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Design and performances of a low cost filtered COFDM system using π-constellations and real channel estimator

P. Bataille, F. Hamon, M. Lecomte and M. Testard,
SACET, Campus de Kerlann, Ecole Louis de Broglie, 35170 Bruz, France

***Abstract* - In order to define a multicarriers modem for High Frequency high rate radio communications we analyze new digital techniques.**
The system originality comes from the using of π-constellations with a low cost soft decision π detector applied to fractionnally spaced multicarrier modulation [1]. We also demonstrate that even if the π-detector is more complex than a classical one it keeps the advantage to easily provide a soft bit information to a soft decoder.

1 Introduction

Most of radio channels and particularly the HF channel are more or less time variant and frequency selective.
The frequency selectivity arises from multi-path propagation and results in InterSymbols Interferences. These ISI can be counteracted using equalizer (DFE ...), or using a coding multicarriers modulation system. In the latter solution, the number of carriers is chosen to get a flat frequency response on each subchannels. Therefore each subchannel doesn't need to be equalized.
The techniques described in this paper are designed for multicarrier modem, therefore "equalization" is reduced to a scalar channel gain compensation on each sub-carrier.
The time selectivity is a consequence of the Doppler effect. It results in signal fading affecting the communication. These signal fades can be bridged by applying diversity techniques. Here, we propose a transmission scheme using a modulation diversity. Contrary to time diversity, modulation diversity has the advantage to not increase the signal bandwidth thus preserving the spectral efficiency. One disadvantage of π-constellations is the complexity of the optimal receiver. As far as we are concerned the receiver used is a low cost sequential suboptimal detector. This detector keeps the performance of the universal detector decreasing highly the complexity. Furthermore, we suggest new method to provide a soft bit information from this detector without increasing sensitively the system cost.
A multicarrier modulation scheme is much more sensitive to frequency offsets than a single-carrier system. Frequency offsets (due to Doppler shifts, or oscillators imperfections) introduce ICI (interchannel interference). The use of a square root Nyquist pulse shapping function avoid overlapping between sub-carriers and allows to reduce this ICI.
In this paper, we define a transmission system in a fading environment using :

- filtered multicarrier modulation.
- π-constellations in which order diversity is equal to the dimension of the constellation.
- a bilinear channel estimator.
- a low cost suboptimal sequential detector which provides a soft bit decision.

The paper outline is as follows. In section II we describe the waveform model with a focus on the π-constellations, and on the bilinear estimation. Next, in section III we describe the soft bit decision detector and we analyze its complexity. Finally, before concluding, we analyze in section IV the simulation results.

2 Waveform model

2.1 Signal and channel model

In an filtered multicarriers modulation system, the transmitted baseband signal is :

$$x(t) = \sum_{n=-\infty}^{+\infty} \sum_{k=0}^{K-1} c_n^k g(t - nT_s) e^{j2\pi f_k t} \quad (1)$$

- M is the number of subchannel.
- T_s is the symbol time.
- c_{nk} is a complex data symbol transmitted during time interval $[n.T_s, (n+1).Ts]$ on the k^{th} subchannel.
- g(t), the shapping function, is the square root of a Nyquist function with a roll of factor α
- f_k is the k^{th} carrier of the base band signal. f_k is defined as: $f_k = \frac{kM}{T_s K}$ where K is the number of carriers and M is the number of samples in a symbol duration.

In the case of multicarrier system filtered with a Nyquist fonction the orthogonality between sub-carrier signals is guaranteed if the sub-carrier spectral support are disjoint: $f_k - f_{k-1} > \frac{1+\alpha}{T_s}$. So the the filtering allow to reduce the ICI but in the other hand it reduces the spectral efficiency with a factor $\frac{1}{1+\alpha}$. In our system the filtering is realise with the polyphase filters technique.

When the number of carriers is well chosen (e.g multi-path delay $T_d << T_s$) each subchannel can be represented by the Rayleigh channel model without ISI.
Thus we assume that the received complex signal of the k^{th} subchannel is :

$$\tilde{y_i} = \tilde{c_i}.\tilde{\alpha_i} + \tilde{n_i} \tag{2}$$

where $\tilde{\alpha_i}$ and $\tilde{n_i}$ are two independent Gaussian complex noise.
Let $\hat{\tilde{\alpha}}_i$ be the channel estimate so the estimate of the coherent received signal is :

$$\tilde{z_i} = \frac{\tilde{y_i}}{\hat{\tilde{\alpha}}_i} = \frac{\tilde{c_i}.\tilde{\alpha_i}}{\hat{\tilde{\alpha}}_i} + \frac{\tilde{n_i}}{\hat{\tilde{\alpha}}_i} \tag{3}$$

The system uses a PSAM (pilot symbol assisted modulation) channel estimation. The channel estimates $\hat{\alpha}_m$ result from a bilinear interpolation of the pilot in the time and frequency grid. The frame is so made of data and pilot periodically inserted in the time and frequency direction.

2.2 Modulation diversity and π constellation

2.2.1 Principle:

The modulation diversity denoted by L is given by the minimal number of different components between two points of the constellation. If the diversity L is equal to the dimension N of the constellation, then one unfaded component is enough to decide on the transmit symbol.
As we have previously noticed, the modulation diversity has the advantage to not increase the signal power or the signal bandwidth. This diversity is carried out from the building of particular N-dimensional signal constellations. The digital transmission of symbols from N-dimensional constellation is seen by the channel as $\frac{N}{2}$ successive complex signal transmission. However the ML detection is realized on N-dimensional signal. So, if we assume diversity and interleaving on the N-component, then the detector increases with N its chances to receive an unfaded component.

2.2.2 The pairwise error probability on the Rayleigh channel and π-constellations:

In this section we introduce a second significant parameter (the first one is the diversity) for the design of constellation adapted to the Rayleigh channel : the product distance. If $x = (x_1, .., x_i, ..x_N)$ and $y = (y_1, .., y_i, .., y_N)$ are two N-dimensional symbols, the product distance d_π of these symbols is defined by :

$$d_\pi = \prod_{i=1}^{N} |x_i - y_i| \tag{4}$$

In a Rayleigh fading channel a pairwise error probability majoration has been given by Divsalar and Simon. This majoration depending on the signal to noise ratio (SNR) denoted by Γ. At high SNR this relation become:

$$p(\mathbf{x} \to \mathbf{y}) \le (\frac{\mathbf{4}}{\Gamma})^{\mathbf{N}} \frac{\mathbf{1}}{\mathbf{d}_\pi(\mathbf{x}, \mathbf{y})^{\mathbf{2}}} \tag{5}$$

where $d_\pi(\mathbf{x}, \mathbf{y}) = \prod_{\mathbf{i=1}}^{\mathbf{N}} |\mathbf{x_i} - \mathbf{y_i}|$ is the product distance between the N-dimensional symbols $\mathbf{x}$ and $\mathbf{y}$. The last relation supposes that $x_i \neq y_i$ with $i\epsilon\{1, .., N\}$. As we can see from 5, two parameters allow to reduce the pairwise error probability majoration :

- $d_\pi(\mathbf{x}, \mathbf{y}) > \mathbf{1}$
- the N-dimension of the constellation which implicitly acts as a N-diversity, as the error probability decreases with Γ^{-N}.

Particular N-dimensional signal constellation have been designed by [2][3] : the π-constellations. In this approach the authors use algebraic tools to build $\mathbf{R^N}$ lattices adapted to Rayleigh fading channel in term of diversity and product distance. The generating matrix of this $\mathbf{R^N}$ lattice is denoted by G.

2.3 Transmission scheme including π-constellations and polyphase filtering

In the system proposed we design π-constellations applying the generating matrix to N-dimensional 32QAM symbols. The transmission scheme is described on Fig. 1.

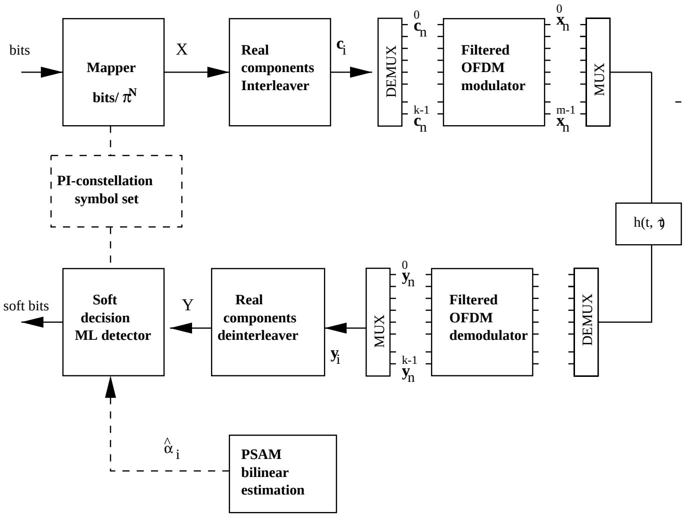

FIG. 1: System transmission scheme

To take advantage of the modulation diversity, we need to decorrelate the symbol component from the fading channel. This independance is achieved by a real component interleaver. At the output of the components deinterleaver we get the N-dimensional vector Y.
The soft decision ML detector which is described in the next section, use the weighted metric

$$D(C|Z, \hat{\alpha}) = \sum_{k=1}^{N} |\hat{\alpha_k}|^2 |z_k - c_k|^2 \tag{6}$$

where Z is a coherent demodulated version of Y and $\hat{\alpha}$ a N-dimensional vector estimate of the channel state.

In order to define the numerical structure of the filtered modulator we take a sampled version of 1 at time $(nTs + mTe)$.

$$x(nM+m) = \sum_{l=0}^{L-1} C_{n-l}^{(nM+m)_N} g(lM+m) \qquad (7)$$
$$C_{n-l}^{(nM+m)_N} = \sum_{k=0}^{N-1} c_{n-l}^{k} e^{j2\pi \frac{k}{K}(nM+m)}$$

where $C_{n-l}^{(nM+m)_N}$ is the inverse Discrete Fourrier transform of the sequence c_{n-l}^{k} over K points. And where the samples $x(nM+m)$ are provide by M polyphase filter [1]. The m^{th} poliphase filter is defined by: $g_m(l) = g(m+lM)$ wich is a decimate version of the prototype filter $g(m)$. The details of the demodulator structure is described in [1].

3 A low cost soft decision detector adapted to π constellation

With this kind of modulation, the number M of points in the constellation increases exponentially with the required L diversity . Also we assume that the universal ML detector yields to prohibitive complexity.
The proposed detector is based on FastML [2][7] algorithm. This algorithm uses the high diversity constellation following property: without disturbance one component of the received point is sufficient to detect the transmited symbol. Therefore, the decoding criteria is fixed on only one component, the least affected by fading. The FastML algorithm operates sequentially, by comparing the least faded received components with the corresponding reference symbol components arranged beforehand. This schedule allows us to decrease the number of symbols and components tested.
Furthermore we demonstrate in the next sections that we can take advantage of the algorithm properties to provide easily a soft bit information to a soft decoder.

3.1 Algorithm description

3.1.1 Algorithm principle:

Let $Z = (Z_1, ..., Z_N)$ be the coherent received signal, and $\{C^i = (C_1^i, ..., C_N^i)\}_{i=1,..,M}$ the possible symbols set. The components of Z are arranged such that Z_1 is the least faded component, and Z_N is the most faded.
All of the C^i are put in order to :

$$|C_1^1 - Z_1| < |C_1^2 - Z_1| < ... < |C_1^M - Z_1| \qquad (8)$$

During the maximum likelihood symbol research, the candidates C^i are evaluated successively from $i=1$ to M until a stopping criteria activation. This criteria is activated when:

$$D(Z, C^i) < d(Z_1, C_1^{i+1}) \qquad (9)$$

where $D(Z, C^i)$ is the weigthed distance between the received symbol Z and the reference symbol C^i, and where $d(Z_1, C_1^{i+1})$ is the partial weighted distance according to the least faded component. Therefore the set of symbols $C^{i+2}, ..., C^M$ is not evaluated by the algorithm.
It is shown in [7] that the FASTML preserves the universal detector optimality. Indeed we have :

$$\forall\ i \in (1, ..., M)\ \ D(Z, C^i) > d(Z_1, C_1^i);$$
$$(8) \Rightarrow d(Z_1, C_1^{i+1}) < d(Z_1, C_1^{i+2}) < ... < d(Z_1, C_1^M)$$
$$(9) \Rightarrow D(Z, C^i) < D(Z, C^j)\ \ \forall\ j \in (i+1, ..., M)$$

Hence, the untreated symbols would not have been chosen.

3.1.2 Soft bit information supplying methods: VFSoftML

With the high spectral efficiency modulation, it is usually complex to provide soft bit information. Indeed, the decision rule is based on symbols that map several bits. An exhaustive method could be the following :
Let the vectors $\Delta 0 = (\Delta 0_0, ..., \Delta 0_{m-1})$ and $\Delta 1 = (\Delta 1_0, ..., \Delta 1_{m-1})$ where $m = log_2(M)$ is the number of bits maps by one symbol. Let $Cb^i = (Cb_0^i, ..., Cb_{m-1}^i)$ be the binary symbol associated to $C^i = (C_0^i, ..., C_{N-1}^i)$. And $(\mathbf{Cb_{0,0}}, ..., \mathbf{Cb_{0,m-1}})$, $(\mathbf{Cb_{1,0}}, ..., \mathbf{Cb_{1,m-1}})$ two m subsets such that:

$$\mathbf{Cb_{0,j}} = (\mathbf{Cb^i} | \mathbf{Cb_j^i} = \mathbf{0}),$$
$$\mathbf{Cb_{1,j}} = (\mathbf{Cb^i} | \mathbf{Cb_j^i} = \mathbf{1}).$$

As we map Cb^i with the reference symbol C^i, we map the subsets $(\mathbf{Cb_{0,0}}, ..., \mathbf{Cb_{0,m-1}})$ with the subsets $(\mathbf{C_{0,0}}, ..., \mathbf{C_{0,m-1}})$. $\Delta 0_i$ and $\Delta 1_i$ are define such that:

$$\Delta 0_j = \min_i D(Z, C_{0,j}^i),$$
$$\Delta 1_j = \min_i D(Z, C_{1,j}^i).$$

Then $\Delta 0_j$ represent the minimal weighted distance between the received symbol and the reference symbols issued from the subset $\mathbf{Cb_{0,j}}$. The soft bit value vector supplied to the binary decoder is defined by:

$$Soft = \frac{\Delta 1 - \Delta 0}{\Delta 1 + \Delta 0}.$$

The complexity of this exhaustive method is prohibitive in the case of large constellation, furthermore it is not adapted to the sequential detection algorithm.

Here after we propose a simplified soft decision detection algorithms.
The aim is to provide values to the vectors $\Delta 0$ and $\Delta 1$ with the lower cost. The basic idea of the VFSoftML is the following : In the FASTML algorithm nd (with $nd << M$) N-dimensional distances are calculated such that:

$$D^{\tau(1)} > .. > D^{\tau(i)} > ... > D^{\tau(nd)}, \qquad (10)$$

where $D^{\tau(1)}$ represents the first calculated distance and $D^{\tau(j)}$ the jth calculated distance.
Furthermore, each distance $D^{\tau(i)}$ is mapped into a binary symbol Cb^j by the relation $\tau(i) = j$. Now, at time $t = t_i$, $D^{\tau(i)}$ is the minimal distance calculated during the algorithm process.

Therefore $D^{\tau(i)}$ can supply the first $m-1$ values of the vector $\Delta 0$, and the last value of the vector $\Delta 1$.
We iterate this process for each N-dimensional distance calculation step crushing the previous values :

$$\Delta 1(\tau(i)) = Cb.D + \overline{Cb}.\Delta 1(\tau(i-1)),$$
$$\Delta 0(\tau(i)) = \overline{Cb}.D + Cb.\Delta 0(\tau(i-1)).$$

This method can not warrant to supply the vectors $\Delta 0$ and $\Delta 1$ with the m needed values. However, we assume that if a value have not been supplied, its complement is given with the maximal fiability. Indeed, an indeterminate value corresponds to a symbol set rejected by the algorithm.
The analysis of the calculation complexity of the system is given in [7].The complexity depends on the constellation size. At every step of the detecting process we evaluate the partial weighted distance : $d(z_i, c_i) = |\hat{\alpha}_i|^2 |z_i - c_i|^2$. Such a calculation is refered as one operation. The Fig. 2 compares the universal hard detector complexity with the VFSoftML complexity. The complexity unit is the partial weighted distance cost.

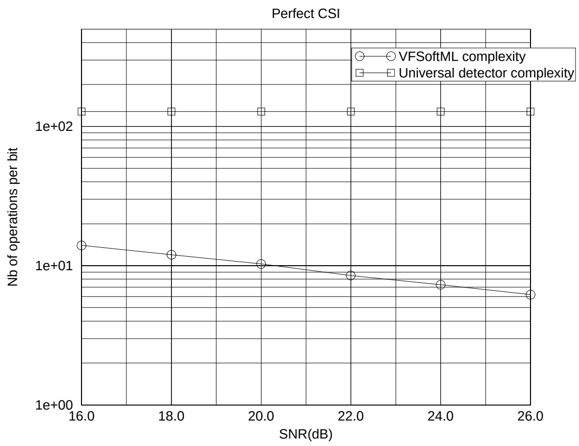

FIG. 2: Comparative estimation of universal and VFSoftML complexity

4 Simulation results

We give the BER resulting from the transmission scheme described in section 2.4. The fading HF channel is a Rice channel characterized by $T_d = 1.10^{-3}s$ and the normalized Doppler band $B_d = 0.1$. The number of sub-carriers is $K = 128$, the roff off factor is $\alpha = 0.25$. The spectral efficiency including the loss introduce by pilots and filtering is $\eta = 3.75 bit/s/Hz$. For the $L = 4$ constellation the generating matrix G is computed in [2]. The initial symbols are issued from a conventional $32QAM$. The channel state information (CSI) is provided by the bilinear channel estimator previously described.
The Fig. 3 compares the BER of a conventional $32QAM$ constellation with their rotated version with diversity $L = 2$ and $L = 4$. With a diversity $L = 4$ we obtain significant gains of 3dB at $BER = 5.10^{-3}$

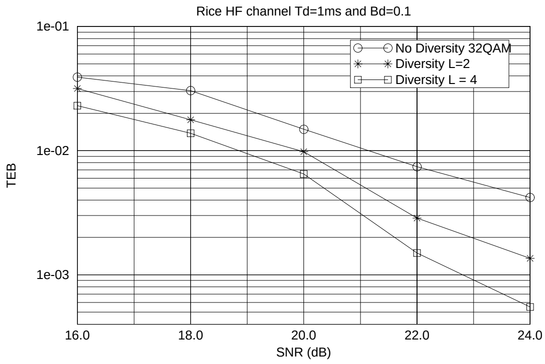

FIG. 3: BER of a $L = 2$ and $L = 4$ constellation, the CSI is given by a bilinear filter $\eta = 3.75 bit/s/Hz$.

5 Conclusion

We have proposed new digital techniques for high frequency communication. The transmission scheme is based on a multicarriers filtered OFDM system, associated to π-constellations and soft decoder.
Modulation diversity allows to fight efficiently against the fading environment without increasing the frequency band.
The design of the receiver allows to pilot any soft decoders.
Complexity analysis and simulation results realized with a real channel estimator demonstrate the interest of such a solution which can be adapted to the material realization constraints.
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References

[1] K. Haj Taieb, Robert Vallet and Daniel Duponteil: "Performance of Fractionally Spaced Multi-Carrier Modulation System", International Conference on Telecommunications, Istanbul Turkey, 14-17 April 1996.

[2] J. Boutros, E. Viterbo, C. Rastello, J.C Belfiore : "Good lattice constellations for both Rayleigh fading and Gaussian channels", IEEE Trans. on Information Theory, vol. 42, no. 2, pp 502-518, March 1996.

[3] Giraud, X., Boutillon, E. and Belfiore, J.C., "Algebraic Tools to build Modulation Schemes for Fading Channels", *IEEE Trans. on Inform. Theory*, vol 43, no 3, pp 938-952, May 1997.

[4] X. Giraud :"Constellations pour le canal à évanouissement", thèse de doctorat, ENST.

[5] M. Leconte, M. Testard : "A model of High Frequencies (H.F) Channel used to design a modem of 9600 bits/s rate in 3kHz of bandwidth",Milcom97, November 1997.

[6] C. Watterson, J. Juroshek, and W. Bensema : Experimental verification of an ionospheric channel model. Technical Report ERL-112-ITS-80, ESSA, 1969.

[7] Hamon, F., "Etude des performances et de la complexité d'un algorithme de détection de type FASTML adapté aux constellations tournées", *Rapport Technique Sacet* , DT-ANVAR-01061998-02-1998, Nov 1998.