Multiple-Source Localization: a new method exploiting the cyclostationarity property

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RESUME

On propose une nouvelle méthode pour estimer le nombre D de sources ponctuelles à longue distance et pour les localiser au moyen d'un réseau rectiligne de $M > D$ capteurs équidistants. La méthode exploite la cyclostationnairité des signaux modulés. Pour appliquer l'approche proposée il faut connaître (s'il y a) la valeur d'une même fréquence de cycle pour tous les signaux; de plus, la matrice de corrélation ciclique des signaux utiles doit être de rang D. La méthode présentée se montre supérieure aux approches classiques lorsque le signal utile est très faible et noyé dans un bruit à bande large et/ou dans une interférence à bande étroite et aussi lorsque le bruit est arbitraire (par exemple, non-stationnaire) et inconnu.

SUMMARY

A new method for detecting the number of radiating sources and for estimating their angles of location by a linear and uniform array is presented. The method exploits the property called cyclostationarity which is exhibited by all the modulated signals. The conditions of applicability of the method are: the existence and the knowledge of a cycle frequency common to each signal source, and the existence and the knowledge of a value of the lag parameter such that the cyclic cross-correlation matrix of the useful signals turns out to be full rank. The main advantage of the proposed method is its immunity to both wideband noise and narrowband interfering signals in weak signal conditions and to arbitrary and unknown interference environments.

1. Introduction

The problem of multiple-source localization consists of detecting the number of radiating sources and of estimating their angles of location.

In many physical problems, with radar, sonar and seismology as examples, the outputs of an array of sensors are collected over some time interval and used to extract the spatial structure of the multiple radiating sources. These are assumed to be located in the far field of the array so that the wavefronts received can be modeled as planewaves.

With reference to narrowband signals impinging on the sensor array most of the methods proposed in the literature [1-4] exploit the eigenstructure of the array covariance matrix (ACM) of the received signal vector to estimate the number of multiple planewaves and their directions of arrival (DOA's). The major drawback of these methods is that the ACM is unknown, and only an estimate from a finite sample size is available. Consequently, the resulting ACM eigenvalues are all different with probability one, and, hence, it can be difficult to determine the number of signals simply by 'observing' the eigenvalues.

The above mentioned methods require that the noise signals are uncorrelated and with the same variance (i.e., spatially white noise). Therefore, these methods can be utilized essentially when the significant noises are those generated internally to the sensor array instrumentations (thermal noise) and the sensors have the same physical characteristics.

In order to take into account external noises (e.g., interferences, jammers, etc.) the multiple-source localization problem has been solved [5,6] with reference to spatially nonwhite noise with stationary characteristics, provided that, however, an estimate of the noise ACM is available.

In adverse noise environments (low signal-to-noise ratios), since a large amount of collect time is required to obtain a satisfactory performance, the assumption of stationary noise is not generally reasonable and, therefore, the previous methods cannot be utilized unless some cumbersome procedure of updating noise statistics is performed.

In the present paper, with reference to arbitrary (not necessarily stationary) sensor noises and interferences, a new method is proposed which does not require the knowledge of the noise and interference covariance matrices in order to estimate the number of the radiating sources and their locations. The approach exploits the spectral correlation property [7] exhibited by the source signals which are reasonably modeled as cyclostationary processes. More precisely, the ACM of the received signal vector has at least a sinusoidal (cyclic) temporal component with a cycle frequency parameter, say α, depending only on the characteristics of the sources, provided that the noises and the interferences do not exhibit cyclostationarity with the same frequency α. The new method is based on the analysis of the eigenstructure of such a cyclic component of the covariance matrix which can be extracted from the ACM of the received signal vector by means of a synchronized averaging [7]. Finally, let us note that a recent paper [8] presents a simplification of the algorithms MUSIC [5] and ESPRIT [6] by exploitation of cyclostationarity property. This method, however, requires that each signal impinging on the sensor array exhibits at least a cycle frequency which is not shared by any other signal.

Section 2 proposes the new method and Section 3 presents simulation results illustrating the performance of the proposed method.

2. The proposed method

Consider a passive linear array consisting of M equispaced and isotropic sensors and assume that D narrowband sources ($D < M$), centered around a known frequency f_0, impinge on the array from the directions of arrival $\theta_1, \theta_2, \ldots, \theta_D$ measured with respect to the array normal.

The narrowband assumption, in the sensor array context, means that the amplitude and phase modulations of any signal of interest do not change significantly during the time it takes for the wavefronts to propagate across the array.

The $M \times 1$ vector of the received analytic signals can be expressed compactly as

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where: $\mathbf{s}(t)$ is the $D \times 1$ vector of the zero-mean analytic signals emitted by the sources as received at the reference point; $\mathbf{n}(t)$ is the $M \times 1$ vector of the zero-mean analytic noise signals; and $\mathbf{A}(\theta)$ is the $M \times D$ matrix of the steering vectors

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_D)] \quad (2)$$

where

$$\mathbf{a}(\theta_k) = [1, \lambda(\theta_k), \ldots, \lambda^{M-1}(\theta_k)]^T \quad (3)$$

(T denotes the transpose) is a Vandermonde-type vector with $\mathbf{a}(\theta_k) \triangle \exp[-j2\pi f_0 \Delta \sin(\theta_k)/c]$, where c is the propagation velocity of the wavefronts in the homogeneous and isotropic medium and Δ is the separation between the sensors ($\Delta \leq c/2f_0$).

The proposed approach to the source-localization problem exploits the cyclostationarity exhibited by all the modulated signals [7], ignored, vice versa, in the traditional methods. The approach is based on the properties of the cyclic cross-correlation function (CCF) defined by:

$$R^{\alpha}_{x_i x_j^*}(\tau) \underline{\Delta} < E[x_i(t+\tau/2)x_j(t-\tau/2)] \exp(-j2\pi\alpha t) > \quad (4)$$

where $< \cdot >$ and $E[\cdot]$ denote the time and ensemble (respectively) average and α is the cycle frequency parameter. For stationary processes the CCF is zero for any $\alpha \neq 0$. For narrowband cyclostationary processes, vice versa, it is proportional to the CCF of the real signals for some discrete values of α related to the periodicities of cyclostationarity [7].

The matrix of the cross-correlation functions between $x_i(t)$ and $x_j^*(t)$, according to eq.(1), can be written as

$$\begin{aligned} \mathbf{R}_{\mathbf{XX}^*}(t,\tau) \;&\Delta\; E[\mathbf{x}(t+\tau/2)\mathbf{x}^T(t-\tau/2)] = \\ &= \mathbf{A}(\theta)\mathbf{R}_{\mathbf{SS}^*}(t,\tau)\mathbf{A}^T(\theta) + \mathbf{R}_{\mathbf{nn}^*}(t,\tau) \end{aligned} \quad (5)$$

where the matrices $\mathbf{R}_{\mathbf{SS}^*}(\cdot,\cdot)$ and $\mathbf{R}_{\mathbf{nn}^*}(\cdot,\cdot)$ are obviously defined, and the assumption that noises and signals are uncorrelated has been accounted for.

Let us assume that all the signals exhibit cyclostationarity at the same frequency[1] $\alpha = 2f_0$, but the noises and interferences do not. It follows that the cyclic cross-correlation matrix (CCM) of the received vector for $\alpha = 2f_0$ is given by:

$$\mathbf{R}^{2f_0}_{\mathbf{XX}^*}(\tau) = \mathbf{A}(\theta)\mathbf{R}^{2f_0}_{\mathbf{SS}^*}(\tau)\mathbf{A}^T(\theta) \quad (6)$$

Equation (6) shows that, by selecting $\alpha = 2f_0$, the contributions to the CCM from arbitrary (not necessarily stationary and/or spatially white) noises and interferences vanish. Such a result allows us to predict for our approach, based on the eigenstructure of the CCM, a satisfactory performance even in strongly adverse interference environments. Moreover, our solution to the problem does not require the knowledge of the noise statistics which is essential, vice versa, in the traditional methods.

Assuming that the matrix $\mathbf{A}(\theta)$ is a full column rank (i.e., the direction vectors are linearly independent of one another) and the signal CCM $\mathbf{R}^{\alpha}_{\mathbf{SS}^*}(\tau)$ is full rank in correspondence of a known value of the lag parameter τ, it results that the rank of the matrix $\mathbf{R}^{\alpha}_{\mathbf{XX}^*}(\tau)$ is just equal to the number D of source signals, provided that $M \geq D$.

In order to evaluate the DOA's of the D signals impinging on the sensor array we will work with the $(D+1) \times (D+1)$ leading principal submatrix of $\mathbf{R}^{2f_0}_{\mathbf{XX}^*}(\tau)$, following a procedure similar to that proposed by Berni [2]. Since the $(D+1) \times (D+1)$ submatrix of $\mathbf{R}^{2f_0}_{\mathbf{XX}^*}(\tau)$ has rank D, it results that there is at least one eigenvalue equal to zero[2] whose corresponding eigenvector, say $\mathbf{u}$, satisfies the equations:

$$\mathbf{a}^T(\theta_k)\mathbf{u} = 0, \quad k = 1, 2, \ldots, D \quad (7)$$

On the assumption of a uniform linear array, $\mathbf{a}^T(\theta_k)\mathbf{u}$ is a polinomial of degree D in the variable $\exp(j2\pi f_0 \Delta \sin\theta/c)$ and, therefore, the roots of (7) allow us to obtain unambiguously the source angles $\theta_1, \theta_2, \ldots, \theta_D$.

Let us note that the proposed method requires the knowledge of the CCM $\mathbf{R}^{2f_0}_{\mathbf{XX}^*}(\tau)$ and, therefore, it is necessary to perform an estimate:

$$\begin{aligned} \hat{\mathbf{R}}^{2f_0}_{\mathbf{XX}^*}(\tau) \;&=\; \frac{1}{2K+1}\sum_{l=-K}^{K} \hat{\mathbf{R}}_{\mathbf{XX}^*}\left(\frac{l}{2f_0(2K+1)}, \tau, f_0\right) \\ &\cdot \exp[-j2\pi l/(2K+1)] \end{aligned} \quad (8)$$

[1]The narrowband signals with carrier frequency f_0 do not exhibit necessarily cyclostationarity at $\alpha = 2f_0$. For example, for quadrature balanced amplitude modulated signals the above assumption does not hold [7].

[2]If a $M \times M$ nonHermitian matrix has rank D, the number L of eigenvalues equal to zero satisfies the relation $L \geq M - D$.

where

$$\hat{\mathbf{R}}_{\mathbf{xx}^*}(t,\tau,f_0) = \frac{1}{2L+1}\sum_{i=-L}^{L}\mathbf{x}\left(t+\frac{\tau}{2}+\frac{i}{2f_0}\right) \cdot \mathbf{x}^T\left(t-\frac{\tau}{2}+\frac{i}{2f_0}\right) \quad (9)$$

is an estimate from a finite sample size of the limit periodic cross-correlation function.

Finally, it is worthwile to emphasize that the crucial point of the proposed method is to find (if it exists) a value of the lag parameter τ such that the matrix $\mathbf{R}_{\mathbf{ss}^*}^{2f_0}(\tau)$ turns out to be full rank. In the traditional methods based on the conventional covariance matrix ($\alpha = 0$), on the assumption that the signals are mutually uncorrelated, it is obvious to choose $\tau = 0$ to assure the nonsingularity of the matrix since the magnitude of the autocorrelation functions peaks at $\tau = 0$. In our approach, vice versa, even if one assumes uncorrelated signals, the choice of τ is not obvious since the magnitude of the cyclic cross-correlation can reach its maximum at $\tau \neq 0$, depending on the type of modulation exhibited by the signals of interest.

The main advantage of the new proposed method is its potential immunity to noise and interfering signals in adverse (weak signals) conditions and arbitrary and unknown interference environments (stationary or nonstationary noises, spatially white or coloured noises).

3. Simulation results

In this section we present simulation results to illustrate the performance of the new method. We refer to a uniform linear array of M sensors with interelement spacing of half the common wavelenght of the D wavefronts impinging from directions $\theta_1, \theta_2, \ldots, \theta_D$. The examples consider amplitude-modulated source signals with modulating signals modeled as zero-mean wide-sense stationary processes obtained by filtering white Gaussian processes by a Butterworth filter of first order with a fractional bandwidth of 0.004. The choice of amplitude modulated signals leads [7] to CCF's whose magnitude peaks at $\tau = 0$ that becomes the obvious selected value for the lag parameter.

The estimation of the rank of the matrix (8) has been performed by evaluating the modulus of the determinants of the leading principal submatrices of increasing order. To simplify the comparison among the determinants all terms of the matrix have been scaled by the factor

$$1/M^2 \sum_{i,j=1}^{M} \mid \hat{R}_{x_i x_j^*}^{2f_0}(0) \mid \quad (10)$$

In the first example we consider an array of $M = 4$ sensors and one source signal with direction of arrival $\theta = -0.5 rad$. The tested signal-to-noise ratios (SNR's) range from -10 dB to 30 dB and the number of samples per period $2K+1$ (see eq. (8)) is assumed (here and in the other examples) equal to 20. Figure 1 shows the modulus of the determinants, say $\mathcal{D}$, as a function of the number $p \underline{\Delta} 2L+1$ of the observed periods $1/2f_0$ with reference to SNR= 0 dB. It turns out that the rank can be correctly determined provided that the number of observed periods is sufficiently large. The time required for the convergence to the correct estimate is a decreasing function of the SNR: in particular the increase from 0 to 10 dB leads to a decrease of the required observation time of one order of magnitude. As regard to the estimate of the source location, the obtained value for an observation of 1000 periods has been: $\hat{\theta} = -0.4928$ for SNR= 0dB.

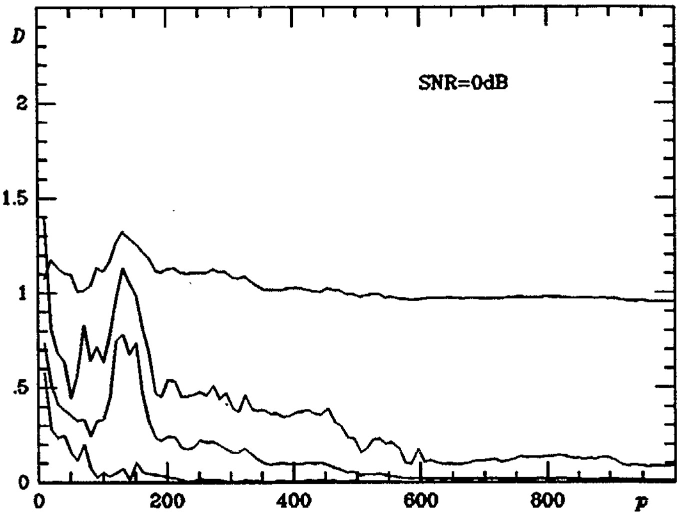

Fig.1 Modulus $\mathcal{D}$ of determinants vs. number p of observed periods for 4 sensors,1 useful signal,and noise (SNR=0dB).

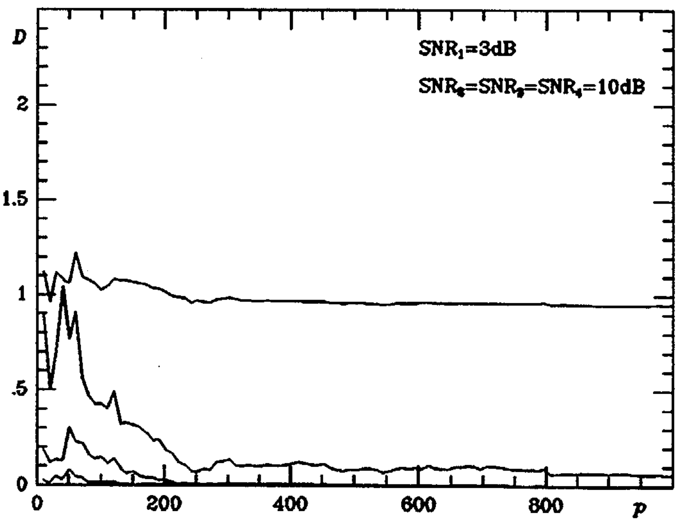

Fig.2 Modulus $\mathcal{D}$ of determinants vs. number p of observed periods for 4 sensors, 1 useful signal, and noise (different SNR's at the sensors).

The proposed method have been tested in operative conditions of different SNR's at sensors providing a satisfactory performance, as evidenced, for example, in Fig.2, where one source signal is considered and the SNR's are assumed to be $SNR_1 = 3$dB, $SNR_2 = SNR_3 = SNR_4 = 10$dB. The estimate of the source location has been $\hat{\theta} = -0.4965$.

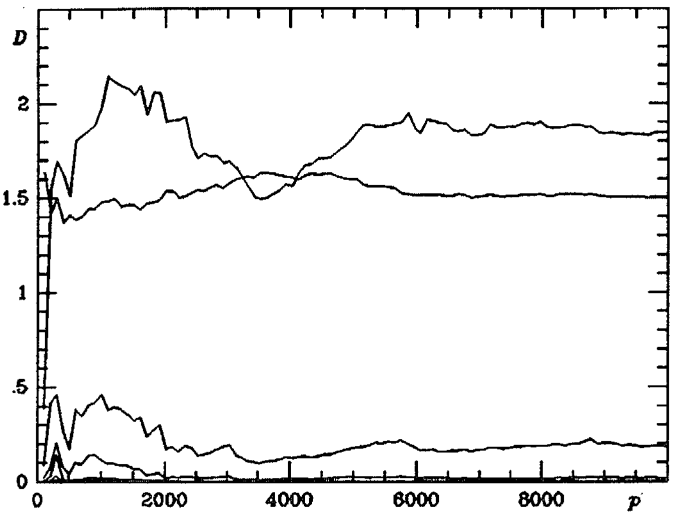

Fig.3 Modulus D of determinants vs. number p of observed periods for 8 sensors, 2 useful signals, noise, and two narrowband interfering signals.

In the final experiment, with reference to $M = 8$ sensors, the simultaneous presence of two useful signals, two narrowband interfering signals (with carrier frequencies $0.8 f_0$ and $0.95 f_0$) and wideband spatially nonwithe sensor noises is accounted for (Fig. 3). The power of all useful and interfering signals has been fixed at 1 unit, whereas the noise power distribution at the different sensors is: 1.0, 1.0,-1.0, 0.9, 1.3, 1.1, 0.8, 1.2. Although a larger number of periods is required in order for interferences and noises to decorrelate the cyclic cross-correlation estimates, the performance is fully satisfactory even in these adverse operative conditions where the traditional methods can be applied only if the noise and interference covariance matrices are available.

4. Conclusions

A new method for locating simultaneously multiple signal sources by a linear and uniform array is presented. The approach exploits the cyclostationarity exhibited by all modulated signals. More precisely, the array covariance matrix of the received signal vector has at least a sinusoidal temporal component with a cycle frequency α depending only on the characteristics of the sources provided that noises and interferences do not exhibit cyclostationarity at the same frequency α.

The method is based on the analysis of such a cyclic component of the covariance matrix. Unlike the traditional methods, the new algorithm does not require the knowledge of the noise and interference covariance matrices in order to estimate the number of the radiating sources and their angular locations.

The conditions required to apply the new approach are: the existence and the knowledge of a cycle frequency common to each source of interest, and the existence and the knowledge of a value of the lag parameter such that the cyclic cross-correlation matrix of the useful signals turns out to be full rank.

The main advantage of the proposed method is its potential immunity to both wideband noise and narrowband interfering signals in adverse (weak signals) conditions and to arbitrary and unknown interference environments (stationary or nonstationary noise, spatially white or coloured noise).

Various numerical experiments have been conducted to observe the algorithm behaviour in different operative conditions with reference to amplitude modulated signals. The results show a satisfactory accuracy in determining the number and the angular location of the signal sources.

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