COHERENT RADAR DETECTION OF SIGNALS WITH UNKNOWN PARAMETERS AGAINST A COMBINATION OF K-DISTRIBUTED AND GAUSSIAN CLUTTER

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RÉSUMÉ

Dans cet article la détection des signaux avec des paramètres inconnues dans un foullis radar non-gaussien est présentée. Le bruit est modelé comme une combination de foullis cohérente avec une distribution K, de foullis cohérente gaussien et de bruit thermique. Deux possibles stratégies de détection d'un cible Swerling I noyé dans le bruit mentionné ci-dessus ont été analysées: la stratégie du Rapport de Vraisemblence Generalisé et le test de Neymann-Pearson. Des simulations Monte Carlo ont été realisées pour évaluer les Caractéristiques Opérationelles des detecteurs. La probabilité de détection, pour une probabilité de fausse alarme donnée a été calculée en fonction du rapport signal-bruit et des paramètres du bruit.

ABSTRACT

In this paper the detection of signals with unknown parameters in non-Gaussian clutter is considered. The disturbance is modelled as a mixture of coherent K-distributed clutter, Gaussian distributed clutter and thermal noise. Two possible strategies to detect a Swerling I target against the above mentioned disturbance have been analyzed: the Generalised Likelihood Ratio Test (GLRT) and the Neyman-Pearson (NP) test. A Monte Carlo simulation has been carried out to evaluate the Receiver Operating Characteristics (ROCs). The probability of detection, for a given false alarm probability, has been calculated as a function of signal-to-noise ratio and of disturbance parameters.

1. INTRODUCTION

The coherent detection of targets against a background of correlated non-Gaussian clutter is a problem that in recent years has gained importance in the radar community. In particular, the detection of known or partially known signals against K-distributed clutter has been tackled in [1-6]. But the papers [1-5] do not consider the presence of a mixture of different types of clutter and the presence of thermal noise, which is always present in radar receiver. As a matter of fact, operational situations of interest refer to the contemporaneous backscattering by ground and sea patches or clouds and sea. The radar returns from ground and clouds may be described by a Gaussian distributed process, while the echoes from sea follow a K-distributed random process. While the paper [6] does not consider partially known targets, it assumes the signal to be deterministic and perfectly known. Objective of the present paper is to remove these limitations. Specifically, two strategies to detect a fluctuating target, according to Swerling-I model, against a mixture of K-distributed and Gaussian distributed disturbance (Gaussian clutter plus white Gaussian thermal noise) are presented. The generalized likelihood ratio test (GLRT) strategy makes use of the m-integrated pulses from the cell under test to estimate the unknown amplitude and initial phase of the useful signal. This strategy is compared with the Neymann-Pearson (NP) strategy, obtained by averaging the conditional likelihood ratio with respect to the *a-priori* distributions of the unknown parameters. Finally, the performance of these receivers is also compared to that of mismatched receivers, designed to take into account only one type of disturbance, either Gaussian or K-distributed.

2. MULTIDIMENSIONAL COHERENT CLUTTER MODEL AND OPTIMUM DETECTOR STRUCTURE

Assume that the radar transmits a train of m pulses, the corresponding sequence of m complex echoes to process is modelled as a m-dimensional complex vector $\mathbf{z}$. In absence of useful signal only the disturbance contributes to the observed sequence, so we have $\mathbf{z}=\mathbf{d}$.

In this paper $\mathbf{d}$ is modelled as the sum of three independent terms: $\mathbf{d}=\mathbf{d}_I+j\mathbf{d}_Q=\sqrt{\tau}\,\mathbf{x}+\mathbf{c}_G+\mathbf{v}$, where $\mathbf{d}_I$ and $\mathbf{d}_Q$ are the vectors corresponding to the in-phase and quadrature components of the complex envelope. $\mathbf{v}$ is the vector representing the Gaussian distributed thermal noise, $\sqrt{\tau}\,\mathbf{x}$ is the product model of K-distributed clutter and $\mathbf{c}_G$ is the Gaussian clutter. $\mathbf{v}$ is a vector of uncorrelated zero mean and variance $2\sigma_v^2$ Gaussian random variables. The clutter term $\mathbf{c}_G$ is characterized by a Gaussian probability density function (pdf) with zero mean, variance $2\sigma_G^2$, and normalized covariance matrix $\mathbf{M}_G$. The m-dimensional complex vector $\mathbf{x}$ (usually named *speckle*) is Gaussian with zero mean, variance 2 and normalized covariance matrix $\mathbf{M}_X$, the variable τ (referred to as *texture*) is Gamma-distributed [3] with mean value μ and order parameter ν (ν controls the deviation from Raleigh statistics). This product model corresponds to the case of pulse-to-pulse complete correlation of the texture [4], the case of partial correlated texture has been tackled in [5]. Given a specific value of τ, the vector $\mathbf{d}$ is the sum of three independent, zero mean, Gaussian vectors. Its conditional covariance matrix is given by:

$$\mathbf{M}_{d|\tau} \triangleq \frac{1}{2}E\{\mathbf{d}\mathbf{d}^H|\tau\} = \tau\mathbf{M}_X + \sigma_G^2\mathbf{M}_G + \sigma_v^2\mathbf{I} \qquad (1)$$

Define $\mathbf{s}$ to be the m-dimensional signal vector embedded in K-distributed clutter plus Gaussian disturbance. In [6] the signal has been assumed completely known. Normally the complex amplitude of the useful signal is unknown to the detector. So, the m-dimensional complex signal vector is more realistically modelled as follows: $\mathbf{s} = \alpha\mathbf{p}$ where $\alpha = Ae^{j\phi}$ is the unknown complex parameter. The amplitude A and the initial phase ϕ are scan-to-scan fluctuating according to the Swerling-I target model (i.e. A is assumed to be Rayleigh distributed and ϕ uniformly distributed in $[0,2\pi]$). $\mathbf{p}$ is a perfectly known complex vector with components $p_k = e^{j2\pi kf_D T}$, T is the radar pulse repetition time and f_D is the target Doppler frequency. It is worth observing that f_D is assumed to be known; but, in practice, a bank of filters is built to determine the Doppler shift of the target. The clutter has been assumed, without loss of generality, with zero-mean Doppler shift. In fact detection performance depends only on the value of the difference between the Doppler frequency of the target and the one of the clutter, but not on their absolute values.

The detection procedure is given by the decision between the two hypotheses H_0 and H_1, absence or presence of the signal in the cell under test, after the vector $\mathbf{z}$ has been received. If we consider the coefficient α as a random variable, the optimum test, in Neyman-Pearson sense, can be obtained by averaging the conditional likelihood ratio $L(\mathbf{z}|\alpha)$ with respect to the distribution of α [1]. Therefore, integrating with respect to A and ϕ and after straightforward manipulations, the detection strategy results

$$\int_0^\infty \left|\mathbf{M}_{d|\tau}\right|^{-1}\left[e^{-q_1(\mathbf{z})/2} - \lambda e^{-q_0(\mathbf{z})/2}\right]p(\tau)d\tau \underset{H_0}{\overset{H_1}{\gtrless}} 0 \qquad (2)$$

where $q_1(\mathbf{z}) \triangleq \mathbf{z}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}\left(\mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{p} + \dfrac{4}{\sigma_A^2}\right) - \left|\mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}\right|^2$ and $q_0(\mathbf{z}) \triangleq \mathbf{z}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}$. $p(\tau)$ is the gamma pdf, σ_A^2 is the signal amplitude variance, and λ is the threshold, set according to the prefixed probability of false alarm.

A different approach to the detection problem could be followed by modelling α as an unknown deterministic parameter, instead of a random variable (this means that we do not make use of the a-priori information about α). In this case, a possible solution is obtained by means of the GLRT approach, whereby the unknown parameters are replaced by their maximum likelihood (ML) estimates under each hypothesis. The decision will be taken by comparing the likelihood ratio, in which the unknown parameter has been replaced by its ML estimate, to the threshold λ, so obtaining the strategy

$$L(\mathbf{z}|\hat{\alpha}_{ML}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad (3))$$

The ML estimate of α is easily obtained [2,4]: $\hat{\alpha}_{ML} = \mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}/\mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{p}$. In this case the detection strategy results

$$\int_0^\infty \left|\mathbf{M}_{d|\tau}\right|^{-1}\left[e^{-q_2(\mathbf{z})/2} - \lambda e^{-q_0(\mathbf{z})/2}\right]p(\tau)d\tau \underset{H_0}{\overset{H_1}{\gtrless}} 0 \qquad (4)$$

where $q_2(\mathbf{z}) \triangleq \mathbf{z}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}\mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{p} - \left|\mathbf{p}^H\mathbf{M}_{d|\tau}^{-1}\mathbf{z}\right|^2$.

It is worth noting that $q_2(\mathbf{z}) = \lim_{\sigma_A^2\to\infty} q_1(\mathbf{z})$, so the strategies (2) and (4) are equal when $\mathbf{p}^H\mathbf{M}_0^{-1}\mathbf{p} >> 1/\sigma_A^2$, namely when SCR is high. In order to underline the advantages of the detectors matched to the proper model of the clutter, we compared the performance of the NP (and GLRT) detector with that of the receiver optimum against only Gaussian clutter or only K-distributed clutter. When the disturbance is supposed to be Gaussian with the same covariance matrix of the actual disturbance

$$\mathbf{M}_d = \frac{1}{2}E\{\mathbf{d}\mathbf{d}^H\} = \mu\mathbf{M}_X + \sigma_G^2\mathbf{M}_G + \sigma_v^2\mathbf{I} \qquad (5)$$

it can be seen that the NP detector for Swerling-I signal is the well known whitening matched filter. The optimum detection strategy is the following

$$\left|\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}\right| \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad (6)$$

Moreover, it can be shown that the corresponding GLRT detector implements the same detection strategy. For a given value of τ, under each hypothesis, the statistic $\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}$ is Gaussian with mean and variance values given by

$$E\{\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}|\tau,H_0\} = E\{\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}|\tau,H_1\} = 0 \qquad (7)$$

$$\begin{aligned} \sigma_{0|\tau}^2 &\triangleq \mathrm{var}\{\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}|\tau,H_0\} = \mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{M}_{d|\tau}\mathbf{M}_d^{-1}\mathbf{z} \\ \sigma_{1|\tau}^2 &\triangleq \mathrm{var}\{\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{z}|\tau,H_1\} = \sigma_{0|\tau}^2 + \sigma_A^2(\mathbf{p}^H\mathbf{M}_d^{-1}\mathbf{p})^2 \end{aligned} \qquad (8)$$

Therefore, the probabilities of false alarm and detection are easily obtained

$$P_{FA} = \int_0^\infty \exp\left(-\lambda/\sigma_{0|\tau}^2\right)p(\tau)d\tau \qquad (9)$$

$$P_D = \int_0^\infty \exp\left(-\lambda/\sigma_{1|\tau}^2\right)p(\tau)d\tau \qquad (10)$$

When the receiver is optimum for K-distributed clutter only (with the same covariance matrix of the actual disturbance) the detection strategies of K-NP detector and of K-GLRT detector can be obtained from (2) and (4), respectively, by simply replacing $\mathbf{M}_{d|\tau}$ with $\tau_K\mathbf{M}_K$, where τ_K is a gamma distributed variable with mean value $\mu_K = \mu + \sigma_G^2 + \sigma_v^2$ and order parameter v. $\mathbf{M}_K$ is such that

$$\mu_K\mathbf{M}_K = \mathbf{M}_d = \mu\mathbf{M}_X + \sigma_G^2\mathbf{M}_G + \sigma_v^2\mathbf{I} \qquad (11)$$

The detection loss of these mismatched detectors can be defined as the incremental SNR required to obtain the same P_D for a fixed P_{FA}, signal and disturbance being equal for the two cases.

3. PERFORMANCE ANALYSIS.

In order to obtain the performance of the NP receiver (2) and of the GLRT receiver (4) we used Monte Carlo simulations because closed-form expressions for the pdfs of the test statistics in (2) and (4) are not available. In the evaluation of the ROCs for the likelihood ratio test of equations (2) and (4) the product $f_D T$ has been set equal to 0.5 and the mean value of the clutter Doppler spectrum has been assumed to be zero. All the simulations have been carried out keeping constant the total clutter-to-noise power ratio: $CNR_{K+G} \triangleq (\mu + \sigma_G^2)/\sigma_V^2\big|_{dB} = 30dB$. The noise power σ_v^2 has been set to unity; the signal-to-noise power ratio SNR and the K-distributed clutter-to-noise power ratio CNR_K have been defined as follows: $SNR \triangleq \sigma_A^2/2\sigma_v^2$, $CNR_G \triangleq \sigma_G^2/\sigma_v^2$. According to a widely assumed model an exponential covariance function has been assumed for the clutter [2]. So, the total disturbance covariance matrix $\mathbf{M}_{d|\tau}$ has elements $m_{ij} = \tau\rho_X^{|i-j|} + \sigma_G^2\rho_G^{|i-j|} + \sigma_v^2\delta_{ij}$, (i,j=1,2,...,m), where δ_{ij} is the Kronecker's delta, ρ_X and ρ_G are the one-lag correlation coefficients (in the simulation we set $\mathbf{M}_X = \mathbf{M}_G$, i.e. $\rho_X = \rho_G$). The assumption $\mathbf{M}_X = \mathbf{M}_G$ greatly reduces the computational effort, but it also has a physical justification. While the texture variable represents the characteristics of the observed scene, the speckle takes into account the coherent sensor effects. In these terms the correlation structure of the speckle is independent of the texture distribution. The two matrices are generated by the same phenomenon (the antenna rotation), so they are almost identical.

Some numerical results are reported in the following figures. Fig. 1 and 2 show the performance of receivers (2) and (4) and, as comparison, the performance of optimum receiver for known signal [6] that implements the following detection strategy:

$$\frac{\int_0^\infty \left|\mathbf{M}_{d|\tau}\right|^{-1} \exp\left[-(\mathbf{z}-\alpha\mathbf{p})^H \mathbf{M}_{d|\tau}^{-1}(\mathbf{z}-\alpha\mathbf{p})/2\right] p(\tau)d\tau}{\int_0^\infty \left|\mathbf{M}_{d|\tau}\right|^{-1} \exp\left[-\mathbf{z}^H \mathbf{M}_{d|\tau}^{-1}\mathbf{z}/2\right] p(\tau)d\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \quad (12)$$

In this case α is a deterministic known parameter instead of a random variable as in (2) or an unknown parameter as for the GLRT detector. In these figures $m=16$, $\nu=0.5$, $SNR=18dB$. Fig.1 refers to the case $CNR_K = 27dB$ ($CNR_G = 27dB$), while Fig.2 refers to $CNR_K = 30dB$, ($CNR_G = -\infty dB$). The curve relative to the NP receiver is very close to the curve relative to the GLRT receiver, that is they have almost equal performance. In Fig.3 and Fig.4 the ROCs of the same three detectors are reported. For each receiver these figures show a couple of curves. The upper curve is relative to $P_{FA} = 10^{-3}$ and the lower curve is relative to $P_{FA} = 10^{-4}$. In Fig.3 $CNR_K = CNR_G = 27dB$ and in Fig.4 $CNR_K = 30dB$.

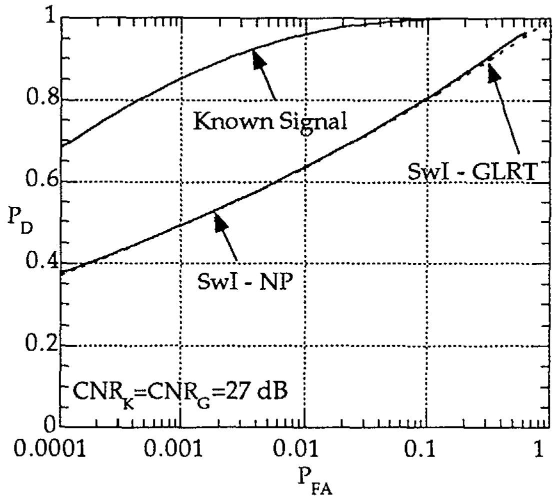

Fig.1- P_D.vs.P_{FA}($SNR = 18dB$, $\nu = 0.5$, $m = 16$, $\rho_X = 0.9$)

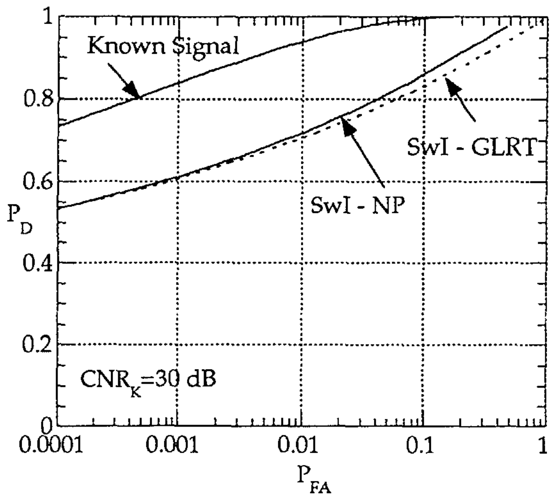

Fig.2- P_D.vs.P_{FA}($SNR = 18dB$, $\nu = 0.5$, $m = 16$, $\rho_X = 0.9$)

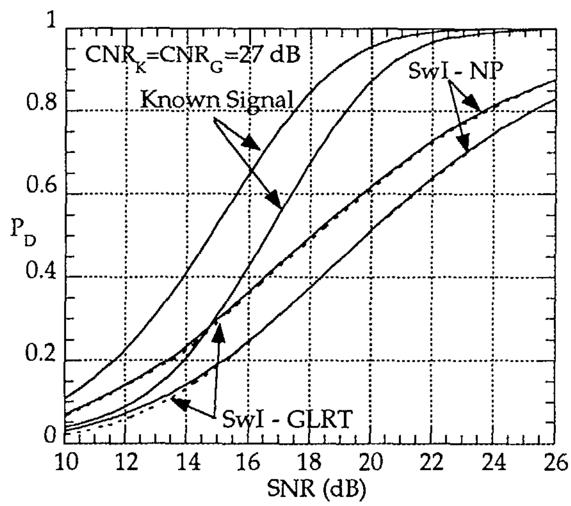

Fig.3 - P_D.vs.SCR. ($\nu = 0.5$, $m = 16$, $\rho_X = 0.9$)

Fig.5 shows the performance of the optimum detector (2) (OPT.DET) and of the detector (6) (GAUSS.DET), matched to Gaussian disturbance only, with the same covariance matrix of the actual disturbance. The advantage of a detector that takes into account the effective presence of both K-distributed clutter and Gaussian clutter is evident, especially for a low signal power.

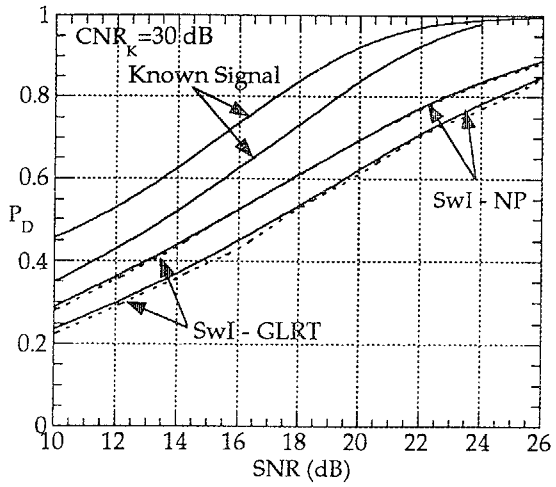

Fig.4 - P_D.vs.SCR. ($v = 0.5, m = 16, \rho_X = 0.9$)

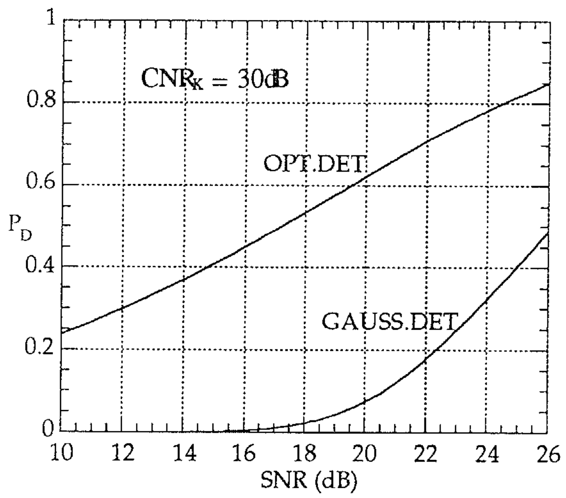

Fig.5 - P_D.vs.SCR ($v = 0.5, m = 16, \rho_X = 0.9$)

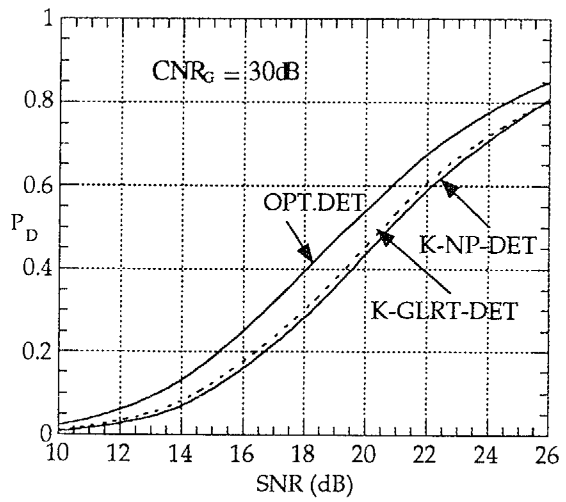

Fig.6 - P_D.vs.SCR ($v = 0.5$, $m = 16$, $\rho_X = 0.9$)

Fig.6 shows that the performance of mismatched detectors NP and GLRT, optimized taking into account the presence of only K-distributed clutter. The input of the analyzed detectors is Gaussian correlated clutter and thermal noise (worse condition). We note that the GLRT receiver outperforms the NP receiver only slightly. This is not in contrast with the theoretical optimality (in Neyman-Pearson sense) of the NP detector, since this detector would be optimum if the actual input was only K-distributed clutter. Moreover, by comparing Fig.5 and Fig.6, we derive that the mismatched K-detector suffers from lower losses than the Gaussian one.

4. CONCLUSIONS

In the present paper two different approaches to the detection of signals with unknown parameters against a mixture of K-distributed and Gaussian disturbance have been considered. It should be noted that the analysis herein includes the effects of receiver thermal noise. The first approach is based on the generalised likelihood ratio test, the second on the Neyman-Pearson test. A mixed numeric-Monte Carlo simulation method has been employed to evaluate the ROCs of the optimum detector and of mismatched detectors. Moreover the detection gain of the processors introduced in this paper, with respect to the detectors matched either only to Gaussian distributed clutter or only to K-distributed clutter, has been derived. Finally, it is worth observing that the detection schemes (2) and (4) are not simple to implement: they involve a heavy numerical integration with respect to the texture variable τ. Besides they strongly depend on the parameters v and μ of the texture distribution and on the correlation structure of the clutter. So no predetermined threshold can be assigned to achieve a given P_{FA}, if they are unknown. In order to overcome these drawbacks, in [7] a linear-quadratic distribution-free detector has been proposed, that offers good performance when the clutter correlation is known. For completely unknown clutter environment, an adaptive detection algorithm, based on higher order statistics, is derived in [8] and its performance analyzed. It would be interesting to extend our analysis to the case of partially correlated texture, according to the model proposed in [5]; this problem is the focus of future research.

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