Improved Subband Coding of Images Using Unequal Length PR Filters

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Abstract

In this paper the choice of subband filters for image compression is addressed. A set of two–band filter banks with perfect reconstruction is introduced which take into account the special statistics of natural images. Their main property is unequal lengths filters, such that the highpass analysis filter is shorter than its lowpass counterpart, while maintaining important properties such as linear phase and maximum regularity. It is shown that these filters outperform the classical subband filters QMF.

Résumé

Dans cet article le choix des bancs de filtres pour la compression d'images est adressé. Une famille de bancs de filtres à deux bandes à reconstruction parfaite est introduite qui tient compte des statistiques d'images naturelles. La propriété principale de ces filtres est une longueur plus courte pour le filtre passe–haut d'analyse que pour son homologue passe–bas tout en maintenant des propriétés importantes telles que la linéarité de la phase et la régularité maximale. Il est montré que ces filtres performent nettement mieux que les filtres classiques tels que QMF.

I Introduction

Subband coding of images has become a domain of intensive researches in recent years. In such a scheme, subband transforms are computed by filtering the input image with a set of bandpass filters and decimating the results, thus obtaining the subband images. The power of this technique resides in its capability to code each subband separately with a bit rate that matches the visual importance of that subband. Subband coding leads to more pleasing image reconstruction than block DCT coding by avoiding blocking artifacts. In addition, it allows progressive multiresolution transmission.

The concept of subband decomposition was introduced first by Crochiere [1] in the context of speech coding. Tremendous studies have been devoted to the theory and design of 2-band or M-band *perfect reconstruction* (PR) filter banks. Applications of subband coding to images was introduced by Woods and O'Neil [2] by means of 2-D separable quadrature mirror filter (QMF) banks.

Image coding applications require filter banks with specific features that differ from the classical PR problem or from the design of filter banks for other applications. It is our conviction that typical statistical and spectral characteristics of natural images have to be taken into consideration. Indeed, images are highly non–stationary sources, and typical natural images have a highly asymmetric power spectrum with respect to $\pi/2$. A good model is a spectrum proportional to f^{-2} [3].

In this paper, a new family of filter banks called Asymmetrical Filter Banks (AFB) is proposed. The analysis lowpass filter is long, while the analysis high-pass filter is very short, thus matching the statistics and non–stationarity of natural images. Maximum regularity is also imposed, an important feature that is well recognized in [4]. The AFBs are almost free of the "ringing effect" which occurs systematically in reconstructed images using QMFs. Also, the computational complexity of the transform implementation is reduced due to the small spatial support of the filters [5]. The performance of the proposed filter banks are compared with the QMF filter banks both in terms of PSNR and subjective visual quality. A detailed analysis of quantization noise and its effects on reconstructed images are given as well.

The paper is organized as follows. Section II briefly reviews the 2–band PR filter bank theory and describes the design of the proposed AFBs. Section III is devoted to the analysis of the quantization noise. Experimental results are given in Section IV. Finally, Section V draws conclusions.

II Asymmetrical Filter Banks

Let $H_i(z)$ be the analysis filters and $G_i(z)$ be the synthesis filters. The input signal is noted $x[n] = \mathcal{Z}^{-1}X(z)$ and the output signal is noted $\hat{x}[n] = \mathcal{Z}^{-1}\hat{X}(z)$. A two-band filter bank ($i = 1, 2$) using this notation is given in Fig. 1. The input/output relationship is then given by

$$\hat{X}(z) = X(z)T(z) + X(-z)A(z) \qquad (1)$$

where the transfer function $T(z)$ is given by $T(z) = H_1(z)G_1(z) + H_2(z)G_2(z)$ and the aliasing term is $A(z) = H_1(-z)G_1(z) + H_2(-z)G_2(z)$. Perfect reconstruction can be achieved by removing the aliasing distortion, $A(z)$, and imposing the transfer function, $T(z)$, to be a pure delay of the form $T(z) = z^{-\delta}$, where δ is the delay of the system. By choosing the synthesis filters as $G_1(z) = H_2(-z)$ and $G_2(z) = -H_1(-z)$ the aliasing component is removed. Under these constraints the system transfer function becomes

$$T(z) = F(z) + F(-z) \qquad (2)$$

where the product filter is $F(z) = H_1(z)H_2(-z)$. Perfect reconstruction is obtained when the product filter is a power-complementary half-band filter. This means that every other sample of $f[n] = \mathcal{Z}^{-1}F(z)$, except $f[\delta]$, is zero.

Most of the classical filter banks such as QMF-filters [6] and CQF-filters [7] impose the constraint on the analysis filters that the magnitude response of the highpass analysis filter is a mirror around $\pi/2$ of its lowpass counterpart. For image compression purposes this constraint is not adequate. Indeed, image statistics have typically a lowpass character and highpass information are typically contours which have a very short spatial extension. Hence, it seems unreasonable to have the same filter characteristics for lowpass and highpass filtering. A shorter length for the analysis highpass filter seems more appropriate. Therefore, the goal of this paper is to design unequal length filters.

Define N_1 the number of samples of the lowpass filter $H_1(z)$ and N_2 the number of samples of the highpass counterpart $H_2(z)$. The assymetric filter bank (AFB) is defined by

$$H_2(z) = Const(z^{-1} - 1)^{N_2 - 1} \qquad (3)$$

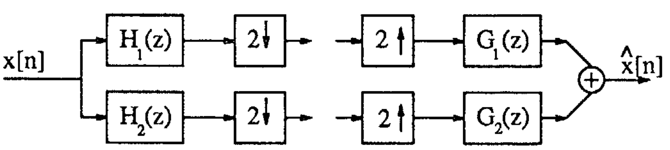

Figure 1: **Two-band analysis/synthesis filter bank system.**

$h_1[n]$	$h_2[n]$
-9.76562500e-03	1.25e-01
2.92968750e-02	-3.75e-01
3.71093750e-02	3.75e-01
-1.89453125e-01	-1.25e-01
-5.07812500e-02	
6.83593750e-01	
6.83593750e-01	
-5.07812500e-02	
-1.89453125e-01	
3.71093750e-02	
2.92968750e-02	
-9.76562500e-03	

Table 1: **Unequal length analysis filters with $N_1 = 12$ et $N_2 = 4$.**

whereas the lowpass analysis filter is obtained by constraining the filter bank to have perfect reconstruction and maximum regularity [5]. From experiments $N_1 = 3N_2$ has been determined to be the best relation between the lengths of the two analysis filters. The filter coefficients for $N_1 = 12$ and $N_2 = 4$ are given in table 1.

III Quantization Noise Analysis

Consider a two-band filter bank. The role of the high-pass analysis filter $H_2(z)$ is to filter the edges of a natural image. The high-frequency subband should contain very low energy. The difference between the proposed filter and the classical QMF filter, which are widely used in practice, is the following. The length of the proposed highpass analysis filter is very short, i.e. only three or four samples. Therefore, there will be very few oscillations of the filtered image around an edge and hence more samples will be equal to zero. The phenomenon is shown in Fig. 2. The edge is modeled as a ramp function. The oscillations are much more localized in the space domain with the proposed filter than with the QMF filter. It is experimentally verified that the high-frequency subbands contain less energy with the proposed filters. Due to their many zero-samples they have also a lower entropy, whereas the entropy of the low-low (LL) subband remains similar both for QMF and AFB. Therefore the total entropy is significantly reduced.

It has already been mentioned that the high-frequency subbands contain many samples having small and/or zero value. On these samples little quantization error is made! The quantization error of these subbands is very localized around the edges. The difference on the error images between the two compared cases is that the errors around the edges are much more

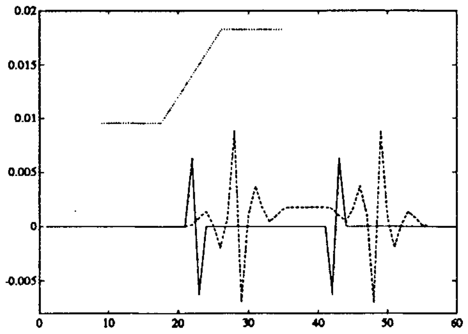

Figure 2: **Illustration of the response of the analysis highpass filter. Top: input signal. (solid line) AFB. (dashed line) QMF16B.**

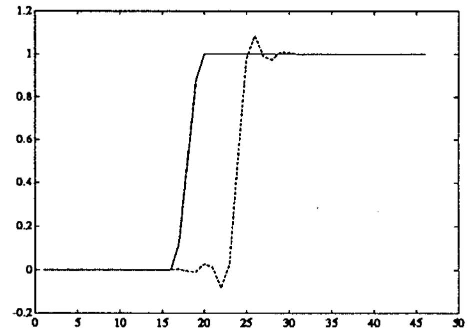

Figure 3: **Step responses of the lowpass synthesis filters. (solid line) proposed filter. (dashed line) QMF16B.**

spread out with the QMF filter. Therefore, a higher error variance in the high–frequency subbands using the QMF filters is expected.

An interesting fact is that the error variance in the LL subband is about the same for the two filters. It contains most of the energy and most of the information. The noise is almost uncorrelated with the original unquantized subband. This can be modeled with a uniform probability density function (pdf) $U[-\Delta/2, \Delta/2]$ for which the variance is

$$\sigma^2 = \frac{\Delta^2}{12}. \tag{4}$$

Hence, the variance of the quantization noise in the LL subband does not depend on the analysis filters [5].

One major disturbing coding error in subband coding is the ringing effect. It appears around the high contrast edges of the original image and is mainly due to the Gibbs phenomenon of the step response of the lowpass synthesis filter [8]. This distortion is spatially localized and is not accurately measured by the PSNR. However, the human visual system is sensitive to this artifact and it is important to eliminate it as much as possible. Fig. 3 shows the step responses of the proposed lowpass synthesis filter and the classical QMF16B filter [6]. There is no oscillation for the proposed filter. In contrast there is a 10% overshoot and some secondary oscillations for the QMF filter, responsible for the ringing effect.

The largest error variance before synthesis is in the LL subband and the quantization error in the high–frequency bands is very small due to the good behavior of $H_2(z)$. Therefore, it is desirable that the synthesis filters reduce the error in the LL subband by keeping the error in high–frequency subbands as small as possible. Defining $E_i(z)$ the quantization noise in the i-th subband, the quantization noise after synthesis filtering, $\xi(z)$, is given by

$$\xi(z) = E_1(z^2)G_1(z) + E_2(z^2)G_2(z) \tag{5}$$

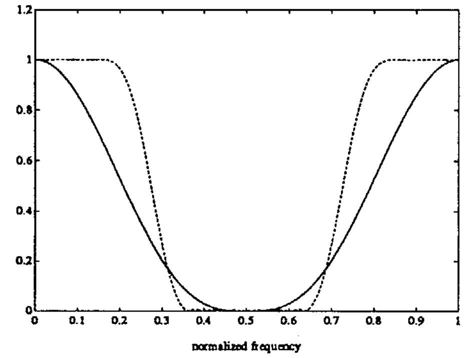

Figure 4: **Frequency responses of the lowpass synthesis filters. (solid line) AFB. (dashed line) QMf16B.**

The upsampled quantization noises are filtered by the synthesis filters and added together to give the total reconstruction error due to the PR property. The reconstruction error is directly related to the frequency responses of the synthesis filters. Consider the lowpass subband. Fig. 4 compares the frequency responses of the two lowpass synthesis filters. The proposed filter cuts off much faster than the QMF filter. This explains the much lower error variance in low–frequency regions with the proposed filters. Note that the human visual system is especially sensitive to errors in these regions.

The power of the AFB is that it reduces as much as possible the zeroth–order entropy after quantization in the analysis stage, whereas the synthesis filters keep the quantization noise as low as possible.

IV Experimental Results

For different quantization steps, the PSNR and the estimated zeroth-order entropy are computed. Fig. 5 shows the objective performance comparison on the image “Lena” 256x256 for the QMF16B and the AFB filter bank for the same pyramidal decomposition of

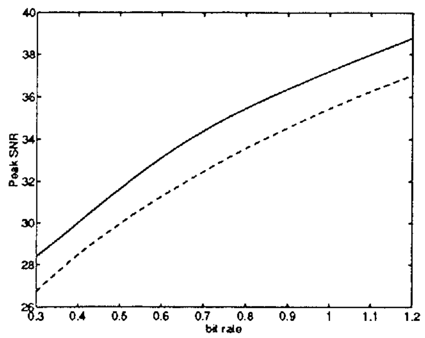

Figure 5: **Objective comparison of two filter banks on the image "Lena" 256x256. (full line) AFB - proposed filter bank. (dashed line) QMF16B.**

order 2. It can be observed that the proposed filter bank has a PSNR of about 2dB higher for the same bit rate than the QMF16B filter bank. Fig. 6 shows two coded images at a bit rate of 0.5 bit/pel. A strong ringing effect due to the QMF filter bank is clearly visible whereas the effect is much weaker with the proposed filter bank. The AFB outperforms the classical one both in terms of objective and in terms of subjective quality.

Figure 6: **Coded "Lena" at 0.5 bit/pel. (top) QMF16B 7 subbands with PSNR of 30.02dB. (bottom) AFB 7 subbands with PSnr of 31.90dB.**

V Conclusions

In this paper a new set of PR filters which takes into account the special statistics of natural images is proposed. Due to the very short spatial support of high-frequency content (edges) in a natural image it is proposed to use a very short highpass analysis filter. The quantization error and its influence on the reconstructed images are analyzed and it is shown that the proposed filter bank has some interesting features which result in a good coding performance with very little ringing effect. In comparison with the standard QMF filter bank, the proposed AFBs outperform the classical filters in terms of visual quality and in terms of objective quality.

References

[1] R. E. Crochiere, S. A. Weber, and F. L. Flanagan, "Digital coding of speech in sub-bands", *Bell System Technical Journal*, pp. 1069–1085, 1976.

[2] J. Woods and S. O'Neil, "Subband coding of images", *IEEE Transactions on Acoustics, Speech, and Signal Processing*, pp. 1278–1288, October 1986.

[3] D. Field, "Relation between the statistics of natural images and the response properties of cortical cells", *J. Opt. Soc. Am.*, vol. 4, pp. 2379–2394, 1987.

[4] Olivier Rioul and Martin Vetterli, "Wavelets and Signal Processing", *IEEE SP Magazine*, pp. 14–38, October 1991.

[5] Olivier Egger and Wei Li, "Improved Image Compression using Asymmetrical Filter Banks", *submitted to IEEE Transactions on Signal Processing*, 1993.

[6] J.D. Johnston, "A Filter Family Designed for Use in Quadrature Mirror Filter Banks", *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, pp. 291–294, April 1980.

[7] M. J. T. Smith and T. P. Barnwell, "Exact Reconstruction Techniques for Tree Structured Subband Coders", *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-34, pp. 434–441, June 1986.

[8] M. J. T. Smith, "FIR and IIR Analysis/Synthesis Systems for Subband Image Coding, Chapter 3", in J. Woods, editor, *Subband Image Coding*. Kluwer, 1990.