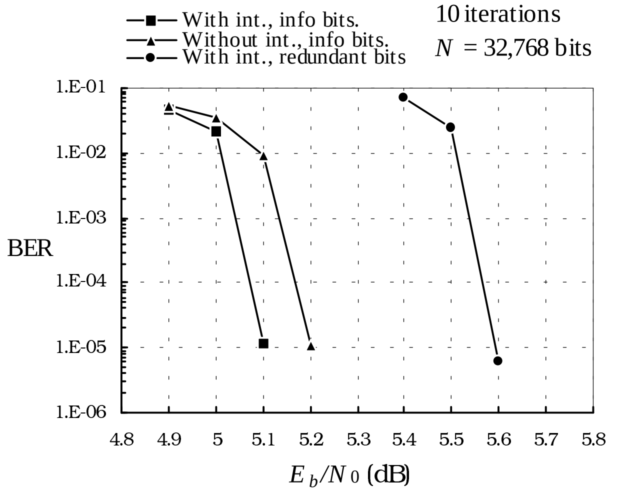

Fig. 6 – BER performance over the Rayleigh channel of a 2-bits/s/Hz BITCM, with and without interleaving function π/π^{-1}, and with the maximum protection given to either information bits or redundant bits.

than with other kinds of BICM schemes.

As an example, in the same conditions, the performance of a BICM using 8-PSK and an 8-state, rate-2/3 convolutional code is degraded by about 2 dB when the interleaving π/π^{-1} is removed [4]. We conclude that correlation between LLR's $\Lambda(u_{k,i})$ has no major effect on error-correction capabilities of BITCM's. This is due to the pseudo-random interleaving embedded in a turbo code, which allows both the second constituent decoder to always operate from uncorrelated samples, and the extrinsic information to be composed of uncorrelated estimates of the information bits. As the number of decoding iterations increases, i.e., the reliability of the extrinsic information is improved, the first constituent decoder relies more and more on this extrinsic information rather than on LLR's $\Lambda(u_{k,i})$. Thus, the effect of an eventual correlation between LLR's at the turbo decoder input tends to disappear after a sufficient number of iterations. From Fig. 6, we also see that the best error-correction capability is obtained when the information bits are the most protected ones. The performance degradation can be quite significant when maximum protection is given to the redundant bits. For this example, the coding gain at 10^{-5} is decreased by 0.5 dB.

5. Conclusions

When $M = 2^{2p}$, we have shown that rectangular QAM modulations are particularly attractive for designing power-efficient BITCM's, over both AWGN and Rayleigh fading channels.

This can be explained by the fact that the association of turbo codes with such modulations is by nature a clever and powerful combination since coding and modulation functions are both optimized for operation in the same SNR region (low SNR). We have also shown that it is crucial to offer the maximum protection to the information bits and the use of an interleaving function π/π^{-1} is actually not really necessary in practice.

Finally, these results are interesting since they clearly indicate that rectangular QAM signal sets are the most attractive constellations for designing BITCMs, which are both power-efficient and extremely simple to implement.

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