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A FAMILY OF LOW-SENSITIVE NONRECURSIVE FILTERS

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RÉSUMÉ

Une famille de FIR filtres à sensibilité restreinte. On propose une structure nouvelle de filtres à réponse impulsionnelle finie. Plusieurs exemples montrent que cette structure est vraiment à sensibilité restreinte.

ABSTRACT

Described is there a family of nonrecursive structures. For a given transfer function, one can choose the structure exhibiting the most advantageous sensitivity properties. The results have been proved by examing numerous filters.

1. INTRODUCTION

The effects of finite wordlength are important also in the case of nonrecursive filters, although usually they are there not so crucial as for recursive filters. This paper deals with reduction of coefficient quantisation effects which are usually measured using the sensitivities of the filter frequency characteristics with respect to the coefficient variations. Considered are FIR structures which exhibit low sensitivity of their amplitude characteristics. Because of the correlation between the effects of coefficient and data quantisation, these structures are also expected to exhibit low output noise [1].

Some authors [2-4] have already proposed to reduce the effects of finite wordlength in FIR filters using the concept of passivity. A generalization [5] of such an approach has been employed by derivation of the structure described in [2]. Here, it is shown that a large family of filter structures can be derived from the basic structure described in [2]. Among the numerous structure versions, for a given transfer function, the optimal in the meaning of a certain sensitivity criterion can be found.

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2. THE STRUCTURE FAMILY

Consider a transfer function of a FIR filter

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n} , \qquad (1)$$

which can be rearranged as

$$H(z) = c \cdot \sum_{n=0}^{N} G_n(z) , \qquad (2)$$

$$G_n(z) = \frac{1}{c} \cdot h(n) z^{-n}, \quad c = h(0)+h(1)+\ldots+h(n).$$

Thus we have a chain $\{G_n\}$. Let $\{P_n\}$ is an arbitrary permutation of $\{G_n\}$. There is

$$P_i = k_i \cdot f_i(z) , \qquad (3)$$

where k_i is a real coefficient being equal to one of the samples of the impulse response $h(\cdot)$,

$f_i(z) = z^{-n}$, where n is a certain nonnegative integer dependent on i.

Of course

$$H(z) = c \cdot \sum_{i=0}^{N} P_i(z) . \qquad (4)$$

The transfer function $P_i(z)$ can be implemented in the structure shown in Fig.1. [2], where

$$d_1 = k_0/(k_0+\ldots+k_N),$$
$$d_2 = k_1/(k_1+\ldots+k_N),$$
$$\vdots \qquad (5)$$
$$d_{N-1} = k_{N-1}/(k_{N-1}+k_N),$$

In order to minimize the number of delays the structure can be rearranged (Fig.2).

The good sensitivity properties of the basic structure [2] as well as of its modifications are related to the existence of a such characteristic frequency for which the overall transfer function is independent from the numerical values of the

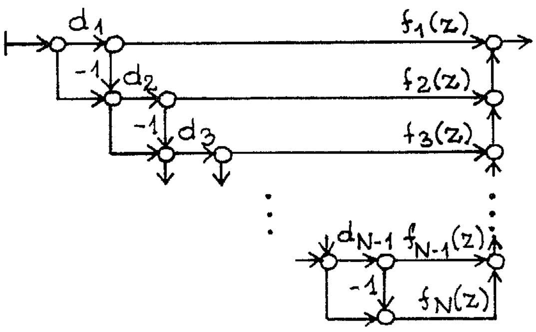

Fig.1. The basic structure

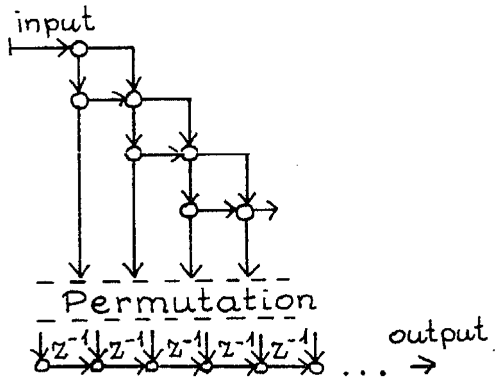

Fig.2. The modified structure

coefficients d_i. The characteristic frequency can be 0 or 0.5 (when the unit sampling frequency is assumed). Thus the structure described is appropriate to implement low- and high-pass filters. In the two-dimensional case the combinations of low- and high-passes are also realizable with a given characteristic frequency in a passband. Existence of a characteristic frequency, where the transfer function sensitivity (not obviously the magnitude sensitivity!) is equal zero, is related to reduced sensitivity in the passband.

As the number of possible permutations of $\{G_n\}$ is considerable, there exists a considerable number of the structure modifications. One among them minimizes a given sensitivity performance index.

In the obtained structure, the numbers of multipliers and delays are canonical. The sensitivity reduction is obtained on the cost of the increased number of additions, similarly as it is often in low-sensitive structures [4-7].

3. PASSIVE STRUCTURES

The concept of passivity has been already used to great advantage in the theory of digital filters [6]. The classical approach is related to the signal power defined as $\gamma\cdot x^2(n)$, where γ is a positive weighting constant and x(n) is a signal sample. Nevertheless, other measures also can be used in order to define the signal power [5,7]. A promising extension is offered by the concept of ℓ_1-power defined by $\gamma\cdot|x(n)|$.

The structure from the Fig.1 is ℓ_1-passive when the condition

$$0 < d_i < 1 \qquad (6)$$

is imposed [7]. In order to preserve this condition, the above design procedure should be modified by making an observation that

$$P_i = |k_i|\cdot \text{sign } k_i \cdot f_i(z), \qquad (7)$$

where sign(·)=-1, 0, or -1 for negative, zero, or positive arguments, respectively.

Now, let $c=|h(0)|+|h(1)|+...+|h(n)|$. (8)

Then

$$d_0=|k_0|/(|k_0|+...+|k_N|),$$
$$d_1=|k_1|/(|k_1|+...+|k_N|),$$
$$\vdots \qquad (9)$$
$$d_{N-1}=|k_{N-1}|/(|k_{N-1}|+|k_N|),$$

fulfill the condition of ℓ_1-passivity. Now, the transfer functions of the branches are

$$\text{sign } k_i \cdot f_i(z). \qquad (10)$$

In this case, the property of ℓ_1-passivity is preserved not only for a particular set of the coefficient values d_i but also for those values from the whole interval (0,1). Such a property called as structural passivity is , in general, related to low magnitude sensitivity [8] for various kinds of filters (low-, high-, and band-pass).

The ℓ_1-passive nonrecursive structures are applicable as building blocks of inherently stable 1-D and 2-D recursive systems [7].

4. EXPERIMENTAL RESULTS

The two permutations which appear in the most natural fashion have been chosen. The first (denoted as A) corresponds to the choice

$$f_i(z)=z^{-i}+z^{-(N-i)}, \qquad (11a)$$

while the second (B) to

$$f_i(z)=z^{-([N/2]-i)}+z^{-([N/2]+1+i)}. \qquad (11b)$$

Considered are also the passive versions (denoted as PA and PB, respectively).

Examined are linear-phase low-pass filters with 14, 20, and 32 taps. The sensitivities for the respective high-pass filters are the same. The sampling frequency is assumed to be unit. The passband edges are fixed to 0.05, 0.15, and 0.25, the transition band widths to 0.025, 0.05, and 0.1. Thus in common we have 27 different equiripple filters designed using the algorithm of McClellan-Parks [9].

Then, examined are also minimum-phase equiripple filters with the parameters as above but only with 14 and 20 taps, i.e., 18 filters. The filters have been designed using the program Mintom [10].

The frequency-dependent sensitivity measures are considered as follows

$$D_m = \max_{a_i} \left\{ \partial |H| / \partial a_i \right\} ,$$

$$D_a = \operatorname{average}_{a_i} \left\{ \partial |H| / \partial a_i \right\} ,$$

$$d_m = \max_{a_i} \left\{ \left[\partial |H| / \partial a_i \right] \cdot a_i / |H| \right\} ,$$

$$d_a = \operatorname{average}_{a_i} \left\{ \left[\partial |H| / \partial a_i \right] \cdot a_i / |H| \right\} ,$$

where $a_i = d_i$ or $h(n)$, respectively.

For the fixed-point arithmetic, the functions D_m and D_a are measures of the effects of coefficient quantisation. D_a has the meaning of the worst-case multiparameter sensitivity. The functions d_m and d_a are of minor importance as they describe the coefficient quantisation effects by the floating-point arithmetic and a substantial dynamic range of the coefficient values.

In order to prove the good properties of the obtained structures, the average values of D_a in the passband (D_{aap}), in the stopband (D_{aas}), and in both these bands (D_{aa}) are calculated as the average from 27 linear-phase filters and 18 minimum-phase filters for 4 versions and the classical transversal structure (T) (cf. Table 1). Table 1 as well as the graphs of sensitivity for two sample filters (Fig. 3 and 4) shows that in most cases the proposed structures are less sensitive.

Table 1

		D_{aap}	D_{aas}	D_{aa}
Linear phase	T	1.3154	1.2766	1.2776
	A	0.7699	0.7056	0.7666
	B	0.2743	0.4793	0.4138
	PA	0.5971	0.6000	0.6103
	PB	0.2108	0.2448	0.2451
Minimum phase	T	0.7049	0.6370	0.6530
	A	0.2417	0.3362	0.3197
	B	0.6427	0.6099	0.6517
	PA	0.1896	0.2930	0.2719
	PB	0.4511	0.5115	0.5023

Nevertheless, the less sensitive structure among the four considered versions is significantly better as the transversal one. It is shown for the averages of D_m in the passband, in the stopband, and in both of them (D_{map}, D_{mas}, D_{ma}), for the maximums of D_a in the two bands (D_{amp}, D_{ams}), for D_{aap},

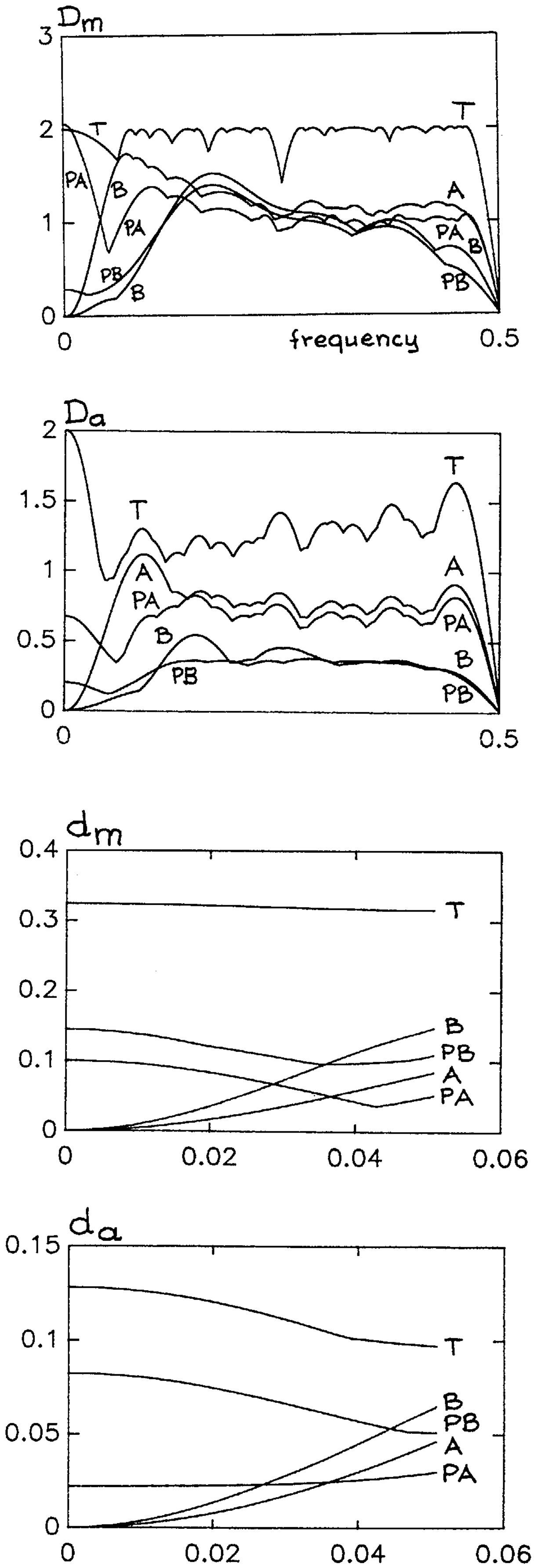

Fig.3. Magnitude sensitivities of the equiripple linear-phase filter (14 taps, passband edge: 0.05, transition band width: 0.1)

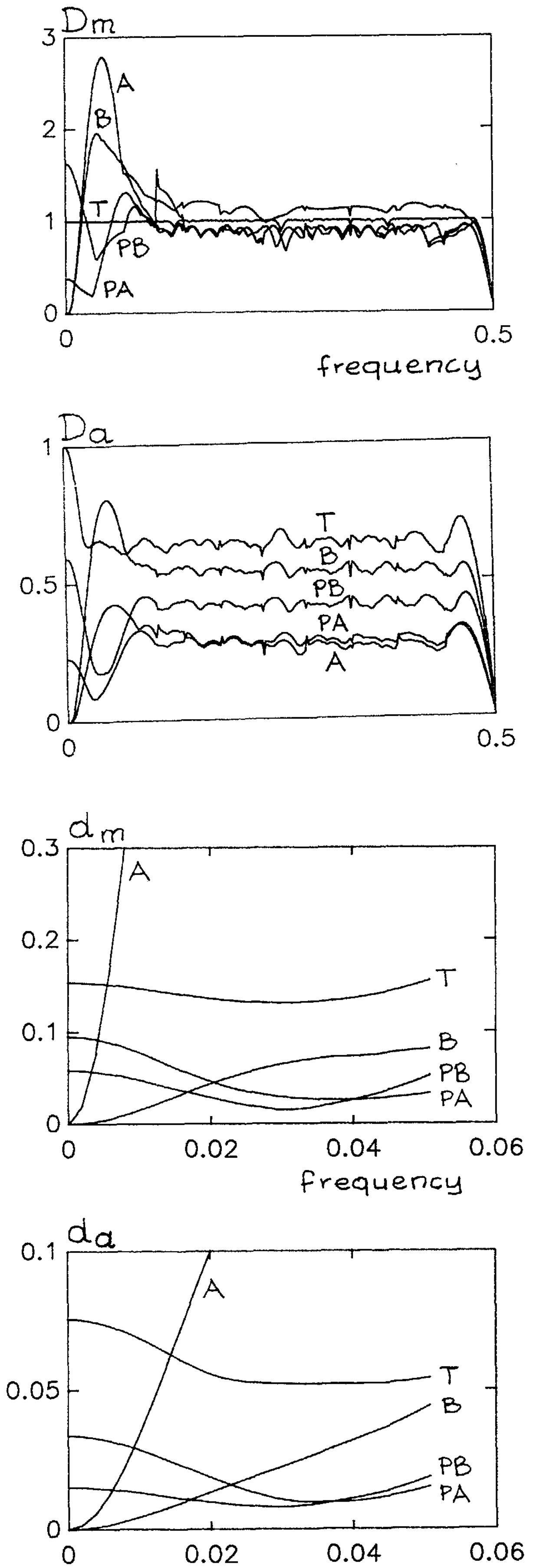

Fig.4. Magnitude sensitivities of the equi-ripple minimum-phase filter (20 taps, passband edge: 0.05, transition band width: 0.05).

D_{aas}, D_{aa} as well as for the averages of d_m and d_a (d_{ma}, d_{aa}) in the passband. In columns N (cf. Table 2) the most advantageous values are presented for comparison to those for the transversal structure (T).

Table 2

	Linear phase		Minimum phase	
	T	N	T	N
D_{map}	1.9520	0.5453	0.9966	0.6166
D_{mas}	1.9098	0.7662	0.9756	0.9365
D_{ma}	1.9279	0.7616	0.9835	0.8585
D_{amp}	1.9961	0.2826	1.0000	0.2915
D_{ams}	1.6007	0.3012	0.8731	0.4049
D_{aap}	1.3154	0.1684	0.7049	0.1808
D_{aas}	1.2766	0.2448	0.6370	0.2862
D_{aa}	1.2776	0.2440	0.6530	0.2667
d_{ma}	0.2491	0.1088	0.3063	0.1053
d_{aa}	0.0770	0.0337	0.0850	0.0257

The above results prove that the proposed family of structures offers large possibilities of sensitivity reduction.

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