ADAPTIVE NOTCH FILTERING WITH HYBRID MAXIMUM LIKELIHOOD ESTIMATOR

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RESUME

Les filtres d'encoche de l'adaptation conventionnel souvent manquent du sens de la présence des sinusoids faibles ou convergence aux frequences faux. Cette article propose une nouvelle algorithm de la vraisemblance maximum quadratique itératif (IQML) et recurrente hybrid (RML). Le resultat de simulation par l'ordinateur est annoncé dans cette artile.

SUMMARY

The conventional adaptive notch filters often fail to sense the presence of weak sinusoids or converge to the wrong frequencies; This paper proposes a new hybrid recursive (RML) and iterative quadratic maximum likelihood algorithm (IQML) to improve its performance. Extensive computer simulation results are done and reported in this paper.

I. INTRODUCTION:

Adaptive notch filters are very interesting in many signal processing applications such as radar, communications, control, biomedical engineering and others. Bhaskar Rao and Kung [1] proposed a constrained adaptive IIR notch filter using 2P parameters where p is the number of notches, the poles and zeros are constrained to lie on the same radial line. More recently Nehorai [2] and Ng [3] have developed an improved p parameter notch filter design using recursive maximum likelihood algorithm (RML), an additional constraint is required that the zeros of its transfer function will also lie on the unit circle and its coefficients will have a mirror symmetric form. When the input sinusoids have large power difference or bad initial vector, the above adaptive notch filters often fail to sense the presence of weak sinusoids or converge to the wrong frequencies.

This paper proposes a new hybrid recursive (RML) and iterative quadratic maximum likelihood algorithm (IQML) to improve the performance. The advantages to estimate the input sinusoidal frequencies and their power levels can be gained with this new hybrid algorithm; Extensive computer simulations are done and reported in this paper.

II. RML (Recursive Maximum Likelihood) ALGORITHM:

The z transfer function of adaptive notch filter with mirror symmetry is given by

$$H(z) = \frac{A(z)}{A(\alpha z)} = \frac{\sum_{i=0}^{2p} a_i z^{-i}}{\sum_{i=0}^{2p} a_i \alpha^{i} z^{-1}}, \quad \alpha \lessapprox 1 \tag{1}$$

where the numerator's coefficients are mirror symmetric, i.e. $a_0=a_{2p}-1$ and $a_i=a_{2p-i}$, $i=1,\dots,p-1$ and the zeros are constrained to lie on the unit circle.

The RML method has been used to update the filter coefficient vector by Nehorai [2] and Ng [3], the exact detail of the RML method can be referred to Reference 3; Due to the limited space, instead we describe the RML algorithm in detail, we like to do some remarks and modification to improve the algorithm. The RML algorithm provides asymptotic approximations of the sinusoidal frequencies to maximum likelihood for sufficiently large data. In practical applications, the factor $\alpha(t)$ has an important effect on the performance of the algorithm. As Reference 2 describes, the initial value of $\alpha(t)$ must be small for the algorithm to be sensitive to the input sinusoids, while the final value must be large and close to 1 to produce narrow notches such that the distortion of the broadband signals may be decreased. Besides, the slow time variation of $\alpha(t)$ at the start of the algorithm can increase the filter's sensitivity, and the slow time variation at the end can improve the algorithm's asymptotic performance (that is, increase the accuracy of the estimation)

The effect of $\alpha(t)$ is more crucial when there is great difference of powers between input sinusoids; In such case, the weak power sinusoids seem to be hidden in the strong ones, if $\alpha(t)$ is not chosen properly, the filter may fail to sense the presence of weak sinusoids.
Curve A in Fig.1 shows the time variation of $\alpha(t)$ proposed by Nehorai [2] and Ng [3], where

$$\alpha(t)=\alpha_0^t[\alpha(0)-\alpha(\infty)]+\alpha(\infty) \tag{2}$$

$\alpha_0=0.99$, $\alpha(0)=0.8$ and $\alpha(\infty)=0.995$

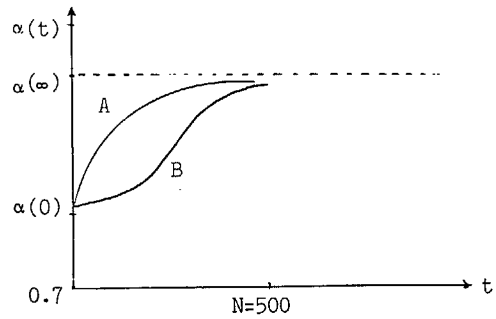

Fig.1 The time-variation curves of $\alpha(t)$.

We propose another time variation type of $\alpha(t)$ which is shown as Curve B in Fig.1. Suppose the number of iterations is N, N is even, then

$$\alpha(t)=\begin{cases} \alpha(0)+\alpha_0^{(\frac{N}{2}-t)}\left[\frac{\alpha(\infty)-\alpha(0)}{2}\right], & t\le\frac{N}{2} \\ \alpha(\infty)-\alpha_0^{(t-\frac{N}{2})}\left[\frac{\alpha(\infty)-\alpha(0)}{2}\right], & t>\frac{N}{2} \end{cases} \tag{3}$$

The sharp variation α(t) at the start of Nehorai's algorithm suggests that this filter has less sensitivity than the new proposed one. We have done some simulation results of the two α(t) curves, we find that Nehorai's curve fails to sense the presence of the weak sinusoids in many cases (Table I and Fig.3(a), 4(a)). The new curve improves the sensitivity greatly in Fig.3(b). However there are still some cases that the new curve fails (see Fig.4(b)).

Note that if both curves work, Nehorai's curve will produce deeper notches since its asymptotic performance is better in which we can find from the α(t) time variation at the end in Fig.1. Small value α(0) increases the filter's sensitivity. However, if the number of iterations is fixed, smaller α(0) will result in less estimated accuracy.

III. ITERATIVE QUADRATIC (IQML) AND HYBRID MAXIMUM LIKELIHOOD ESTIMATORS:

The IQML algorithm in [4] can be combined to estimate the parameter vector of the notch filters, this batch processing algorithm provides more accurate parameter estimations than other existing current algorithms especially when short data length (Snapshots) is used [4]. However if large measured data set is available, IQML computation becomes time consuming, we may prefer RML algorithm for its computation efficiency. In this section we suggest a new hybrid algorithm combined IQML with RML; By using short data IQML estimator, we avoid the problem caused by α(t) in using RML alone, while by using RML, we get a more efficient estimator than IQML used alone for large data estimation.

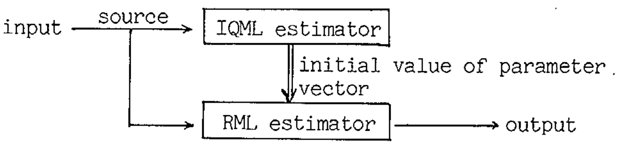

Fig.2 Hybrid Maximum Likelihood Estimator.

Fig.2 shows an algorithm, where we use an IQML estimator to estimate the input sinusoidal frequencies roughly, then we transfer the estimated frequencies to a RML estimator as initial values of the filter's parameter vector. By setting the initial values of the filter's parameter in the neighborhood of input sinusoidal frequencies, we can solve the sensitivity problem caused by α(t). Besides, since a rough frequency estimation in IQML algorithm is enough for the initialization, we use only a few data snapshots for the IQML estimator and use the rest measured data for iterations in RML algorithm to get more accurate estimations. Therefore the initial recursive calculations of IQML are not large and the whole algorithm is more efficient than using IQML estimator alone.

Some additional advantages can be gained with this new hybrid algorithm. If the length of the adaptive notch filter is less than required, some sinusoids won't be removed. Although in practical applications when the number of sinusoids is unknown, we can use a large filter length to cancel such residual sinusoids, the overdetermined length not only goes to a waste of computation and hardware, but also produces additional trivial notches which will cause some ambiguity and extra distortions for the signal.

The conventional algorithm in Reference [1] and [2] assumes the number of input sinusoids is known a priori. Therefore the order of the filter can be set to match the required length. In our algorithm, the IQML estimator has the ability to estimate the number of input sinusoids and their powers, thus there is no need for such assumption.

If the order of IQML estimator, 2No, is greater than or equal to the required length 2Ns, where Ns is the number of input sinusoids, i.e. No≥Ns, then for the actual input sinusoidal frequencies, the IQML estimator indicates their real power estimations; as for those additional pseudo-sinusoidal frequencies, it will result in very weak estimated powers. We can set a proper threshold of the estimated power, for example -6 dB, to exclude the pseudo-sinusoidal information, and transfer only the true information of input sinusoids to the RML estimator. This not only saves some hardware also has more efficient computation in updating the parameters.

IV. COMPUTER SIMULATIONS:

Fig.4(c), Fig.4(d) and Table II show some successful simulation results in which both α(t) curves fail previously. Since the filter's sensitivity is not so crucial in this hybrid algorithm, we choose large values of α(0)=0.8 to increase its accuracy. The data length of snapshort is 10 and the number of data points used in IQML is 19 before switch to RML, from the results we find 200 iterations for the hybrid algorithm are enough to form the notches to remove input sinusoids (Fig.4(c)); Also it has more deeper notches with 500 iterations in Fig.4(d).

V. CONCLUSIONS:

This paper discusses in detail the adaptive notch filtering with hybrid maximum likelihood estimator, it overcomes the conventional problems with failure to sense the weak sinusoids or convergence to the wrong frequencies. This new hybrid algorithm has some additional advantages to estimate the input sinusoidal frequencies and their power levels, the order of the filter can be set quite correctly to match the required length. Extensive computer simulation results are done and reported in this paper.

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[4] Y. Besler and A. Macovski, "Exact maximum likelihood parameter estimation of superimposed exponential signals in noise," IEEE Trans. on Acoustics, Speech, Signal Proc., Vol.ASSP-34, No.5, pp.1081-1089, Oct. 1986.

TABLE I Computer Simulation of RML Algorithm with Two $\alpha(t)$ Curves.

	I/P Sinusoids (freq. SNR)	$\alpha(0)$	$\alpha(t)$ - time varying curve: Nehorai est. freq.	$\alpha(t)$ - time varying curve: New est. freq.	The filter power gain at input sinusoid frequency: Nehorai	The filter power gain at input sinusoid frequency: New
Fig.3	0.125, 20 0.35, 3	0.8	0.1249896 0.5235937	0.1249961 0.3500121	-49.6 Fig.3(a) 0.2*	-58.0 Fig.3(b) -48.3
	"	0.7	0.1249931 0.3500206	0.1249963 0.3500805	-53.1 -43.6	-58.7 -31.8
	0.125, 10 0.35, 20	0.7	0.4186735 0.3499829	0.1250102 0.3499924	0.2* -45.2	-49.7 -42.5
	0.125, 10 0.35, 30	0.7	0.5354736 0.3499883	0.4329209 0.3499924	0.2* -48.5	0.2* -52.2
	0.6, 20 0.85, 6	0.7	0.6000627 0.6447833	0.5999982 0.8502223	-34.0 0.2*	-64.2 -23.0
	"	0.6	0.5999881 0.8501392	0.5999689 0.850228	-48.3 -27.1	-40.1 -23.0
Fig.4	0.125, 3 0.35, 20	0.7	0.65922562 0.3499917	0.6407779 0.3499983	0.2* Fig.4(a) -51.6	0.2* Fig.4(b) -65.0
	0.6, 20 0.85, 3	0.7	0.6000079 0.5753609	0.5999755 0.6527046	-51.9 0.2*	-42.1 0.2*

ps: "*" denotes the sinusoidal frequency that adaptive notch filter fails to sense.
Number of iterations: 500

TABLE II Computer Simulation of Hybrid IQML + RML Algorithm with No=Ns.

	I/P Sinusoids (freq. SNR)	IQML est. freq.	IQML + RML iteration	IQML + RML est. freq.	The filter power gain at input sinusoid frequency
Fig.4(c)	0.125, 3 0.35, 20	0.1406902 0.3516122	200	0.1243600 0.3500385	-14.0 Fig.4(c) -38.2
Fig.4(d)	"	"	500	0.1249875 0.3500068	-48.1 Fig.4(d) -53.3
	0.125, 10 0.35, 30	0.1314404 0.3504702	200	0.1246884 0.3500116	-20.1 -48.6
	"	"	500	0.1249893 0.3500018	-49.3 -64.7
	0.6, 20 0.85, 3	0.5993693 0.8504055	200	0.6001100 0.8504893	-29.1 -16.2
	"	"	500	0.5999868 0.8502190	-47.5 -23.1

ps: The number of snapshots used for IQML algo. is 10.
$\alpha(0)$ for RML is 0.8.

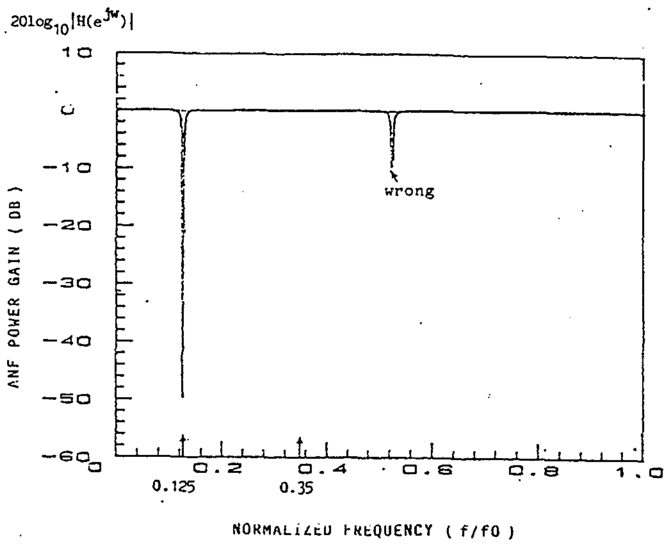

Fig. 3(a) The frequency magnitude response of RML adaptive notch filter with Nehorai's $\alpha(t)$ curve, Notice that the sinusoidal freq. 0.35 is not sensed with 500 iterations. (The experimental parameters are listed in Table I.)

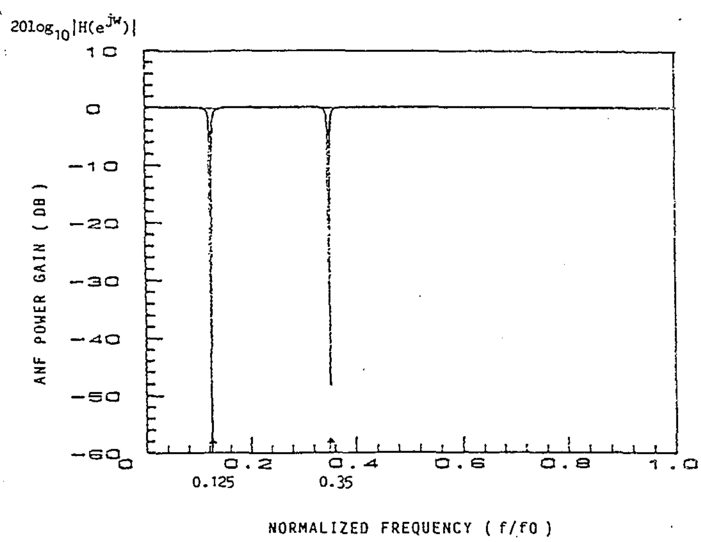

Fig. 3(b) The frequency magnitude response of RML adaptive notch filter with new $\alpha(t)$ curve; it correctly senses the two input sinusoidal frequencies with 500 iterations. (The experimental parameters are listed in Table I.)

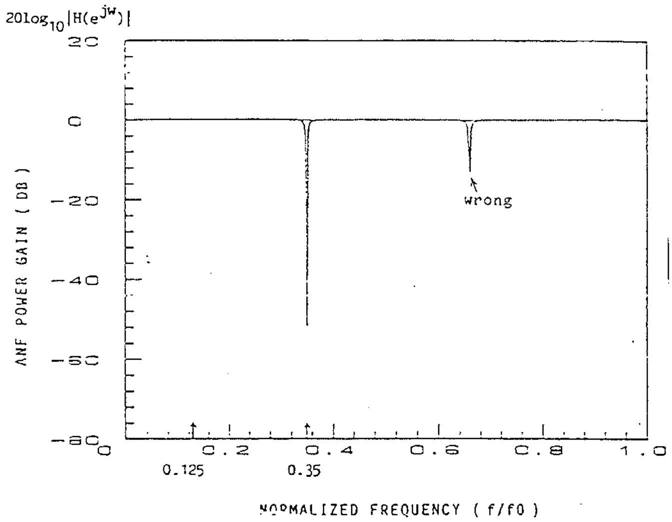

Fig. 4(a) The frequency magnitude response of RML adaptive notch filter with Nehorai's $\alpha(t)$ curve, the sinusoidal frequency 0.125 is not sensed with 500 iterations (see Table I).

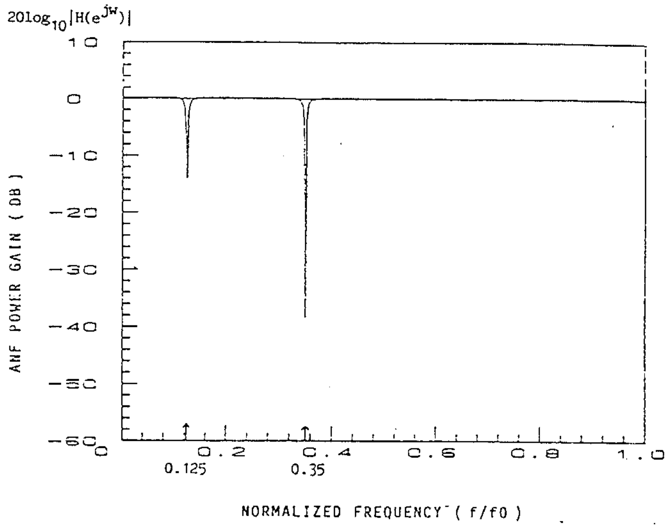

Fig. 4(c) The frequency magnitude response of hybrid IQML + RML adaptive notch filter, it correctly senses two input sinusoidal frequencies with 200 iterations (see Table II).

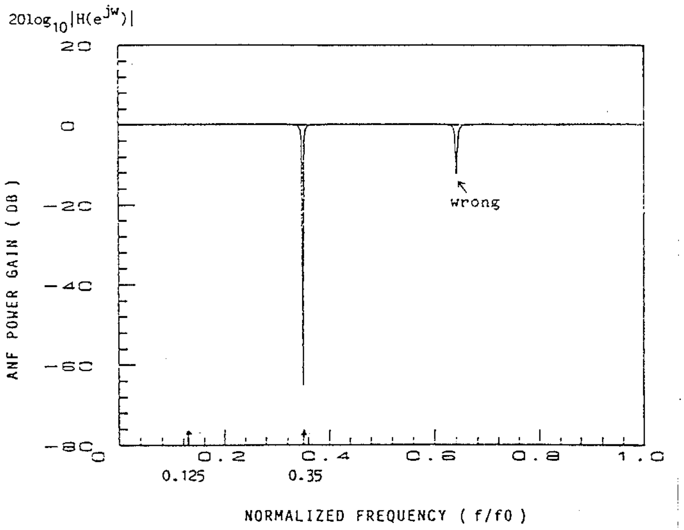

Fig. 4(b) The frequency magnitude response of RML adaptive notch filter with New $\alpha(t)$ curve, the sinusoidal frequency 0.125 is still not sensed with 500 iterations (see Table I).

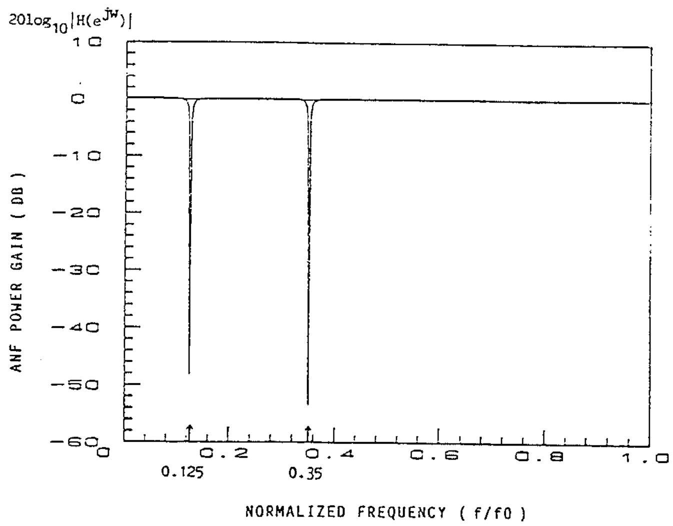

Fig. 4(d) The frequency magnitude response of hybrid IQML + RML adaptive notch filter, it correctly sense two input sinusoidal frequencies with 500 iterations (see Table II). Notice it has much more deeper notches than in Fig. 4(c).