ANALYSIS AND SYNTHESIS OF QUASI-PERIODIC TEXTURES USING COMPLETE TRANSLATION INVARIANT TRANSFORM

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Résumé:

Un modèle de textures quasi-périodiques basé sur une transformation invariant par translation est présenté. Cette transformation est construite à partir de la transformation de Fourier et est complète au sens que si deux signaux ont la même transformée alors l'un est une version permutée de l'autre. Cette transformation est appliquée localement sur l'image à travers une fenêtre. L'attraction d'attributs est faite dans l'espace transformé par un moyennage non-linéaire. Cet espace est ensuite modélisé par un réseau de fréquences généré par les deux fréquences fondamentales. La variation des périodicités sont caractérisés simplement par deux matrices de covariance. Le nombre de paramètres est de l'ordre de 50. Finalement, une méthode de synthèse simple est présentée et quelques résultats de synthèse de textures réelles sont montrés.

Summary:

A model of quasi-periodic textures built on a Fourier based translation invariant transform is presented. This transform is complete in the sense that if any two signals have the same transform then they are translated versions of each other. The transform is locally applied to the texture image using a smoothing window. Feature extraction is done in the transform-space by a nonlinear averaging method. The transform space is modeled by a network of points so that each of them represents an harmonic component of the texture. Only two covariance matrices are used to characterize the variations of the periodicities. The number of parameters of the model is approximately 50. A very simple fast synthesis method is proposed and syntheses of real textures from the Brodatz album show the validity of the model.

Keywords: Quasi-Periodic Texture, Complete Translation Invariant Transform, Analysis, Synthesis, Modeling.

1. INTRODUCTION

Texture modeling is an important area in image processing and is traditionnally divided into two branches: stochastic and structured models [1,2]. This paper adresses the later and particularly the class of quasi-periodic textures. Two main approaches for modeling this class of textures have been investigated. In the first approach, Matsuyama et al. [3] used a global Fourier transform to analyse the texture image. This approach works well when the texture is perfectly periodic which is generally not the case for real textures. In the second approach, Volet assumes that texture is locally periodic (globally, it may be periodic or not) [4]. These assumptions correspond better to real textures and for this reason, we will adopt the same assumptions in our work: the textures are locally periodic or quasi-periodic. On the other hand, it is natural in a texture problem to attempt to introduce translation invariant transforms (TIT). There exist two main families of TITs: R-, B-, Q-transforms [5,6] and Fourier based transforms [7,8,9]. The former is not complete whereas the later has recently been made complete [9]. This paper deals with the use of complete TIT in texture modeling.

Section 2 recalls the definition of 2-dimensional TIT and shows its application to texture analysis. The next section deals with the modeling of quasi-periodic textures based on the analysis results. Section 4 presents a simple synthesis method and shows some syntheses results.

2. ANALYSIS

First, let us define the two-dimensional TIT [9] in the case of two non-colinear reference frequencies.

Definition 1. The two-dimensional TIT of a real discrete signal x(k,l) has two components: its magnitude which is equal to the 2-DFT magnitude and its phase which is a modified version of the 2-DFT phase with respect to two reference frequencies (m1,n1) and (m2,n2). Let $X(m,n)=|X(m,n)|.\exp\{j.\Phi(m,n)\}$ be the 2-DFT of x(k,l). The 2D-TIT is defined by:

$$TIT(x(k,l))= |X(m,n)|.\exp\{j.\Psi(m,n)\}; \ \Psi(m,n)= \Phi(m,n) -\lambda_1.m - \lambda_2.n \ (\mathrm{mod}\ 2\pi) \tag{1}$$

with

$$\lambda_1, \lambda_2 \ / \ \Psi(m1,n1) = \Psi(m2,n2) = 0 \ (\mathrm{mod}\ 2\pi) \tag{2}$$

$$\lambda_1 = \frac{n2.\Phi(m1,n1) - n1.\Phi(m2,n2)}{m1.n2 - m2.n1} + \frac{n2.\alpha - n1.\beta}{m1.n2 - m2.n1} 2\pi; \ \alpha, \beta \in Z \tag{3}$$

$$\lambda_2 = \frac{m1.\Phi(m2,n2) - m2.\Phi(m1,n1)}{m1.n2 - m2.n1} + \frac{-m2.\alpha + m1.\beta}{m1.n2 - m2.n1} 2\pi; \ \alpha, \beta \in Z \tag{4}$$

In this paper, we will only use a particular solution for λ_1, λ_2 given by $\alpha,\beta = 0$. The general case is discussed in [10].

This transform may be used in texture analysis as illustrated in Fig. 1. For any position in the texture image, we extract an sub-image by using an analysis window and then apply definition 1 to the extracted sub-image. In other terms, we observe locally the texture image through the analysis window.

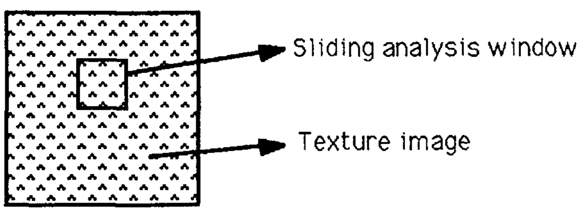


Fig. 1. Local analysis of the texture image through a sliding analysis window.

On the other hand, a perfectly periodic texture has all its energy concentrated at frequencies that satisfy [3,4]:

$$f= p.\mathbf{F}_1+q.\mathbf{F}_2;\ p,q\in Z. \qquad (5)$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are two fundamental frequencies representing the periodicities of the texture. These two fundamental frequencies are related to the two fundamental periods by [10]:

$$<\mathbf{F}_1,\mathbf{A}_1>=1, <\mathbf{F}_2,\mathbf{A}_2>=1, <\mathbf{F}_1,\mathbf{A}_2>=0, <\mathbf{F}_2,\mathbf{A}_1>=0. \qquad (6)$$

It can be proven [10] that if 1) the texture is periodic, 2) the analysis window is such that for any two positions in the texture, the first sub-image is a permuted version of the second one and 3) the two fundamental frequencies are choosen as reference, i.e. $\mathbf{F}_1$= (m1,n1) and $\mathbf{F}_2$=(m2,n2) then for any position in the texture, the application of the 2D-TIT gives the same result. In addition, there exists no other texture which may give the same result.

In practice, textures are not perfectly periodic and thus imply some adaptations:
- the use of a smoothing window (gaussian like) to reduce border effects and
- the reference frequencies are average fundamental frequencies of all sub-images.

The general analysis scheme becomes: the texture image is observed through an analysis window at different positions. For each position, we obtain an extracted sub-image, compute the two-dimensional DFT and square the magnitude to obtain the power spectrum. The power spectrum of all sub-images are averaged and give rise to the average power spectrum. The average power spectrum is used to detect the average fundamental frequencies of the texture image. These average fundamental frequencies are then used as the reference frequencies for definition 1. For each sub-image, the magnitude part of the transform is already computed (power spectrum). Our task is reduced to computing its phase part $\Psi(m,n)$ (modified phase) from the DFT phase. This sequence of operations provides us a set of N TITs of N sub-images. The final step is to define an averaging method to combine these N TITs. The average magnitude is already defined (average power spectrum), the average phase is given by:

$$\bar{\Psi}(m,n)=\mathrm{Arg}\ \{\bar{P}(m,n)\};\ \bar{P}(m,n)=(1/N).\sum_{i=1}^{N}\cos(\Psi_i(m,n)) +j.(1/N).\sum_{i=1}^{N}\sin(\Psi_i(m,n)) \qquad (7)$$

It can be verified that:

$$0\leq |\bar{P}(m,n)| \leq 1.$$

$|\bar{P}(m,n)| = 0$ corresponds to the case where $\Psi_i(m,n)$ (i=1,...,N) are randomly or regularly distributed on the unit circle.

$|\bar{P}(m,n)| = 1$ corresponds to the case where $\Psi_i(m,n)$ (i=1,...,N) are identical.

$\bar{P}(m,n)$ *is thus a very interesting quantity, since its argument is a mean value of the phases and its magnitude represents the consistancy of phase (over different sub-images at a given frequency* (m,n) *).*

The fundamental frequencies correspond to local maxima in the Fourier domain and their detection has been the subject of many works [3,4] and thus will not be discussed here.

<u>Illustration of analysis steps.</u>

The input image is taken from the Brodatz album [11], its size is 256x256 (Fig.2.a shows a part of 128x128). The analysis window is 32x32 cosinusoidal separable type. Fig.2.b shows 16 sub-images extracted at random positions in order to avoid the inherent periodicities of the texture and Fig.2.c their DFT magnitude. The average power spectrum and the fundamental frequencies are presented in Figs.2.d and 2.e. Finally, Fig.2.f shows the inverse DFT of the average TIT.

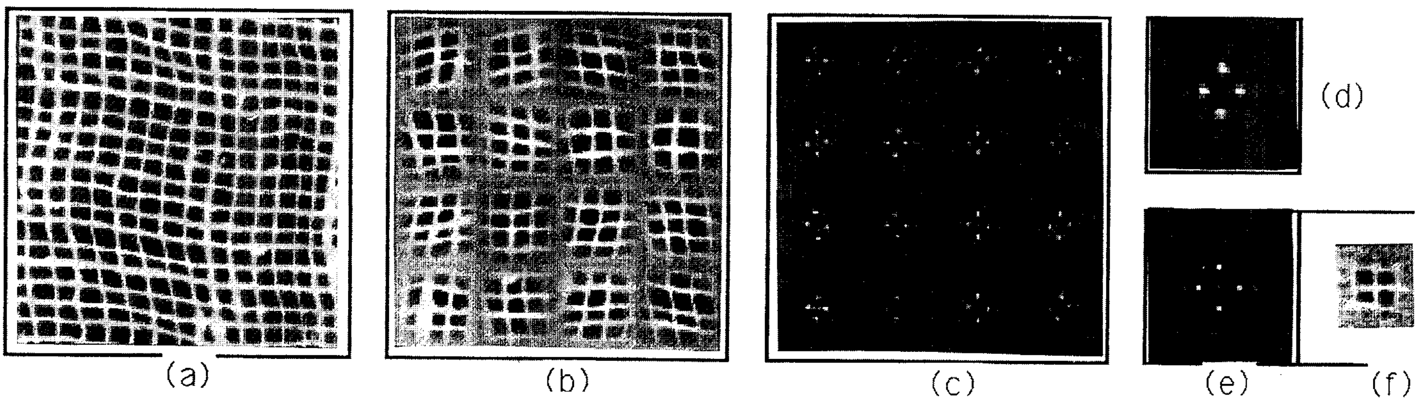


Fig. 2. Illustration of different analysis steps.

3. MODELING

The FT of a periodic texture is completely characterized by two fundamental frequencies and the complex values (equivalently magnitude and phase) at the network spanned by these two fundamental frequencies. The variation of each fundamental frequency from one sub-image to another may be roughly characterized by a covariance matrix with three components. The above considerations lead us to modeling a texture image by two types of parameter:

1) Parameters related to the two fundamental frequencies: their components and their covariance matrix.
2) Parameters related to magnitude and phase at the frequencies lying on the network generated by the two fundamental frequencies.

3.1. Parameter estimation

The two fundamental frequencies have already been determined in the analysis phase. Our task now consists of estimating their covariance matrix and the magnitude and phase at the frequencies lying on the network generated by these two fundamental frequencies.

Let us consider a real signal having only one frequency $(\mathbf{f}_0+\mathbf{f}_x)$, where $\mathbf{f}_x$ represents the variation of the central frequency $\mathbf{f}_0$. This variation is characterized by its density distribution $p(\mathbf{f}_x)$. The Fourier magnitude of this real signal in half-plan representation is:

$$|X(\mathbf{f})| = \sqrt{A}.\delta[\mathbf{f}-(\mathbf{f}_0+\mathbf{f}_x)]. \quad (8)$$

Let $W(\mathbf{f})$ be the FT of the analysis window, the power spectrum of the signal extracted by this window is easily obtained using the convolution equation:

$$A.|W[\mathbf{f}-(\mathbf{f}_0+\mathbf{f}_x)]|^2 \quad (9)$$

The average power spectrum $Y(\mathbf{f})$ may thus be expressed as:

$$Y(\mathbf{f}) = E\{\ A.Z[\mathbf{f}-(\mathbf{f}_0+\mathbf{f}_x)]\ \} = \int_{\mathbf{f}_x \in D_p} A.Z[\mathbf{f}-(\mathbf{f}_0+\mathbf{f}_x)]\ .\ p(\mathbf{f}_x)\ .\ d\mathbf{f}_x \quad (10)$$

where $Z(\mathbf{f}) = |W(\mathbf{f})|^2$ and D_p is the definition domain of $p(\mathbf{f}_x)$. If A is a constant then it can be verified that [10]:

$$\mathbf{B}_Y \approx \mathbf{C}_p + \mathbf{B}_Z \quad (11)$$

$$A \approx S_Y / S_Z \quad (12)$$

where $\mathbf{B}_Y$, $\mathbf{B}_Z$, S_Y and S_Z are given by:

$$\mathbf{B}_Y = (1/S_Y).\int_{\mathbf{f}\in D_Y} (\mathbf{f}-\mathbf{f}_0).(\mathbf{f}-\mathbf{f}_0)^t.\ Y(\mathbf{f}).\ d\mathbf{f}\ ;\ S_Y = \int_{\mathbf{f}\in D_Y} Y(\mathbf{f}-\mathbf{f}_0).\ d\mathbf{f} \quad (13)$$

$$\mathbf{B}_Z = (1/S_Z).\int_{\mathbf{f}\in D_Z} \mathbf{f}.\mathbf{f}^t.\ Z(\mathbf{f}).\ d\mathbf{f} \text{ and } S_Z = \int_{\mathbf{f}\in D_Z} Z(\mathbf{f}).\ d\mathbf{f} \quad (14)$$

and $\mathbf{C}_p$ is the covariance matrix of $p(\mathbf{f}_x)$.

If we apply eqs. (11) and (12) at a fundamental frequency, they allow us to estimate the covariance matrix and the energy associated with this frequency. If we apply eq. (12) to any harmonic frequency, we obtain an estimate of the energy associated with this frequency. For the phase at any harmonic frequency, it is sufficient to take the phase of the nearest sample. The phases at fundamental frequencies are zero by definition of TIT.

The integration domains must be local either for estimation of covariance matrix or for estimation of energy. The integration domain for estimation of the covariance matrix is circular and that for estimation of energy is a parallelogram such that the total energy over the Fourier domain is preserved. The precision of the covariance matrix estimation has been verified on a synthetic (perfectly periodic) texture. All components of the two matrices are found to be smaller than 10^{-2} for frequencies of approximately 6. This corresponds to a relative error of $\sqrt{(10^{-2})}/6 \approx 1.6\%$.

3.2. Reconstruction and number of parameters of the model

Inspired from the analysis phase and the parameter estimation method, we propose the following reconstruction method: the FT of the reconstructed sub-image is given by the sum of gaussian functions located on the network generated by the two fundamental frequencies. The weigth of each function is proportionnal to the corresponding energy extracted in last paragraph:

$$X'(\mathbf{f}) = \sum_{h_1=0}^{H} \sum_{h_2=-H}^{H} A(h_1,h_2).\exp\{\bar{\Psi}(h_1.\mathbf{F}_1+h_2.\mathbf{F}_2)\}.G(\mathbf{f}-h_1.\mathbf{F}_1-h_2.\mathbf{F}_2) \quad (15)$$

where $G(\mathbf{f})=\exp\{-(1/2).\mathbf{f}^t.\Sigma^{-1}.\mathbf{f}\}$; $\Sigma=\sigma\mathbf{I}$; $\mathbf{I}$ is the identity matrix. The bandwidth σ of the gaussian function is the same as that of the analysis window. The value of H depends obviously on the texture. For all textures used in our work, H=3 is visually sufficient. The total number of parameters of the model is:

$$\begin{aligned} N_p = \ & 2.\,[\,H^2+(H+1)^2\,]-3+ && (\text{ magnitudes and phases, minus 3 because } \bar{\Psi}(0)=\bar{\Psi}(\mathbf{F}_1)=\bar{\Psi}(\mathbf{F}_2)=0) \\ & 2.\,2+ && (\text{ 2 fundamental frequencies }) \\ & 2.\,3 && (\text{ 2 covariance matrices }). \\ N_p = \ & 57 \quad (H=4) && \end{aligned} \quad (16)$$

4. SYNTHESIS

The main idea of this method is to synthesize a texture image by first generating different sub-images and then by putting them together in an appropriate way.

The generation of sub-images works as follows. Let s(k,l) be the sub-image reconstructed in the last section, $\mathbf{A}_1$ and $\mathbf{A}_2$ be its fundamental periods, s'(k',l') be the sub-image to be generated and $\mathbf{A'}_1$ and $\mathbf{A'}_2$ be its fundamental periods. Let us recall that $(\mathbf{A}_1,\mathbf{A}_2)$ and $(\mathbf{A'}_1,\mathbf{A'}_2)$ have their equivalences $(\mathbf{F}_1,\mathbf{F}_2)$ and $(\mathbf{F'}_1,\mathbf{F'}_2)$ in Fourier domain given by eqs. (6). The two covariance matrices allow us to generate $(\mathbf{F'}_1,\mathbf{F'}_2)$ from $(\mathbf{F}_1,\mathbf{F}_2)$ using conventionnal random vector generator. $(\mathbf{A'}_1,\mathbf{A'}_2)$ are then calculated from $(\mathbf{F'}_1,\mathbf{F'}_2)$ using the eqs. (6). The determination of s'(k',l') is now possible by considering that s' is a deformation of s: s'(k',l')=s(k,l) with (k',l') related to (k,l) by:

$$(k'|l')^t = (\mathbf{A'}_1|\mathbf{A'}_2).\mathbf{z} \text{ and } (k\ |\ l)^t = (\mathbf{A}_1\ |\mathbf{A}_2\ ).\mathbf{z} \text{ thus } (k\ |\ l)^t = (\mathbf{A}_1\ |\mathbf{A}_2\ ).(\mathbf{A'}_1|\mathbf{A'}_2)^{-1}.(k'|l')^t. \quad (17)$$

For a given integer couple (k',l'), k and l are generally not integers and the determination of s'(k',l') requires the interpolation of s(k,l).

Now, we have a set of sub-images with random fundamental periods. The final step in the synthesis method consists of putting these sub-images together by taking into account the actual values of fundamental periods of adjacent sub-images. This idea is presented in Fig. 3. Let us assume that we have put together a number of sub-images, particularly sub-images number 1 and 2 and are now considering sub-image number 3. Thus, the available information is the position and fundamental periods of sub-images 1 and 2:

$$\{ X^{(1)}, Y^{(1)} \} ; \{ A'_1{}^{(1)}, A'_2{}^{(1)} \} \text{ and } \{ X^{(2)}, Y^{(2)} \} ; \{ A'_1{}^{(2)}, A'_2{}^{(2)} \}$$

and the fundamental periods of sub-image 3 (randomly generated): $\{ A'_1{}^{(3)}, A'_2{}^{(3)} \}$.
The problem is to determine $\{ X_3, Y_3 \}$. One simple solution is given below:

$$\{ X^{(3)}, Y^{(3)} \} = 0.5.\{ 0.5 [A'_1{}^{(3)} + A'_1{}^{(1)}] + \{X^{(1)}, Y^{(1)}\} + 0.5 [A'_2{}^{(3)} + A'_2{}^{(2)}] + \{X^{(2)}, Y^{(2)}\}\} \quad (1\text{ξ}$$

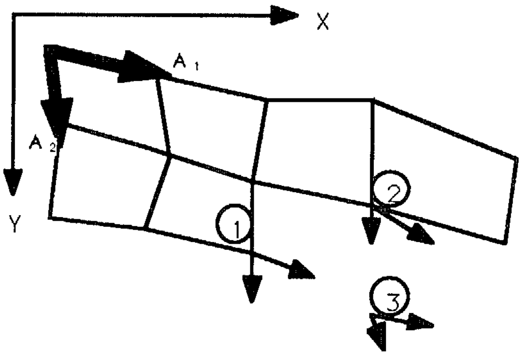
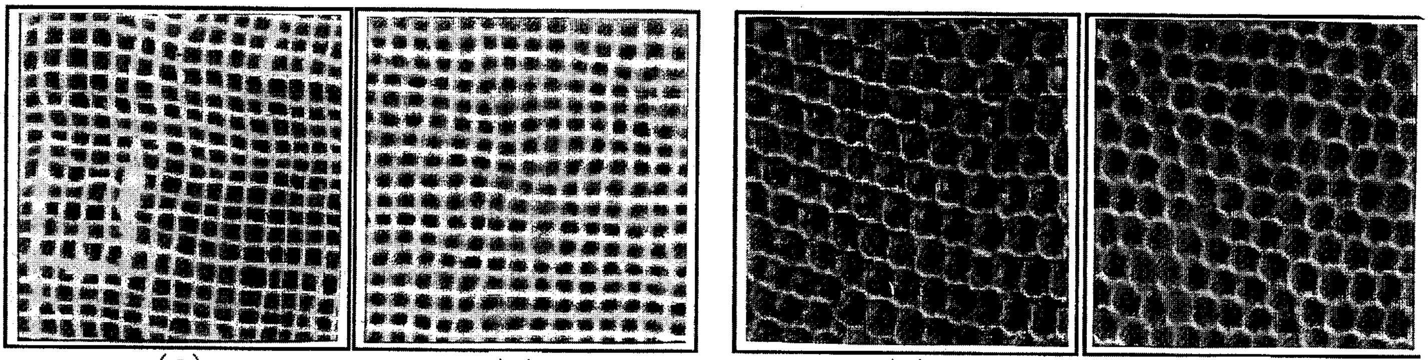


Fig. 3. Synthesis principle.

Two syntheses of real textures taken from Brodatz album [11] are shown in Fig. 4.



(a) (b) (c) (d)

Fig. 4. (a) and (c): original textures. (b(and (d): synthesized textures.

5. CONCLUSIONS

A simple and compact model of quasi-periodic textures has been proposed. This model is built on a Fourier based complete translation invariant transform. The phase is explicitly used without beeing tied to any spatial reference. This property is fulfilled probably for the first time.

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