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OPTIMUM STRUCTURE OF M-ARY MODULATED SIGNALS FOR ERROR DETECTING CODES

Giuliano Benelli

Dipartimento di Ingegneria Elettronica, Università di Firenze,
via S.Marta 3, Firenze, Italy.

RESUME

L'intégration de la modulation et de la codage de canal dans un système de communication a canal rétractif permit de méliorer les performances. Dans cette communication on analyse un nouvel méthode pou le combination de la codage de canal dans une modulation avec M symboles . Cette méthode gagnes meilleur performances relativement à semblables systèmes.

SUMMARY

In some ARQ schemes, the combination of modulation and channel coding improves the performance of a communication system. This paper describes a new method for the integration of an error detecting code with a M-ary modulation .The described method permits to improve both the error probability and the throughput of an ARQ protocol with respect to other similar schemes.

1. Introduction

Automatic Repeat Request (ARQ) techniques are largely used today in order to reduce the error probability in a communication system. In fact, these techniques are more flexible than the Forward Error Correction (FEC) schemes and, in addition, offer a lower implementation complexity [1],[2]. The main disadvantage of the ARQ techniques lies in their low throughput, particularly for medium and high error rates in the communication channel. An interesting approach is the introduction of a memory in the decoding operation [3]. In this case the received vectors, even those detected in error, are retained, to facilitate the correct decoding of the transmitted codeword .

A recent paper [4] proposes the integration of the channel coding and modulation in an ARQ protocol . In this way a significant improvement in the performance of a communication system using the ARQ techniques can be achieved, particularly for high error rates in the communication channel. However, the method proposed in [4] can be usefully applied only to continuous-phase modulations or similar schemes, having a phase continuity between successive time-signaling intervals.

Over the last few years, a great attention has been focused on non-binary modulation schemes, such as M-ary PSK, QAM, AM/PSK. These schemes permit achieving higher information rates at the expense of a higher implementation complexity. However, their performance can be enhanced by combining channel coding in the modulation process.

This paper describes a new method for the combination of the modulation and the error-detecting code in an ARQ protocol. The mapping rule can be applied to any M-ary modulation scheme with M>2, even in the case in which no phase continuity between successive time-signaling intervals is introduced.

2. ARQ scheme with combined M-ary modulation and channel coding

The general block-diagram of the ARQ scheme here considered is shown in Fig. 1. The source generates symbols from a finite alphabet $A=\{a_1,a_2,...,a_N\}$ with N elements. These symbols are encoded through a code C. Before transmission, each symbol c_i is sent to a modulator having N different waveforms. The modulator associates to c_i the waveform $s_{i,p}(t)$ in the interval $[(i-1)T,iT]$,if $c_i=a_p$, with T the time-signaling interval. The form of $s_{i,p}(t)$ depends on the modulation scheme used.

The performance of a communication system depends on the Euclidean distances between the modulates signals $s_{i,p}(t)$. The integration of the modulation operation in the error-detecting code of an ARQ

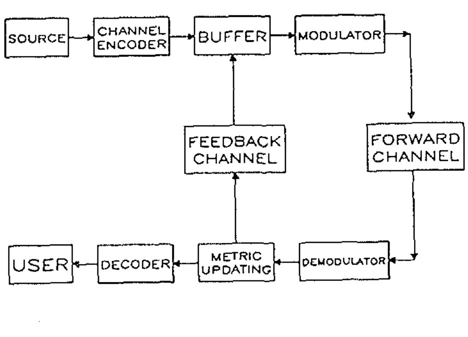

Fig. 1

protocol make it possible to improve significantly the Euclidean distances, as it has been shown in [4] for Continuous Phase Modulations (CPM) .

In this paper two new methods for the combination of the modulation operation and an error detecting code, in order to improve the Euclidean distances among modulated signals in the case of non-binary modulation, are presented.

The first method, denoted as MARQ1, uses a suitable memory at the receiver side to improve the performance of the ARQ protocol. Let us consider the transmission of a codeword $\underline{c}$. The codeword is divided into s subblocks, each b symbols long. The i-th subblock is denoted by $\underline{c}_i=\{c_{i,p}\}$ and can assume 2^b configurations. Each component $c_{i,p}$ is a symbol of the B alphabet and can be transformed into a vector with m components from the A alphabet. Therefore, vector $\underline{c}$ can be transformed into a vector $\underline{c}'$ with nm components defined on the A alphabet. The i-th subblock $\underline{c}_i$ is transformed in a vector $\underline{c}'_i$ for $1 \leq i \leq s$ with $k_i=mb$ components.

The subblock $\underline{c}'_i$ is encoded through a suitable rule Φ in a vector $\underline{v}_i=\Phi(\underline{c}'_i)$, defined on the alphabet A and composed of $n_i=qk_i$ components, with q an integer greater or equal to 2. The vector $\underline{v}_i$ is divided in q frames, each composed of k_i symbols,i.e. :

$$\underline{v}_i = [\underline{v}_i(1),\underline{v}_i(2),......,\underline{v}_i(q)]$$

It is also assumed that the transformation Φ is invertible and that it is possible to recover $\underline{c}'_i$ from the exact knowledge of any frame $\underline{v}_i(j)$. The conditions which must satisfy code C_i to meet this requirement have been given in [5]. However, these conditions are satisfied by many classes of codes and therefore do not limit significantly the choice of code C_i. It is also assumed for simplicity that $\underline{v}_i(1)=\underline{c}'_i$, but this hypothesis is not generally necessary.The vector $\underline{v}_i(p)$ for $1 \leq p \leq q$ can be transformed into a vector $\underline{v}'_i(p)$, defined on the alphabet B, and composed of b symbols.

During the j-th transmission of a message $\underline{c}$ the transmitter sends the vector $\underline{w}_j$ given by :

$$\underline{w}_j = [\ \underline{v}'_1(p),\ \underline{v}'_2(p),.....,\ \underline{v}'_s(p)\ ]$$

with p an integer defined as :

$$p = \begin{cases} j \bmod q & \text{if } j \neq dq \\ q & \text{if } j = dq \end{cases}$$

and d an integer. For the hypothesis being considered, it results $\underline{w}_1=\underline{c}$. The vector $\underline{w}_j$ is sent to the modulator, which associates to it a signal vector $\underline{s}_j(t)=\{s_{j,u}\}$,of nT seconds duration. The vector is divided into s subblocks , each bT seconds long and can assume 2^b configuration; the u-th configuration of the i-th subblock is denoted by $\underline{s}_{j,i}(u)$.

The received signal $\underline{r}_j(t)$, nT seconds long, is divided into s subblocks $\underline{r}_j(i)$ for $1 \leq i \leq s$. Naturally, we can write :

$$r_j(i)=[r_{(i-1)q+1}(t),r_{(i-1)q+2}(t),..,r_{iq}(t)]$$

where $r_u(t)$ represents the received signal in the u-th time signaling interval.

Let us consider the first transmission of $\underline{c}$. The vector $\underline{r}_1(t)$ is divided into s subblocks $\underline{r}_1(i)$. The demodulator evaluates the Euclidean distances $d^2_{i,p}(1)$ between the $\underline{r}_1(i)$ and the p-th configuration of $\underline{s}_{1,i}(p)$ for $1 \le p \le 2^b$. The Euclidean distances $d^2_{i,p}(1)$ are stored in a distance vector $D_i(1)$, with p-th components $D_{i,p}(i) = d^2_{i,p}(1)\}$.

Now let us consider the j-th transmission of $\underline{c}$ for $j \ge 2$. The receiver evaluates the Euclidean distance $d^2_{i,p}(j)$ between $\underline{r}_j(i)$ and $\underline{s}_{j,i}(p)$ and updates the distance vector in the following way:

$$D_{i,p}(j) = D_{i,p}(j-1) + d^2_{i,p}(j)$$

The vector $D_i(j)=\{D_{i,p}(j)\}$ with 2^b components is termed the cumulative vector of the Euclidean distances. Once the vector $D_i(j)$ has been constructed, the receiver chooses as i-th subblock $(1 \le i \le s)$ of $\underline{c}$ the p-th configuration yielding :

$$D_{i,p}(j) = \max_{\substack{1 \le l \le s \\ l \ne p}} D_{i,l}(j)$$

When all the s subblocks have been decoded, the vector $\underline{c}=(\underline{c}_1,\underline{c}_2,\ldots,\underline{c}_s)$ is sent to the C decoder. If $\underline{c}$ is a codeword of C, i.e. no error is detected in $\underline{c}$, then it is assumed as the transmitted codeword and a positive acknowledgment (ACK) is sent to the transmitter. Conversely, a negative acknowledgment (NACK) is forwarded to the trasnmitter, which provides a new copy of the message.

The performance of the proposed ARQ schemes depends significantly on the choice of the mapping rule $\Phi[\underline{v}_i(j-1)]$, because it influences the Euclidean distances between the signals vectors $\underline{v}_i(j)$.

In the following two mapping rules , which afford significant improvement in Euclidean distances, are described.The first rule, denoted M1, uses the structure of a binary block code to improve the Euclidean distances in the signal space. Let us to consider a systematic block code C_1 defined on the A alphabet and having a codeword length n_1 with k_1 information symbols. In this case the mapping function Φ is represented by the encoding procedure of code C_1. The vectors $\underline{v}_i(l)$ for $2 \le l \le q$ defined in (4) represent pieces of the redundant symbols associated to the k_1 information symbols $\underline{v}_i(1)$. Let us consider the j-th transmision of a message . If j=1 the transmitter send $\underline{v}_i(1)$, i.e. the information symbols $\underline{c}_i$, and the transmitted vector is $\underline{c}$. If $j=2e+1$, where $e \ge 0$ is an integer, a redundancy vector composed by the vectors $\underline{v}_i(p)$ for $2 \le p \le s$ is transmitted.

The second mapping rule, denoted M2, has been derived in an empirical way. It presents low demodulation complexity, but it allows achieving high Euclidean distances in many cases. Let us consider the case in which the i-th symbol c_i is equal to the p-th symbol a_p of the alphabet A $(1 \le p \le M)$. During the j-th transmission the rule M2 associates to $c_i=a_p$ the u-th signal $s_{i,m}(t)$ given in (1), with

$$u = [q-1+(p-1)j] \bmod M$$

As an example of the application of the M2 rule , we shall consider the case of a PSK with M=8. The signal set associated to the first transmission of M symbols is shown in Fig. 2.a, where each alphabet symbol is near the corresponding signal. Fig. 2.b shows the signal associated to each symbol of alphabet A during the second transmission . In the example, phase zero is alwyas associated to the symbol $c_i=a_1$.

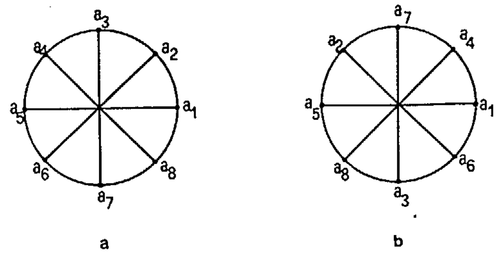

Fig. 2

3. Results and comparisons

As previously stated, the increase in the implementation complexity resulting from the MARQ1 and ARQ1 protocols with respect to the classical ARQ protocols is generally high, particularly when the M1 mapping rule is used. However, the MARQ1 and the ARQ1 methods also offer high performance for high or medium error rates in the communication channel and permit appreciable extension of the region over which the ARQ protocol can be usefully applied.

The bit error probability and throughput of the proposed ARQ schemes depend on the Euclidean distances and, in particular, on the minimum Euclidean distance $d^2_m(j)$ in the signal space. Therefore, results for $d^2_m(j)$ are first presented.

In the M1 mapping rule, the Euclidean distances depend upon the choice of code C_1. Only cyclic codes as C_1 was considered herein. A computer search was performed to determine the code C_1 giving the higher minimum Euclidean distance for fixed n_1 and k_1. Because the code C_1 has a code rate of 1/q, the increase in the Euclidean distance between the j-th and (j+1)-th transmission is the same as that between the (q+j)-th and (q+j+1)-th transmissions.

Fig. 3 shows the minimum Euclidean distance in the case of a PSK modulation with M=8. Curves b and c refer to the M1 mapping rule using as C_1 the code (12,6) and (15,3) resepctively, while curve d to the M2 mapping rule. The improvement achieved by using the M1 mapping rule is further enhanced. As an example, for j=2, while MARQ scheme gives $d^2_m(2)=1.758$, the M1 mapping rule with a (12,6) C_1 code gives $d^2_m(2)=6$ with a gain of 6.27 db. The minimum Euclidean distance achieved using the M2 mapping rule is represented by curve b. In this case it is obtained $d^2_m(2)=6$, as in the previous case. As can be seen in the

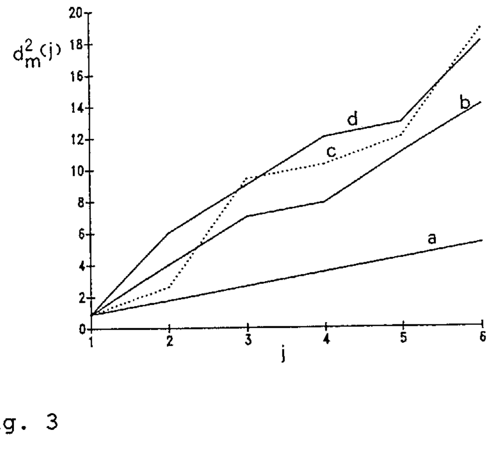

Fig. 3

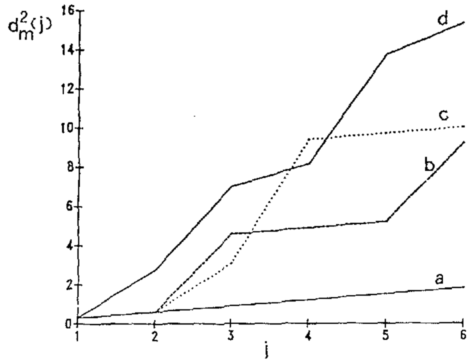

Fig. 4 -

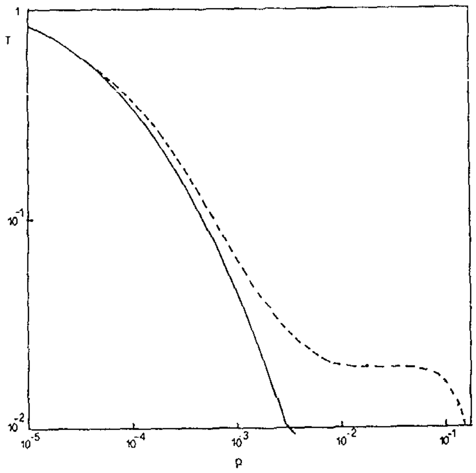

Fig. 5

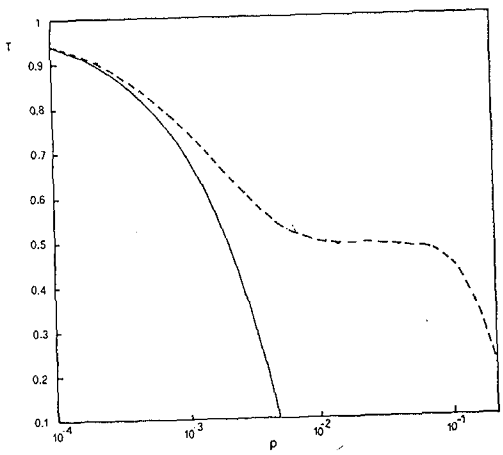

Fig. 6

figure, the M2 mapping rule gives high Euclidean distances and, in some cases, outperforms other more complex structures using the M1 rule.

Similar results are given in Fig. 4 for a PSK modulation with M=16. Curves b and c refer to the M1 mapping rule using as C_1 the code (8,4) and (16,4) resepctively, while curve d to the M2 mapping rule.

The increase in the Euclidean distance permits improving the throughput and reducing the bit error probability of an ARQ protocol. To evaluate throughput , the code C is assumed a perfect error-detecting code, i.e., a code able to detect all the errors introduced by the transmission channel. Fig. 5 shows the throughput of a Go-Back-N protocol versus the channel error probability p, by assuming a round-trip delay equal to the time required to transmit N=50 codewords. Similarly, Fig. 6 shows the throughput of a selctive protocol versus p. In Fig. 5 and 6 a PSK modulation with M=8 has been considered; continuous curves represent the throughput of a classical protocol, while dotted cirves refer to the MARQ protocol using the M2 mapping rule.

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