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VECTOR QUANTIZATION OF RANDOM MODEL SOURCES

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RESUME

Résumé

La qualité de la quantification vectorielle (VQ) utilisant l'algorithme de Linde, Buzo et Gray et la mesure de la distance Euclidienne a été évaluée en mesurant le rapport signal sur bruit de quantification (SNR) pour des processus stochastiques stationnaires. L'évaluation était faite pour des dimensions d'un à dix et pour un nombre des intervalles de quantification 2, 4 et 8 - conforme à 1, 2 et 3 bits par échantillon. Nous avons étudié des processus stochastiques avec des distributions des amplitudes gaussiennes, laplaciennes et suivant fonctions K_0 ou gamma avec des types des correlations différentes.
Les résultats montrent que la qualité de la VQ est fortement déterminée par le type de correlation mais qu'elle depend seulement d'une façon insignifiante de la distribution. Avec le processus gaussien on obtient toujours - c'est à dire avec tous les types de correlations - des valeurs SNR les plus grands qui étaint au maximum 2 dB sous la "rate distortion function". Pour les processus statistiquement indépendants les valeurs SNR dépendent d'une manière significative de la statistique d'amplitude. L'ordre des valeurs SNR d'un quantificateur vectoriel des dimensions plus grandes sont en accord avec les conclusions de la théorie d'information. Quand on augmente la dimension les valeurs SNR arrivent aux limites théoretiques.
Finalement nous nous avons occupés de la question, si la méthode VQ est en fait la méthode la plus efficace. Au-delà nous avons étudié la convergence de l'algorithme de construction du dictionnaire.

SUMMARY

Summary

The performance of vector quantization (VQ) based on the LBG algorithm and Euclidian distance measure was evaluated for dimensions up to 10 and for quantization rates up to 3 by signal-to-quantization-noise ratio (SNR) measurements for stationary random processes. The investigations included random processes with Gaussian, Laplacian, K_0-, and Γ-distributed amplitudes without memory and with memories of different types. The results show that the performance of VQ is significantly determined by the memory type but depends only slightly on the amplitude distribution. With all memory types the highest SNR-values are obtained for the Gaussian process. These SNR-values approach to the rate distortion functions within 2 dB. For memoryless processes the SNR-values are strongly dependent on the amplitude statistics. The order of the SNR-values for vector quantizers of higher dimensions is in accordance with that predicted by information theory. With increasing dimensions the SNR-values tend towards the theoretical limits. Furthermore for uniform, Gaussian, and Laplacian statistically independent random processes the problem how far a VQ is an optimum quantizer is discussed as well as the convergence behaviour of the codebook design.

VECTOR QUANTIZATION OF RANDOM MODEL SOURCES

Introduction

In information theory stationary stochastic processes with specific amplitude distribution densities and memory structures are considered as models of information sources. Those models offer the possibility of an analytical, at least a numerical treatment of basic signal processing problems such as quantization, prediction, encoding, or level-crossing behaviour.

In this study the recent concept of vector quantization (VQ) is analyzed with widely used random sources. The performance of VQ based on the Linde-Buzo-Gray algorithm [1] and on the Euclidian distance measure has been evaluated by signal-to-quantization-noise ratio (SNR) measurements. The results are compared to the SNR-values obtained by the well known optimum scalar quantization [2]. For Gaussian processes the SNR-values also are compared with the rate distortion functions.

With some statistically independent random processes the optimality of VQ is discussed by comparisons of computer simulation data with analytical results. Additionally, the convergence behaviour of the codebook design algorithm is treated.

Simulation Experiments

Pseudo random sources with uniform (U), Gaussian (G), Laplacian (L), K_0-, and Γ-amplitude distribution densities - as illustrated in Fig. 1 - have been implemented on an HP-1000 minicomputer. Sources without memory and with memories characterized by the linearly (LIN), exponentially (RC), or oscillatorily (BW) decaying autocorrelation functions (ACF) of Fig. 2 have been included. Vector quantizers for dimensions n from 1 up to 10 and a quantization rate q of 1, 2, or 3 bit per sample - with a maximum of 1024 codebook entries - have been simulated on a FPS-120B array

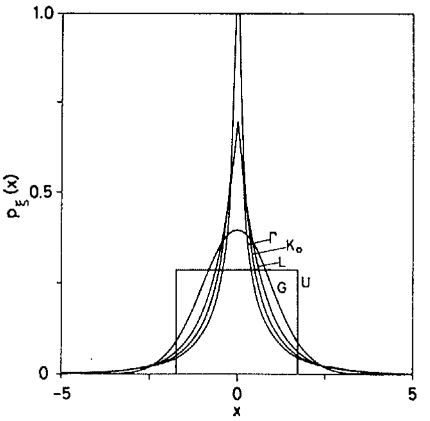


Fig. 1 Amplitude densities of model processes

processor. The codebooks have been designed on the basis of the Euclidian distance measure for full search applying the product code method and the algorithm of Linde, Buzo, and Gray [3]. A high source production rate and an efficient full search codebook design have been achieved by microprogramming of the main algorithms. Each of the random number sequences comprised more than 2 million samples processed at a minimum rate of 10 kHz

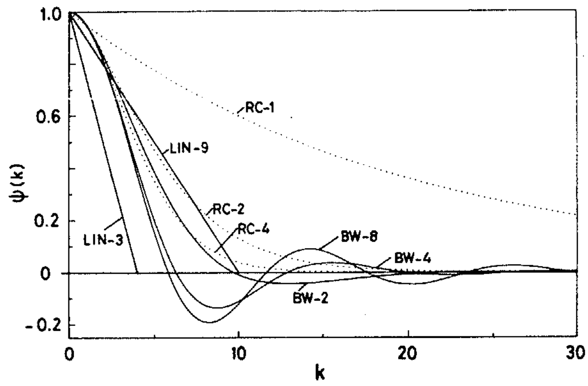


Fig. 2 Autocorrelation functions $\psi(k)$ of the model processes

for all of the source models. These sequences have been vector quantized and the SNR-values have been analyzed as functions of the dimension n and of the bit rate q. Furthermore with some of the statistically independent random processes the development of the codebooks has been investigated in detail for n=2. The codebook structure has been determined and the corresponding SNR-values have been measured after each step during the iterative codebook optimization.

Results

For Gaussian-, Laplacian-, and K_0-distributed random processes with the above defined memories the measured SNR-values are given in Table 1. Typical results are illustrated in Fig. 3 and Fig. 4. In Fig. 3 the SNR-values of Gaussian random processes are depicted as functions of n and q for the different memory types. Fig. 4 demonstrates the influence of the amplitude statistics on the SNR for one specific memory structure.

The SNR-values range from 2 dB to 17 dB with q=1 and from 7 dB to 23 dB with q=2 depending on the source type and on the VQ dimension n. Starting at the values of the scalar quantizer the SNR-values first increase strongly with increasing n but obviously tend towards saturation for high n. The results show that the performance of VQ is significantly determined by the

Table 1 SNR-values in dB of vector quantized model processes with different memory types

	q	n	ACF LIN-3	LIN-9	RC-1	RC-2	RC-4	BW-2	BW-4	BW-8
Gaussian	1	1	4.40	4.40	4.39	4.40	4.40	4.40	4.40	4.39
		2	6.41	7.90	8.53	8.53	8.53	8.52	8.53	8.53
		3	7.08	9.29	11.06	10.38	10.25	10.32	10.21	10.19
		4	7.55	10.04	12.23	11.63	11.53	11.58	11.55	11.55
		5	7.84	10.56	12.80	12.46	12.53	12.59	12.92	13.05
		6	8.09	10.86	13.20	13.05	13.12	13.19	13.74	14.00
		7	8.31	11.16	13.53	13.49	13.76	13.70	14.43	14.74
		8	8.51	11.37	13.81	13.85	14.18	14.12	15.17	15.55
		9	8.66	11.51	14.02	14.14	14.56	14.45	15.66	16.33
		10	8.83	11.69	14.24	14.45	14.95	14.77	16.16	16.88
		10	*9.82*	*12.56*	*15.26*	*15.68*	*16.44*	*16.68*	*17.98*	*18.82*
	2	1	9.27	9.27	9.26	9.27	9.27	9.27	9.27	9.27
		2	11.46	13.44	14.95	14.95	14.95	14.95	14.96	14.96
		3	12.36	14.82	16.88	17.55	17.93	17.72	18.16	18.22
		4	12.88	15.66	17.86	18.90	19.71	19.26	20.72	21.14
		5	13.26	16.16	18.51	19.82	21.09	20.29	22.38	22.88
		5	*15.02*	*17.83*	*20.06*	*21.51*	*23.10*	*22.11*	*24.77*	*25.39*
	3	1	14.48	14.47	14.45	14.46	14.46	14.46	14.47	14.48
		2	16.98	18.84	20.41	20.44	20.36	20.38	20.43	20.40
		3	17.90	20.43	22.42	23.20	24.17	23.54	25.26	25.64
		3	*21.52*	*22.87*	*24.80*	*25.57*	*26.55*	*25.89*	*27.77*	*28.31*
Laplacian	1	1	3.01	3.00	3.00	3.00	3.00	3.00	3.00	3.00
		2	5.52	6.59	7.01	7.01	7.02	7.02	7.03	7.03
		3	6.21	8.49	10.04	9.51	9.36	9.44	9.30	9.28
		4	6.80	9.31	11.33	10.51	10.33	10.41	10.31	10.35
		5	7.11	9.88	12.20	11.47	11.54	11.50	11.72	11.77
		6	7.50	10.25	12.67	12.15	12.18	12.28	12.76	13.01
		7	7.74	10.59	13.04	12.71	12.78	12.84	13.39	13.68
		8	7.95	10.83	13.33	13.12	13.37	13.37	14.14	14.48
		9	8.11	11.02	13.55	13.44	13.77	13.72	14.80	15.37
	2	1	7.51	7.50	7.49	7.50	7.51	7.51	7.52	7.52
		2	10.20	12.16	13.68	13.71	13.72	13.72	13.70	13.74
		3	11.22	13.79	15.89	16.39	16.63	16.47	16.76	16.79
		4	11.92	14.69	16.96	17.93	18.68	18.28	19.68	20.05
K_0	1	1	2.38	2.38	2.39	2.38	2.38	2.38	2.38	2.38
		2	5.17	6.13	6.48	6.49	6.49	6.50	6.49	6.50
		3	5.94	8.25	9.68	9.23	9.06	9.13	8.97	8.96
		4	6.60	9.04	11.06	10.18	9.99	10.09	9.94	9.87
		5	6.98	9.70	11.94	11.22	11.17	11.22	11.36	11.38
		6	7.35	10.10	12.44	11.88	11.92	12.03	12.48	12.76
		7	7.59	10.46	12.82	12.45	12.54	12.60	13.16	13.40
		8	7.81	10.71	13.16	12.93	13.17	13.18	13.90	14.19
		9	8.04	10.91	13.42	13.28	13.55	13.54	14.63	15.09
	2	1	6.89	6.85	6.88	6.90	6.89	6.85	6.89	6.89
		2	9.67	11.80	13.31	13.32	13.32	13.36	13.31	13.33
		3	10.89	13.50	15.58	15.97	16.16	16.14	16.29	16.32
		4	11.66	14.46	16.71	17.61	18.37	18.01	19.38	19.74

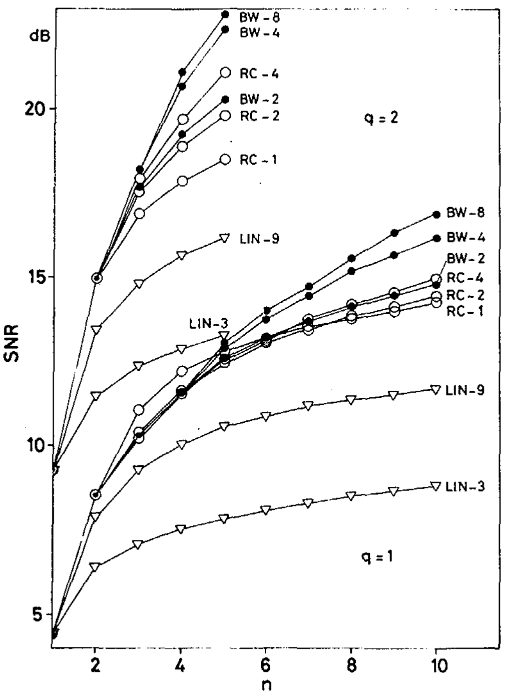


Fig. 3 SNR-values in dB of Gaussian processes with different memory types vs. n for q-bit quantization

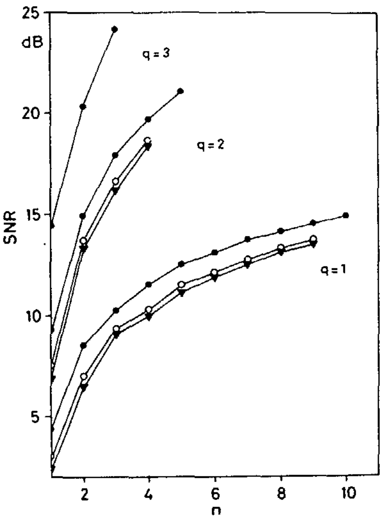


Fig. 4 SNR-values in dB vs. n for q-bit quantization of Gaussian (•), Laplacian (○), and K_0 (▼) processes (memory type: RC-4)

memory type but depends only slightly on the amplitude distribution. With all memory types the highest SNR-values are achieved by the Gaussian process. These SNR-values approach closely to the corresponding limits of the rate distortion functions, which are also given in Table 1 (in italics).

On the contrary the SNR-values for the memoryless sources are strongly dependent on the amplitude statistics. Table 2 summarizes the results. Fig. 5 illustrates the performance of vector quantizers with q=1. As can be seen the SNR-values increase monotonically with increasing dimension n. While up to n=3 the order of the SNR-values for the different amplitude distributions does not agree with the statements of the rate-distortion theory, for dimensions n≥4 the order is

Table 2 SNR-values in dB for vector quantized memoryless sources

q	n	G	L	K_0	Γ
1	1	4.39	3.00	2.42	2.21
	2	4.39	3.66	3.35	3.27
	3	4.39	4.45	4.56	4.72
	4	4.61	4.63	5.08	5.42
	5	4.76	4.78	5.19	5.55
	6	4.80	4.98	5.40	5.77
	7	4.88	5.21	5.69	6.09
	8	4.98	5.27	5.92	6.43
	9	5.06	5.35	5.98	6.60
		6.02	*6.54*	*7.54*	*8.47*
2	1	9.30	7.53	6.88	6.69
	2	9.54	8.77	8.80	8.98
	3	9.92	9.53	9.89	10.35
	4	10.17	9.98	10.53	11.14
	5	10.38			11.72
		12.04	*12.67*	*13.74*	*15.12*
3	1	14.59	12.61	11.93	11.76
	2	15.24	14.18	14.29	14.62
	3	15.68	15.18	15.50	16.09
		18.06	*18.69*	*19.87*	*21.58*

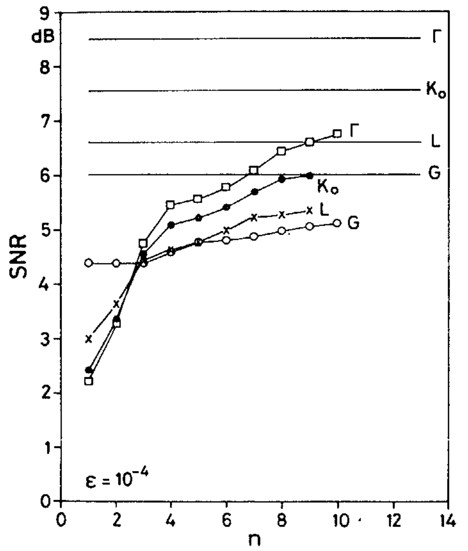
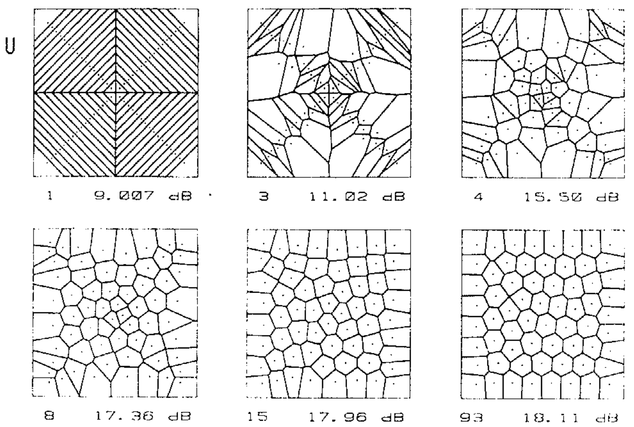


Fig. 5 SNR vs. n of 1 bit vector quantized memoryless random processes; —— limits of rate distortion theory

reversed and then is in accordance with that predicted by information theory. With further increasing n the SNR-values tend to the theoretical limits [4].

Finally the convergence behaviour of the codebook design algorithm as well as the optimality of VQ was analyzed. Typical results for n=2 and n=3 or 1 are presented in Fig. 6 and Fig. 7, respectively. Fig. 6 shows the development of the codebooks during the iteration procedure with the random processes for each of the distributions investigated here. In each case the iteration was initiated with the same codebook and was continued until the SNR-values became constant. Firstly the rapid convergence of the codebook design seems to be remarkable. After 15 iterations the algorithm leads to configurations of the codebook vectors which are very close to the corresponding final configurations. Referring to the SNR-values one observes a similar behaviour. After the 15th iteration step the SNR-values are less than 1 dB below their maximum.



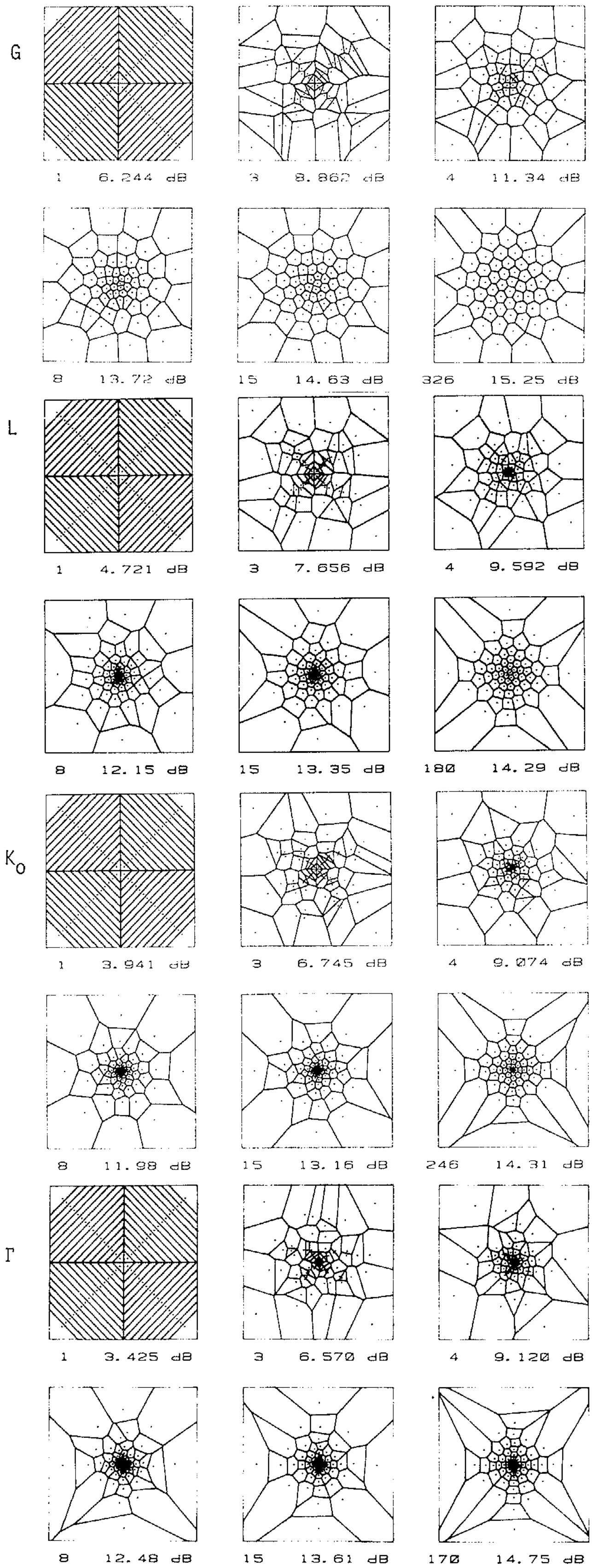


Fig. 6 Development of codebook structure (n=2, q=3) for uniform (U), Gaussian (G), Laplacian (L), K_o, and Γ process. SNR-values in dB and numbers of iteration step.

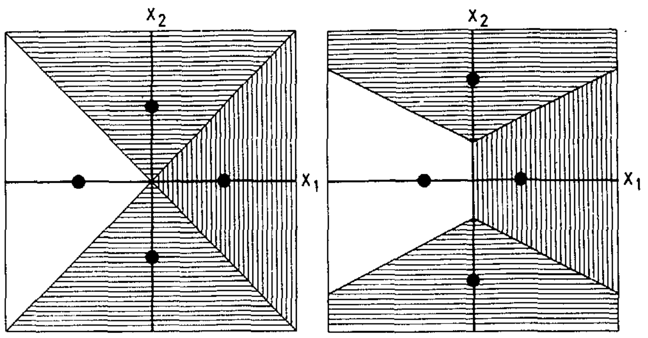


Fig. 7 Optimum codebook configuration of a 2-dimensional 1-bit-VQ for Gaussian (left) and Laplacian process (right)

A fundamental problem is the question whether these maximum values are optimum. An answer to this question requires the solution of the multidimensional generalized Max quantizer optimization problem. In the particular case of 2-dimensional 1-bit-quantization of Gaussian and Laplacian random processes (with variance 1) analytical solutions could be derived. The optimum codebook vectors together with their corresponding partitions are depicted in Fig. 7. While for the Gaussian process a rotational invariant configuration results where all codebook vectors have identical lengths of $2/\sqrt{\pi}$, yielding a SNR-value of 4.396 dB, for the Laplacian process a less symmetrical configuration with the codebook vectors (0.826;0), (0;1.457), (-0.826;0), (0;-1.457), and a SNR-value of 3.663 dB is optimum. In both cases these optimum solutions have been obtained by VQ applying the LBG-algorithm, thus verifying the optimality of this codebook design method.

Conclusion

The investigations reported here have shown that

- with correlated random processes the performance of VQ is significantly determined by the memory type but depends only slightly on the amplitude distribution; Gaussian processes yield the highest SNR-values.
- with statistically independent random processes VQ can be considered to be optimum; in agreement with information theory VQ leads to amplitude distribution dependent SNR-values increasing in the order G-, L-, K_0-, and Γ-distribution.

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