SIGNAL INTERCEPTION:
A UNIFYING THEORETICAL FRAMEWORK FOR FEATURE DETECTION

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In this paper, a unifying approach to the design and analysis of quadratic detectors for signal interception is presented. The methods of detection that are incorporated in this unification include radiometry, delay-and-multiply chip-rate-feature detection, filter-and-square carrier-feature detection, dual-channel-correlation detection, ambiguity-plane feature detection, Wigner-Ville time-frequency-plane feature detection, spectral-correlation-plane feature detection, liklihood-ratio detection for weak signals, maximum-deflection detection, and optimum spectral-line-regeneration detection. The unification reveals the fundamental role that spectral correlation and spectral-line-regeneration play in the signal interception problem. It also suggests the spectral-correlation plane approach as a general approach to interception that offers great flexibility as well as inherent tolerance to one of the most challenging problems in interception, namely, accomodating unknown and changing noise level and interference activity.

I. INTRODUCTION

Interception of communications is attempted for a variety of reasons including reconnaissance, surveillance, and other intelligence gathering activities, as well as position fixing, identification, and communications jamming. For example, interception plays an important role in arms control verification and in prevention of illegal operations such as drug smuggling. Typically, the interceptor has knowledge of no more than the communicator's frequency band, modulation format, and modulation characteristics such as bandwidth and hop rate or chip rate. It is commonly held that the most appropriate approaches to the detection task for signal interception must be based on radiometry, that is, measurement of received energy in selected time and frequency intervals (cf. [1]-[5]). It is also commonly recognized that such radiometric methods can be highly susceptible to unknown and changing noise levels and interference activity. There have been many proposals for methods of countering such complications, including various approaches to adjusting or adapting threshold levels, and adaptive filtering, cancelling, and directional nulling of interfering signals. But these problems remain as the most serious impediment to signal detection and other signal interception tasks (cf. [4]-[6]).

The radiometric approach to detection is based on the use of stationary random processes as models for the signals to be intercepted. However, for the purposes of signal interception, the signal of interest is more appropriately modeled as a cyclostationary random process, that is, a random process whose probabilistic or statistical parameters vary periodically with time [7]. Since the message contained in the modulated signal is unknown, it is usually modeled as a stationary random process (discrete-time or continuous-time). This stationarity coupled with the periodicity of sine wave carriers, pulse trains, repeating spreading codes, etc. results in a cyclostationary model for the signal. However, these cyclostationary signals typically do not exhibit spectral lines, because the spectral lines of the unmodulated carriers and/or pulse trains are spread out over relatively broad bands by the stationary random modulation.

The purpose of this paper is to use the unifying framework of the spectral correlation theory of cyclostationary signals to present a broad treatment of weak random signal detection that clearly reveals the relationships among the variety of detectors that have been proposed, or are in the development stage, or are in use, and to present several arguments with supporting results that favor cyclic-feature detection over energy detection for accommodating the problems associated with unknown and changing noise levels and interference activity. Cyclic features result from the characteristic property of cyclostationarity called regenerative periodicity, which means that spectral lines can be regenerated from the signal with the use of appropriate quadratic transformations [8].

II. CYCLOSTATIONARITY AND SPECTRAL CORRELATION

If there is more than one source of regenerative periodicity in a signal and the periods are not all commensurate, then the process is called almost cyclostationary since its parameters are almost periodic functions of time (that is, sums of periodic functions with incommensurate periods) [7],[8]. For example, the autocorrelation function

$$R_x(t+\tau/2, t-\tau/2) = E\{x(t+\tau/2)x(t-\tau/2)\} \qquad (1)$$

for the signal $x(t)$, by virtue of the (almost) cyclostationarity of $x(t)$, will be a (almost) periodic function of the variable t, and will therefore admit the Fourier series representation

$$R_x(t+\tau/2, t-\tau/2) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{i2\pi\alpha t} , \qquad (2)$$

where the sum is over integer multiples of fundamental frequencies (reciprocals of periods), such as carrier frequency, baud rate, chip rate, hop rate, and their sums and differences. The Fourier coefficients $R_x^{\alpha}(\tau)$, which depend on the lag parameter τ, are given by the formula

$$R_x^\alpha(\tau) = \lim_{Z\to\infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} R_x(t+\tau/2, t-\tau/2) e^{-i2\pi\alpha t} dt. \quad (3)$$

(If there is only one period, say T, then Z can be chosen equal to T, and the limit in (3) can be omitted.) If x(t) is a cycloergodic process [7] (which it always will be if an appropriate model is used), then after substitution of (1) into (3), the expectation operator can be omitted to obtain

$$R_x^\alpha(\tau) = \lim_{Z\to\infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} x(t+\tau/2)x(t-\tau/2) e^{-i2\pi\alpha t} dt. \quad (4)$$

(When (4) is used in place of (3), the limit $Z\to\infty$ cannot be omitted when there is only one period.) The function $R_x^\alpha(\tau)$ is called the cyclic autocorrelation function. For $\alpha=0$, it reduces to the conventional autocorrelation function $R_x^0(\tau)$. Whereas $R_x^0(\tau)$ can be seen from (4) to be the dc component of the lag-product waveform $x(t+\tau/2)x(t-\tau/2)$ for each value of τ, $R_x^\alpha(\tau)$ can be seen to be the ac component corresponding to the sine wave frequency α.

The Fourier transform

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-i2\pi f\tau} d\tau \quad (5)$$

is called the cyclic spectral density function. For $\alpha=0$ it reduces to the conventional power spectral density function, that is, the spectral density of time-averaged power. However, for $\alpha\neq 0$, it can be shown [7]-[9] that $S_x^\alpha(f)$ is the density of spectral correlation, that is, the density of correlation between spectral components at the frequencies $f+\alpha/2$ and $f-\alpha/2$. Specifically, it is shown in [7]-[8] that

$$S_x^\alpha(f) = \lim_{T\to\infty} \lim_{Z\to\infty} \frac{1}{TZ} \int_{-Z/2}^{Z/2} X_T(t, f+\alpha/2) X_T^*(t, f-\alpha/2) dt \quad (6)$$

where $X_T(t,f)$ is the complex envelope of the narrowband spectral component with center frequency f and bandwidth on the order of 1/T,

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(u) e^{-i2\pi fu} du \; . \quad (7)$$

Thus, $S_x^\alpha(f)$ is also called the spectral correlation function, and it follows from the preceding discussion that spectral correlation is a characteristic property of cyclostationarity of the autocorrelation.

The most common approach to modeling signals for interception studies (cf. [4],[5]) is to ignore cyclostationarity by either (i), introducing a random phase variable Θ uniformly distributed over one period of the cyclostationarity (or a sum of such phases, one for each period of an almost cyclostationary process) so that $x(t+\Theta)$ becomes stationary [7], [10]--that is only the α=0 term in (2) is then non-zero--or (ii), using the time-average approach based on (4), and simply ignoring the $\alpha\neq 0$ averages. This approach is appropriate if there is no desire to exploit cyclostationarity, and it leads to the popular conclusion that the radiometer is essentially the optimum detector (cf. [7].) Therefore the adoption in this paper of the cyclostationary model marks the point of departure of this work from many previous studies of signal interception. Nevertheless, since the stationary model is a special case of the cyclostationary model, the results derived in this paper include the more conventional results as special cases.

III. DETECTION BY SPECTRAL-LINE REGENERATION

Perhaps the most straightforward interpretation of most cyclic-feature detectors is that they use a nonlinearity to regenerate a spectral line from the noisy modulated signal, and then use a bandpass filter, DFT, or other methods of spectral analysis to detect the presence of the regenerated spectral line which is masked by continuous spectral components due to noise, interference, and the signal of interest itself. Since the signal-to-noise ratio (SNR) is often very low in an interception environment, the lowest order nonlinearity is usually chosen since an SNR of less than 0dB becomes increasingly lower as the order of the nonlinearity used is increased. Thus, most feature detectors use quadratic nonlinearities. (An exception is carrier regeneration for balanced QAM signals, such as QPSK, since these require higher order nonlinearities for spectral-line regeneration.) Hence, most feature detectors are quadratic time-invariant systems which can always be represented by

$$y_\alpha(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_\alpha(u,v) x(t-u) x(t-v) du dv \; , \quad (8)$$

where α represents the frequency of the spectral line to be regenerated from x(t). For example, for the dual-channel-correlation feature detector, we have

$$k_\alpha(u,v) = \int_{-\infty}^{\infty} h_1(u-w) h_2(v-w) g_\alpha(w) dw \quad (9)$$

where $h_1(t)$ and $h_2(t)$ are the impulse-response functions of bandpass filters with center frequencies f_1 and f_2, and $g_\alpha(t)$ is the impulse-response function of a bandpass filter with center frequency $\alpha = f_2 - f_1$. Similarly, for the delay-and-multiply chip-rate-feature detector, we have (9) with $h_2(t) = h_1(t-\tau_0)$, where the delay τ_0 is typically chosen to be half the chip interval, $h_1(t)$ is the impulse-response function of a band-selection filter, and α is the chip rate. Also, for the filter-squarer carrier-feature detector, we have (9) with $h_2(t) = h_1(t)$, and α is the doubled carrier frequency.

The approach in the past has been to choose the particular detector structure, and then to optimize its parameters, such as the delay τ_0 and prefilter bandwidths (cf. [11]). An alternative approach that puts these ad hoc detectors into better perspective is to analytically solve for the kernel $k_\alpha(u,v)$ in the general representation (8) that regenerates the strongest possible spectral line at some appropriate frequency α for a specific signal type. This same objective applies not only to the design of feature detectors for interception but also to the design of synchronizers that operate on the basis of using a phase-lock loop to lock on to the phase of a regenerated carrier or clock signal.

It has recently been shown [8], [12] that the particular kernel that maximizes the SNR of the regenerated spectral line for a cyclostationary signal s(t) in additive stationary Gaussian noise and interference n(t), $x(t) = s(t) + n(t)$, is specified by

$$k_\alpha(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_\alpha(f,\nu) e^{i2\pi(fu-\nu v)} du dv, \qquad |u|,|v| < T/2 \; , \quad (10)$$

where T is the collect time of the detector and

$$K_\alpha(f,\nu) = \frac{S_s^\alpha(f-\alpha/2)^*}{S_n^0(f) S_n^0(f-\alpha)} \delta(f-\nu-\alpha), \qquad \alpha\neq 0. \quad (11)$$

(In the derivation of (10)-(11), it is assumed that T is large enough that a spectral window of width 1/T will resolve $S_s^\alpha(f)$ and $S_n^0(f)$.) Thus, the optimum spectral line regenerator is completely specified by the spectral correlation function for the signal and the power spectral density for the noise plus interference. Furthermore, the maximized value of SNR is given by

$$SNR_{max}^\alpha = \frac{T}{2} \int_{-\infty}^{\infty} \frac{|S_s^\alpha(f)|^2}{S_n^0(f+\alpha/2) S_n^0(f-\alpha/2)} df, \qquad \alpha\neq 0, \quad (12)$$

where SNR^α is defined to be the ratio of the power in the regenerated spectral line to the power in the band of width 1/T centered at frequency α, due to the noise plus interference. Substitution of (11) into (10) and the result into (8) yields

$$y_\alpha(t) = \int_{-\infty}^{\infty} S_s^\alpha(f)^* S_{z_T}^\alpha(t,f)df(e^{i2\pi\alpha t}), \quad (13)$$

where $S_{z_T}^\alpha(t,f)$ is the cyclic periodogram for z(t),

$$S_{z_T}^\alpha(t,f) \triangleq \frac{1}{T} Z_T(t,f+\alpha/2)Z_T^*(t,f-\alpha/2), \quad (14)$$

and $Z_T(t,f)$ is the frequency-weighted finite-time Fourier transform of x(t) (cf. (7))

$$Z_T(t,f) = X_T(t,f)/S_n^0(f). \quad (15)$$

In conclusion, if the noise is white ($S_n^0 = N_0$), then the maximum-SNR spectral-line-regeneration detector measures the cyclic periodogram of the received data x(t) and correlates it (over f) with the ideal cyclic spectral density (spectral correlation) function for the signal to be detected s(t). On the other hand if there is strong narrowband interference as well as white noise, then the measurement $X_T(t,f)$ is first notched by division by $S_n^0(f)$ to obtain (15). Then the cyclic periodogram is formed and it is correlated with the ideal cyclic spectral density function for the signal. For weak signals, $S_s^0(f) \ll S_n^0(f)$, we have $S_n^0(f) \cong S_x^0(f)$ and $S_x^0(f)$ can be measured from the received data x(t).

For specific signal types, the general form of the optimum detector can often be reduced to more familiar forms. For example, it is shown in [12] that for PSK type signals (13) reduces to a narrowband-rejection filter for narrowband interference removal, followed by a matched filter, followed by a squarer and a bandpass filter. This reveals that the optimum prefilter for the filter-squarer detector is a matched filter, and that although the optimum delay for the delay-and-multiply detector with no prefilter is half the chip interval, the optimum pre-filter is a matched filter and, when this is used, the optimum delay is zero.

IV. DETECTION BY CYCLIC SPECTRAL ANALYSIS

Although such familiar forms as a matched-filter-squarer are intuitively appealing, they are not necessarily as flexible as the general form (13). For example, if the particular signal type of interest is not known sufficiently well to obtain a good approximation to the spectral correlation function $S_s^\alpha(f)$ used in (13) as a weighting function before integration over f, then it can be replaced with a simple window such as a rectangle, whose width Δf is chosen to be as large as possible without exceeding the widths of features expected to be present in $S_s^\alpha(f)$, and whose center f is a variable parameter. The resultant detection statistic is

$$y_\alpha(t,f) = \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{z_T}^\alpha(t,\nu)d\nu \; . \quad (16)$$

This frequency-smoothed cyclic periodogram is a standard estimate of the frequency-weighted ideal cyclic spectral density function $S_z^\alpha(f)$. In fact, it can be shown [8] that

$$\lim_{T\to\infty} y_\alpha(t,f) = \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} \frac{S_x^\alpha(\nu)}{S_n^0(\nu+\alpha/2)S_n^0(\nu-\alpha/2)} d\nu. \quad (17)$$

Moreover, it can be shown that the variance of $y_\alpha(t,f)$ is inversely proportional to $T\Delta f$ [8]. Furthermore, if n(t) exhibits no cyclostationarity with cycle frequency α (or no spectral correlation with frequency separation α), then $S_n^\alpha(f) \equiv 0$ and therefore $S_x^\alpha(f) \equiv S_s^\alpha(f)$. In conclusion, if $y_\alpha(t,f)$ were measured and graphed as a time-variant surface above the (f,α) plane, then the presence of recognizable spectral correlation features could be used to detect signals of interest and also to classify them according to modulation type. This approach to detection and classification based on cyclic spectral analysis was first proposed in [13]. A variety of methods for cyclic spectral analysis are described in [8] and [14]. The unique spectral correlation (cyclic spectrum) surfaces for a wide variety of modulation types are calculated and graphed in [7], [8], [15], [16].

V. AMBIGUITY-PLANE AND WIGNER-VILLE-TIME-FREQUENCY-PLANE METHODS OF DETECTION

Having revealed the central role played by spectral correlation and cyclic spectral analysis in the spectral-line-regeneration approach to detection, it is a simple matter to interpret the ambiguity-plane approach and the Wigner-Ville time-frequency-plane approach in terms of optimum cyclic-feature detection. Like the three conventional feature detectors considered in Section III, these two approaches are also ad hoc. They have not been derived from any objective design criterion. Nevertheless, when properly modified, they can be made equivalent to the optimum spectral-line-regeneration detector. This follows directly from the facts that the ambiguity function can be obtained by inverse Fourier transformation of the cyclic periodogram $S_{x_T}^\alpha(t,f)$ in the f variable and the Wigner-Ville function can be obtained by inverse Fourier transformation of $S_{x_T}^\alpha(t,f)$ in the α variable. (This is explained in [8], [9]). In both cases t is the time-index for evolution of these two-dimensional surfaces.

It is emphasized that the smoothing operation used in the spectral correlation plane (or cyclic-spectral-analysis) method derived from the optimum spectral-line-regeneration detector in Section IV is important since the variance of the measured spectral correlation function is inversely proportional to the product of the collect time T and the smoothing window width Δf. (In fact, the SNR performance of the cyclic-spectral-analysis detector is proportional to $T\Delta f$ provided that Δf is not too large.) Thus, methods based directly on the Wigner-Ville function without smoothing in f, or on the ambiguity function without weighted integration in τ, would not be expected to perform comparably. Furthermore, the characteristic of the Wigner-Ville function of adding contributions from <u>all</u> values of α is a probable source of poor performance for two reasons: (i), the signal of interest makes its primary contributions to the detection surface at only values of α equal to its cycle frequencies, and (ii), even the contributions at the cycle frequencies of the signal should not in general be added directly without proper phase compensation. This is explained in the next section.

VI. LIKELIHOOD-RATIO DETECTION

To gain further insight into the potential and the limitations of the optimum spectral-line-regeneration detector and its cyclic-spectral-analysis adaptation, we describe the relationship between the spectral-line-regeneration detector and the likelihood-ratio detector.

It is explained in [8] and [12] that the statistic for likelihood-ratio detection (for each time interval [t-T/2, t+T/2]) of a weak zero-mean (almost) cyclostationary signal in additive stationary Gaussian noise and/or interference is closely approximated by

$$y(t) = \sum_\alpha \int_{-\infty}^{\infty} S_s^\alpha(f)^* S_{z_T}^\alpha(t,f)df, \quad (18)$$

which is simply the sum over all cycle frequencies of the complex envelopes of the maximum-SNR spectral-line detection statistics (13). Thus, this optimum multi-cycle detector measures the cyclic periodograms of the received corrupted signal x(t) for all cycle frequencies α contained in the signal to be detected s(t), correlates these (over f) with stored replicas of the ideal cyclic spectral densities of s(t), and then adds up these correlations. (If the signal were modeled as stationary, then only the α=0 term would remain and this would yield the optimum radiometer, cf. [7]). Unfortunately, even if the modulation type and its parameter values (e.g., carrier frequency and chip rate for a BPSK signal) are known, the optimum multi-cycle detector (18) cannot be implemented without knowledge of the phase of the signal because the quantities $S_s^\alpha(f)$ depend on this phase. Incorrect phases of the individual terms in (18) can result in destructive interference rather than constructive interference when the sum over α is performed.

Although (18) might not be practical for implementation, it lends further support to the cyclic spectral analysis approach to detection since this approach enables the user or an automated algorithm to exploit more than one cycle frequency of the signal of interest. This can be beneficial not only for detection, but also for identification of signal-modulation type as well as for signal analysis (parameter estimation). In fact, it is shown in [12] that maximization of (18), with respect to signal parameters on which $S_s^\alpha(f)$ depends, yields their weak-signal maximum-likelihood estimates.

VII. MAXIMUM-DEFLECTION DETECTION

An alternative approach to arriving at the multi-cycle detector (18) as a detector with optimality properties is based on a performance measure called deflection. Specifically, the deflection is a measure of output SNR that is particularly appropriate for weak-signal detection. For a detection statistic y(t), the deflection is defined by

$$d(t) \triangleq \frac{|E\{y(t)|s(t)\text{present}\} - E\{y(t)|s(t)\text{absent}\}|}{(\text{var}\{y(t)|s(t)\text{absent}\})^{1/2}} \quad (19)$$

It is shown in [7] that the quadratic transformation that maximizes deflection is identical to the optimum multi-cycle detector (18). It is also shown in [7] that the value of the maximized deflection is given to a close approximation by the sum of maximized SNRs,

$$d^2(t)_{max} \cong \sum_\alpha SNR^\alpha_{max} \quad (20)$$

where SNR^α_{max} is given by (12).

VIII. A FUNDAMENTAL DISTINCTION BETWEEN RADIOMETERS AND CYCLE DETECTORS

The radiometer output contains a spectral line at $\alpha = 0$ regardless of whether or not the signal is present, but the cycle detector (13) contains a spectral line at $\alpha \neq 0$ only if the signal is present. Thus, the radiometer must distinguish between the strength of the spectral line at $\alpha = 0$ due to signal plus noise and/or interference, and the spectral line at $\alpha = 0$ due only to noise and/or interference, whereas the cycle detector need only distinguish between the presence and absence of a spectral line at $\alpha \neq 0$. This follows directly from the formula [8], [12]

$$P_y^\alpha = \int_{-\infty}^{\infty} K(f+\alpha/2, f-\alpha/2) S_x^\alpha(f) df \quad (21)$$

for the spectral-line power of any quadratic detector (K is the double Fourier transform of the kernel k, cf. (8) and (10)) and the fact that

$$S_x^\alpha(f) = \begin{cases} S_s^\alpha(f), & \alpha \neq 0, \text{ signal present} \\ S_s^0(f) + S_n^0(f), & \alpha = 0, \text{ signal present} \\ 0 & \alpha \neq 0, \text{ signal absent} \\ S_n^0(f), & \alpha = 0, \text{ signal absent,} \end{cases} \quad (22)$$

where it is assumed that the noise plus interference n(t) does not exhibit cyclostationarity with cycle frequency α, $S_n^\alpha(f) \equiv 0$. This greatly complicates the problem of setting the threshold level to be used with the radiometer and renders the radiometric approach to detection inherently more susceptible than the cycle detector to unknown and/or changing noise and interference, especially for weak signals.

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