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A METHOD OF BLIND EQUALIZATION

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RESUME

Aveugle égalisation, ou ce qu'on appele Rexercice à Data est la fonction significative à transmettre data en très vite. Ce problème est fondamentalement à identifier le linéaire système inconnu sans mesurer système prise de puissance. Même si le système prise de puissance est IID, il devient très difficile à égaliser aveuglement en cas généraux où le système n'est pas causal. Quelques travaux précédents presentaient le plan à réduire en minimum

$$E(y_k-\gamma \mathrm{sign}(y_k))^2,$$

où y_k est à égaliseur débit de puissance. Le renouvellement des palamètres de l'égaliseur s'établissait en on-line operation, où ils sont corrigés en petite value à chaque k chance à recevoir le signal.

En ce papier, l'aveugle égaliseur d'off-line opération après enregistrer la longue suite de systeme débit de puissance est presenté. Cette off-line operation va être banale dans autres champs sauf transmettre data où l'information faut etre opérée en l'heure réelle. Ce l'off-line algorithme accomplit le application à résoudre

$$E(y_{k-\ell}|y_k>0)=0 \quad (\ell\neq 0).$$

Resultantement, cette idée conduit à la méthode d'enlever la dépendance entre égaliseur débits des puissances y_k.

Section 1, on décrit l'histoire d'aveugle égaliseur. Section 2, on discute de deux idées ; la première est à égaliser la fonction de probabilité densité d'égaliseur débit de puissance a cela de systeme prise de puissance, et la deuxième est à enlever la dépendance entre égaliseur débits des puissancese. En section 3, on propose l'off-line algolithme. En section 4, notre off-line algorithme est confirmé en cas limité. Section 5, on montre quelque résultats de la simulation de computeur.

SUMMARY

Blind equalization, or called Retrain on Data is significant function in high speed data transmission. This problem is basically to identify the unknown linear system without measuring system input. Even if system input is IID, it becomes very difficult to blind-equalize in general cases where the system is not causal. Several previous works presented the scheme to minimize

$$E(y_k-\gamma \mathrm{sign}(y_k))^2,$$

where y_k is equalizer output. The updating of equalizer tap-weights was established in on-line operation, where they are corrected by small increment at every kth signal receiving chance.

In this paper, an off-line blind equalization processed after recording a long time sequence of system output is reported. Such an off-line operation will be conventional in other fields except data transmission where the information must be processed in real time. The present off-line algorithm performs a contraction mapping to solve

$$E(y_{k-\ell}\,|\,y_k>0)=0 \quad (\ell\neq 0).$$

Resultantly, this idea leads to a method of removing time-dependency in equalizer output y_k.

Section 1 describes a past history of the blind equalization. Section 2 discusses the concept of two ideas; the first is to equalize probability density function of equalizer output with that of system input, and the second is to remove time-dependency in equalizer output. Section 3 proposes an off-line algorithm. In section 4, the present off-line algorithm is confirmed for the limited case. Section 5 shows the several results of computer simulation.

1. Introduction

Blind equalization, compelled to converge to an optimum equalizer state without referring the sending date sequence, is of interest to research engineers in several fields : image processing ; signal detection ; biophysics, in particular, neuro-physiology ; and statistical mathematics. Starting from the original work in this subject of the high speed data transmission by Y.Sato[1], A.Benveniste framed out the theoretical structure and his work is appreciated highly.[2] The blind equalization is called "Retrain on data" in CCITT V.27 and V.29 recommendations, then, with it high speed data modems must be equipped. For the data transmission cases, the original work achieved a blind equalization with theoretical treatment and computer simulation under limited situations, where the sending sequence is independent and has identically uniform distribution (IID) through the data scrambling and multi-level coding. The extended and general work by A.Benveniste derived theorems for blind equalizable functionals having the steepest descent lines which converge to the optimum, and they mean to lighten the restrictions for sending sequence distribution and distortion of linear system. A primal difficulty comes from nonminimum phase property of linear system, since the system is not causal. In order to overcome it, new techniques except application of the classical adaptive linear prediction to the autoregressive systems had been required.

Denote the distribution of the system input of IID by ν and the equalizer output by y_k, the theorem by A.Benveniste says "If the distribution of y_k is ν too, the linear system must be transparent and y_k must be equal to the system input with time shift ambiguity, where ν is not Gaussian." Every previous work intended to force implicitly the equalizer output to depend on the distribution ν, beside directly aiming intersymbol interference reduction or time independency in the sequence y_k. The problem of blind equalization remains a large misterious part. Questions are, for example, what is solved for the whole permitted distribution ν associated with blind algorithms and, furthermore, what kind of knowledge of ν is substantial for this problem. It is evident that, by means of any schemes from standpoints of minimizing the correlation $E\{y_k\, y_{k-\ell}\}(\ell\neq 0)$, the intersymbol interference cannot be removed. It should be remarked that the independency as

$$P(y_{k-\ell}\,|\,y_k=\text{constant})=P(y_k)$$
$$(\text{for all } \ell\neq 0 \text{ and the constant is arbitrary})$$

is equivalent to vanishing intersymbol interference. This paper presents a new type of off-line algorithm to realize the above equations. To simplify the algorithm, the estimated averages of each $y_{k-\ell}$ when $y_k>0$ are used to adjust the equalizer parameters, and iterative contraction toward the optimum is accomplished. The computer simulation for several typical models are added.

2. A concept of blind equalization

Presuming that sending data sequence a_k in Figure 1 is time independent and has identical distribution, two strategies stated below have the same destination that the total responce satisfies

$$t_0 \neq 0, \quad t_\ell = 0 \ (\ell\neq 0) \qquad (t_\ell=\sum_n h_{\ell-n} w_n)\,. \quad (1)$$

(1) The first strategy is to equalize the probability density function (p.d.f.) of equalizer output y_k with that of a_k.

(2) The second is remove the time dependency in y_k, in sense that y_k is strictly uncorrelated : the p.d.f. of $y_{k-\ell}$ ($\ell\neq 0$) is not warped when the subsequence $y_{k-\ell}, y_{k-\ell+1}, \cdots, y_k$ is selected using any rules on the value of y_k.

The schemes of algolithm design based on above strategies are different in two cases of application where p.d.f. of a_k is a priori known and unknown. In the first application, we can establish the more reliable blind equalization using knowledge of the p.d.f.. Along the first strategy of p.d.f. matching, for example, the maximum likelihood estimation $\hat{a}_k$ of a_k after receiving y_k will be derived by evaluating the y_k histogram. Then the equalizer tap-weight updating schemes employing the estimated steepest descent of $(y_k-\hat{a}_k)^2$ as in decision directed manner is one of the possible blind equalization. In general application cases where the p.d.f. of a_k is unknown, we cannot take any efficient information of a_k from a kth sampling value of y_k. Major question in these situations must be for what class of p.d.f. a presented robust and simple algorithm possesses the desired destination.

The every previous works treated the simple algorithm which minimizes $E(y_k-\gamma\,\text{sign}(y_k))^2$ as

$$w_\ell = w_\ell - \alpha x_{k-\ell}(y_k - \gamma\,\text{sign}(y_k)) \,, \qquad (2)$$

where

$$\gamma = E|a_k| \,/\, E(a_k^2) \,. \qquad (3)$$

Y.Sato proved that this converges to Eq.(1) in case of data communication where a_k is uniformly distributed and initial peak distortion $\sum_{i\neq 0}|t_i|/|t_0|$ is less than 1. The extended work by A.Benveniste derived that the algorithm of Eq.(2) is one of the possible algorithm permitted for broader class of a_k p.d.f.. The original work started from an idea that: since the uniformly distributed a_k is regarded as binary data added by uniformly distributed source noise, the conventional binary adaptive equalizer Eq.(2) is expected to smooth out the source noise and converge to Eq.(1). From the distribution matching principle suggested later by A.Benveniste, to minimize $E(y_k-\gamma\,\text{sign}(y_k))^2$, which means to minimize the dispersion around some constant γ in the positive region of y_k, should identify p.d.f. of y_k with that of a_k. To give further discussion for the concept of blind equalization, decompose $E(y_k-\gamma\,\text{sign}(y_k))^2$ as follows,

$$E(y_k-\gamma\,\text{sign}(y_k))^2 = E(y_k^2) - 2\gamma E|y_k| + \gamma^2 . \qquad (4)$$

From this formula, it is seen that the minimization is equivalent to maximizing under power constraint

$$E(y_k^2) = \text{constant} \,. \qquad (5)$$

To see details of $E|y_0|$ (denote $\bar{y}_0$), describe it by using $E\{a_\ell\}$ (denote $\bar{a}_\ell$) when $y_0>0$ where we employ ensemble average when k=0 for the simplicity of notation. Then we have

$$\bar{y}_\ell = \sum_i \bar{a}_{\ell-i}\, t_i \qquad (i=-\infty,\cdots,\infty) \qquad (6)$$

and

$$\bar{y}_0 = E|y_0| \,. \qquad (7)$$

Providing that the p.d.f. of a_k (denote ν) has zero-mean and is symmetric, $\bar{a}_\ell$ for $y_0>0$ is written as

$$\bar{a}_\ell = \int_{-\infty}^{\infty} a_\ell \nu(a_\ell) \int_{-\infty}^{a_\ell t_{-\ell}} p(x)dx \, da_\ell \,, \qquad (8)$$

where $p(x)$ is a normalized p.d.f. of all t_i except t_ℓ and its variance is determined by $1-t_\ell^2$ from unit power constraint. Let us introduce the following expression, which is not strictly correct, but, will be accepted in limited case discussed later.

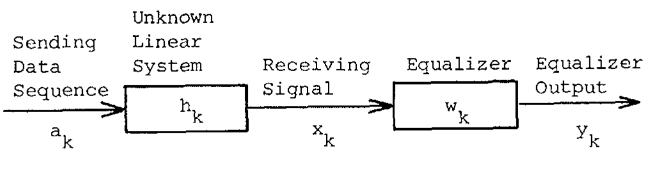

Fig.1 Model of equalization system

$$\bar{a}_\ell = \mu(t_\ell) \tag{9}$$

then

$$\bar{y}_\ell = \sum_i \bar{a}_{\ell-i} t_i = \sum_i \mu(t_{-\ell+i}) t_i \ . \tag{10}$$

It is noted that $\mu(t_\ell)$ is monotonic and increasing function of t_ℓ and $\mu(0)=0$. Substituting power constraint $t_0^2=1-t_i^2$ to Eq.(10), $\bar{y}_0$ is derived as

$$\bar{y}_0 = \frac{\mu(t_0)}{t_0} - \sum_{i\neq 0} \left(\frac{\mu(t_0)}{t_0} - \frac{\mu(t_i)}{t_i} \right) t_i^2 \ . \tag{11}$$

Out of the second term of Eq.(11), ith component when $t_i=0$ is to be omitted. From Eq.(11), it is concluded that the condition

$$\frac{\mu(t_0)}{t_0} - \frac{\mu(t_i)}{t_i} > 0 \quad (\text{for all } i\neq 0) \tag{12}$$

is necessary and sufficient in order that $\max E|y_0|$ gives the desired destination $t_i=0$ ($i\neq 0$). Furthermore, the condition (12) leads to another important result, i.e., the mean square distortion of a_ℓ is smaller than that of t_i.

$$D_a \left(= \frac{\sum_{i\neq 0} \mu(t_i)^2}{\mu(t_0)^2}\right) < D_t \left(= \frac{\sum_{i\neq 0} t_i^2}{t_o^2}\right) \ . \tag{13}$$

To find the p.d.f. of a_k satisfying the global condition (12) is the problem to give breakthrough toward blind equalizability and is remained as future problem. It is easy to examine for the limited case of local behavior around the solution $t_i=0$ ($i \neq 0$) and $t_0=1$. At first ,when t_0 tends to 1, denoting delta function by $\delta(x)$,

$$\frac{\mu(1)}{t_0} = \lim_{t_0\to 1} \int_{-\infty}^{\infty} a_0 \nu(a_0) \int_{-\infty}^{a_0 t_0} p(x)dx\, da_0 \,/t_0$$

$$= \frac{\int_{-\infty}^{\infty} a_0 \ (a_0) \ . \int_{-\infty}^{a_0} \delta(x)dx\, da_0}{\int_{-\infty}^{\infty} \nu(a_\ell) \ . \int_{-\infty}^{\infty} \delta(x)dx\, da_\ell}$$

$$= 2 \int_0^{\infty} a_0 \ (a_0) da_0$$

$$= E(a_0) \text{ when } a_0>0 \ . \tag{14}$$

The second, when all t_ℓ tend to 0 and t_0 tends to 1,

$$\frac{\mu(0)}{t_\ell} = \lim_{t_\ell\to 0} \int_{\infty}^{\infty} a_{-\ell}\nu(a_{-\ell}) . \int_{\infty}^{a_{-\ell}t_\ell} p(x)dx\, da_{-\ell} \,/t_\ell$$

$$= \lim_{t_\ell\to 0} \int_{\infty}^{\infty} a_{-\ell}\nu(a_{-\ell}) \, \mathbf{P}(a_{-\ell}t_\ell)/t_\ell da_\ell$$

$$= \lim_{t_\ell\to 0} \int_{\infty}^{\infty} a_{-\ell}\nu(a_{-\ell}) \{1/2 + \mathbf{P}'(0) a_{-\ell}t_\ell\}/t_\ell da_\ell$$

$$= 2\nu(0)E(a_k^2) \ . \tag{15}$$

If $\nu(x)$ is uniform p.d.f. defined in $[-V,V]$,then

$$\text{Eq.(14)} = \frac{1}{2} V \quad \text{and Eq.(15)} = \frac{1}{3} V \ , \tag{16}$$

are derived and it is said that the condition (12) is satisfied in the neighborhood of the solution. For the Gaussian p.d.f.,

$$\text{Eq.(14)=Eq.(15)} = \frac{1}{2\sqrt{2\pi}} \sqrt{E(a_k^2)} \tag{17}$$

is derived. Therefore, from Eq.(11) it is said that the ambiguity of t_i is remained and $\max E\,|y_0|$ cannot have the desired destination.

Now, apart from the first strategy of p.d.f. matching, let us recommend a following simple algorithm along the second strategy

$$\bar{y}_\ell = 0 \quad (\ell\neq 0) \quad \text{for } y_0>0 \ . \tag{18}$$

Equation (10) remarks that the ℓth expectation $\bar{a}_\ell$ has the same polarity of t_ℓ. From this fact, according that Eq.(18) is satisfied and namely the sequence of expectation $\bar{a}_\ell$ is the inverse of sequence t_ℓ, it is mostly expected that we have the trivial solution $t_\ell=0$ ($\ell\neq 0$) .

3. Off-line algorithm

After receiving long sequence of signal x_k, the present algorithm adjusts equalizer tap-weight w_ℓ so that time averages $\bar{\bar{y}}_\ell$ ($\ell\neq 0$) vanish, where

$$\bar{\bar{y}}_\ell = \sum_{k=1}^{L} \text{sign}(y_k) y_{k-\ell} \ . \tag{19}$$

Let $\bar{\bar{x}}_\ell$ be time average of $x_{k-\ell}$ when $y_k>0$, i.e.,

$$\bar{\bar{x}}_\ell = \sum_{k=1}^{L} \text{sign}(y_k) x_{k-\ell} \ , \tag{20}$$

then

$$\bar{\bar{y}}_\ell = \sum_{n=-N}^{N} w_n \bar{\bar{x}}_{\ell-n}$$

$$= \sum_n w_n \sum_i h_{\ell-n-i} \bar{\bar{a}}_i$$

$$= \sum_i \left(\sum_n h_{\ell-n-i} w_n \right) \bar{\bar{a}}_i$$

$$= \sum_i t_{\ell-i} \bar{\bar{a}}_i \tag{21}$$

While $\bar{\bar{a}}_i$ and $\bar{\bar{x}}_\ell$ are evaluated by time averaging, the equalizer is freezed. Hence, both of them are functional of equalizer tap-weight w_ℓ . From this reason, we must perform w_ℓ optimization, for example, repeating to solve w_ℓ satisfying $\bar{\bar{y}}_\ell=0$ ($\ell\neq 0$) for the averaged $\bar{\bar{x}}_{k-\ell}$ and to average $x_{k-\ell}$ when $y_k>0$ under the solved w_ℓ . Thus,an off-line procedure is described as

Step 1. calculate time average

$$\bar{\bar{x}}_\ell = \sum_{k=1}^{L} \text{sign}(y_k) x_{k-\ell}$$

Step 2. solve w_n to satisfy

$$\left.\begin{aligned} \sum_{n=-N}^{N} \bar{\bar{x}}_{\ell-n} w_n &= 0 \quad (\ell\neq 0) \\ \sum_{n=-N}^{N} \bar{\bar{x}}_{-n} \ w_n &= 1 \end{aligned}\right\} \tag{22}$$

and return to step 1.

Through this procedure, the following equivalent one is implicitly carried.

Step 1. calculate time average

$$\bar{\bar{a}}_\ell = \sum_{k=1}^{L} \text{sign}(y_k) a_{k-\ell} \tag{23}$$

Step 2. solve t_i to satisfy

$$\left.\begin{aligned} \sum_i \bar{\bar{a}}_{\ell-i} t_i &= 0 \quad (\ell\neq 0) \\ \sum_i \bar{\bar{a}}_{-i} t_i &= 1 \end{aligned}\right\} \tag{24}$$

and return to step 1.

Figure 2 illustrates this implicit procedure, where T_r and A_r show responses t_i and $\bar{\bar{a}}_i$ respectively obtained after rth iteration times.

To investigate the convergence of the above procedure, normalize $\bar{\bar{a}}_\ell$ and t_ℓ by their peaks as $\bar{\bar{a}}_\ell/\bar{\bar{a}}_0$ and t_ℓ/t_0 so that the convergence problem can be separately treated for peaks $\bar{\bar{a}}_0$ or t_0 and for distortion D_t or D_a.

Rewrite the interation procedure in Fig. 2 as

$$\Delta T_{r-1} \longrightarrow \Delta A_{r-1} \longrightarrow \Delta T_r \longrightarrow \Delta A_r \longrightarrow \Delta T_{r+1} ,$$

where above functions are defined after each peaks are subtracted from T_r, A_r, etc. and devide them by their peaks. Hence, acounting that time sequenes are all real valued, we have, for example

$$\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \Delta T_r d\omega = \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \mathrm{Re}[\Delta T_r] d\omega = 0 , \quad (25)$$

$$D_t^r = \frac{T}{2\pi}\int |T_r|^2 d\omega \equiv \|\Delta T_r\|^2 . \quad (26)$$

Now, we must confirm the inequalities

$$\cdots > D_t^{r-1} > D_t^r > D_t^{r+1} > \cdots .$$

Denote peak of $(1+\Delta T_r)^{-1}$ by $1+\delta t_0^r$, then

$$\delta t_0^r = \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \mathrm{Re}\left[\frac{-\Delta T_r}{1+\Delta T_r}\right] d\omega . \quad (27)$$

Therefore,

$$D_a^{r-1} = \left[\left\|\frac{\Delta T_r}{1+\Delta T_r}\right\|^2 - (\delta t_0^r)^2\right]/(1+\delta t_0^r)^2 . \quad (28)$$

The deviation of peak value from 1, δt_0^r, is approximately given by

$$\delta t_0^r \simeq \sum_{i\neq 0} t_i t_{-i} . \quad (29)$$

Perform the similar derivation for $\Delta A_r \longrightarrow \Delta T_{r+1}$, and taking account that the distortion reduction ratios through $\Delta T_{r-1} \longrightarrow \Delta A_{r-1}$ and $\Delta T_r \longrightarrow \Delta A_r$ are given by $\bar{\mu}_{r-1}$ (<1) and $\bar{\mu}_r$ (<1) respectively, the reduction ratios from δt_0^r to δa_0^r, whose polarities are the same, is approximated in order of $\bar{\mu}_r$ from Eq.(29), and we obtain roughly

$$D_t^{r-1} \simeq \bar{\mu}_{r-1}^{-1}[D_t^r - (\delta t_0^r)^2]/(1+\delta t_0^r)^2 \quad (30)$$

$$D_t^{r+1} \simeq [D_a^r - (\delta \bar{\bar{a}}_0^r)^2]/(1+\delta \bar{\bar{a}}_0^r)^2$$

$$= [\bar{\mu}_r D_t^r - \bar{\mu}_r^2(\delta t_0^r)^2]/(1+\bar{\mu}_r^2 \delta t_0^r)^2 . \quad (31)$$

Resultantly, in situations where D_t^r is larger than $(\delta t_0^r)^2$, it can be said that $D_t^{r+1} < D_t^{r-1}$.

For the peaks t_0 and a_0, the recursive formula is

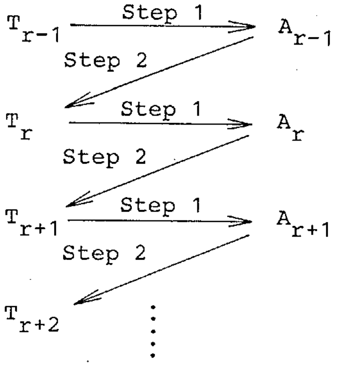

Fig.2 Implicit procedure of the off-line algorithm

$$t_0^r = \frac{1}{\bar{\bar{a}}_0^{r-1}} + \delta_r$$

$$\bar{\bar{a}}_0^r = \mu_0^r t_0^r$$

$$t_0^{r+1} = \frac{1}{\bar{\bar{a}}_0^r} + \delta_{r+1}$$

$$\bar{\bar{a}}_0^{r+1} = \mu_0^{r+1} t_0^{r+1} \quad (32)$$

$$t_0^{r+2} = \frac{1}{\bar{\bar{a}}_0^{r+1}} + \delta_{r+2}$$

$$\left(\delta_r = \int \frac{-(\Delta A_r/\bar{\bar{a}}_0^r)}{1+(\Delta A_r/\bar{\bar{a}}_0^r)/\bar{\bar{a}}_0^r} d\omega \right) .$$

From these

$$t_0^{r+2} = \frac{t_0^r}{(\mu_0^{r+1}/\mu_0^r)+\delta_{r+1} t_0^r} + \delta_{r+2} . \quad (33)$$

In Eq.(33), if δ_r tends to zero and μ_0^{r+2}/μ_0^{r+1} tends to 1, it is said that t_0^{r+2} converges to some finite value.

4. Analysis in case of uniform p.d.f. and $|t_0| > \sum_{i\neq 0}|t_i|$

The uniqueness that $t_\ell=0$ ($\ell\neq 0$) satisfying $\bar{y}_\ell=0$ ($\ell\neq 0$) is resulted from the following derivations (refer Appendix A for several derivations in this section)

$$\bar{a}_0 = \mu(t_0)t_0 , \quad \mu(t_0) = \frac{V}{2t_0}(1-\frac{1}{3}D_t) , \quad (34)$$

$$\bar{a}_\ell = \mu(t_{-\ell})t_{-\ell}, \quad \mu(t_\ell) = \frac{V}{3t_0} \quad (\ell\neq 0) , \quad (35)$$

Where $D_t = \sum_{i\neq 0} t_i^2 / t_0^2$, and

$$\bar{y}_0 = \frac{Vt_0}{2}(1+\frac{1}{3}D_t) \quad (36)$$

$$\bar{y}_\ell = \frac{Vt_\ell}{6}(1-D_t) + \frac{V}{3t_0}\phi_\ell \quad (\ell\neq 0) \quad (37)$$

$$\left(\phi_\ell = \sum_k t_k t_{k-\ell} \right) .$$

From above equations forcing $\bar{y}_\ell$ to zero and normalizing t_ℓ by the peak t_0

$$t_\ell = -\frac{2\phi_\ell}{t_0(1-D_t)} \quad (\ell\neq 0) \quad (38)$$

are obtained, and in frequency domain

$$T(\omega) = K_1|T(\omega)|^2 + K_2 \quad (39)$$

$$K_1 = -\frac{2}{t_0(1-D_t)} \quad (40)$$

$$K_2 = \frac{(3+2D_t)}{1-D_t} \quad (41)$$

is derived. From Eq.(39), it is concluded that $T(\omega)$ must be real valued constant.

According to the investigation of the convergence, at first in step 1, the mean square distortion decreases by the mapping t_ℓ to $\bar{a}_\ell$ as

$$D_a = \frac{4D_t}{9(1-\frac{1}{3}D_t)^2} , \quad (42)$$

whose curve is shown in Fig. 3. In step 2, for the normalized T_r and A_r, their inverses are roughly approximated by

$$T_r^{-1}(\omega) = \frac{1}{1+\Delta T_r(\omega)}$$
$$\simeq 1-\Delta A_{r-1}(\omega) \qquad (43)$$

and similarly

$$A_r^{-1}(\omega) \simeq 1-\Delta T_{r+1}(\omega) \ , \qquad (44)$$

where $\Delta A_{r-1}(\omega)$ and $\Delta T_{r+1}(\omega)$ are already normalized as have not non-zero constant offset.

Permitting above approximation, it is shown that $\|\Delta A_{r-1}\|>\|\Delta T_{r+1}\|$ and then $\|\Delta T_{r-1}\|>\|\Delta T_{r+1}\|$, and that $\|\Delta T_r\|$ converges to zero (See Appendix B).

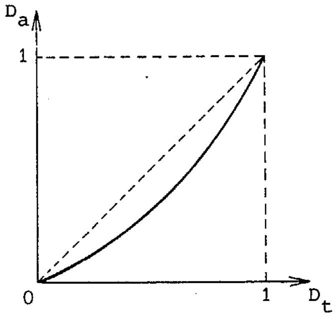

Fig.3 Curve of D_t-D_a

5. Computer simulations

Several computer simulations are performed, assuming simplified response of unknown system. The response was fixed as

$h_{-2}=0.2$, $h_{-1}=-0.3$, $h_0=1.0$, $h_1=0.5$, $h_2=0.2$,

and the number of tap-weights was 7. Time-averaging was calculated over 20000 samples.

Figure 4 shows the curves of D_t for the uniform p.d.f., and Figure 5 shows trajectory of $T_r(\omega)$ at every even times of iteration. Samely, Figures 6 and 7 are for triangle p.d.f.. In triangle cases, the degradation of convergence is seen in Fig. 6. The reason of this degradation is imagined that $\bar{y}_\ell$ ($\ell\neq0$) converge too fast to zero and in neighborhood of the solution the solved tap-weights have ambiguity caused by inaccuracy of 20000 times time-averaging. To overcome this, the following polarity detection was employed,

$$\psi(y_0) = \begin{cases} 1 & \text{if } y_0>\Delta \\ 0 & \text{if } \Delta>y_0>-\Delta \\ -1 & \text{if } -\Delta>y_0 \end{cases} \qquad (45)$$

Its simulation result is shown in Fig. 8. The convergence is more rapid than that of Fig. 6.

6. Conclusion

An off-line algorithm based on idea of removing time-dependency in equalizer output was presented. Through derivations of the algorithm, several problems what is substantial for blind equalization was discussed and some of them was remained for future problem. They are

(1) To solve the broad class of p.d.f. in explicit form satisfying the condition (12).

(2) To confirm whether maximizing E $(|y_0|)$ under power constraint implies equivalently $\bar{y}_\ell=0(\ell\neq0)$ or not.

(3) To give a perfect proof of the convergence of the off-line algorithm for some general cases of p.d.f..

(4) To find a general method to accelerate the convergence as tried in Fig. 8.

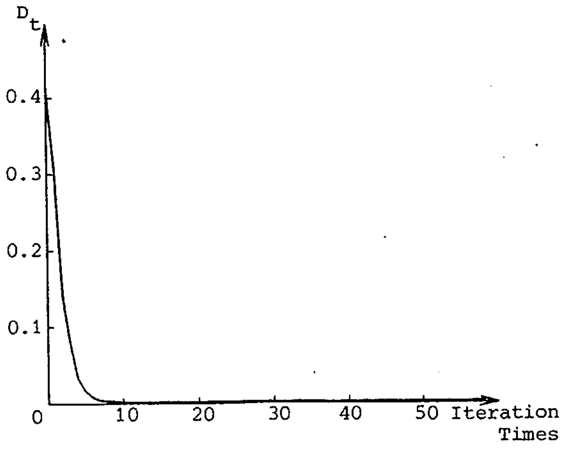

Fig.4 Evolution of D_t for uniform p.d.f.

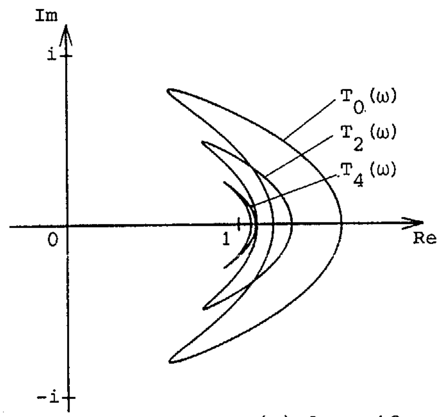

Fig.5 Trajectory of Tr(ω) for uniform p.d.f.

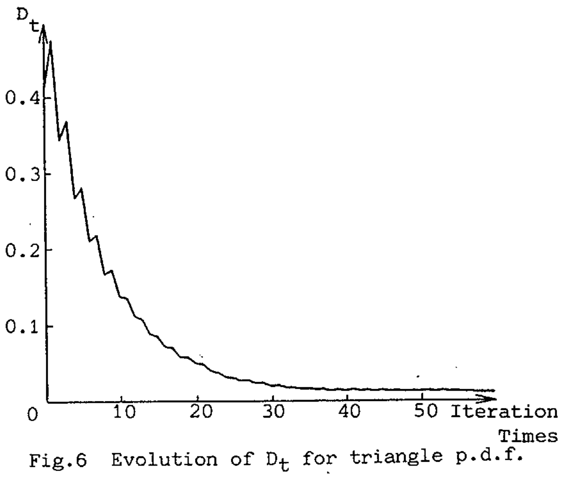

Fig.6 Evolution of D_t for triangle p.d.f.

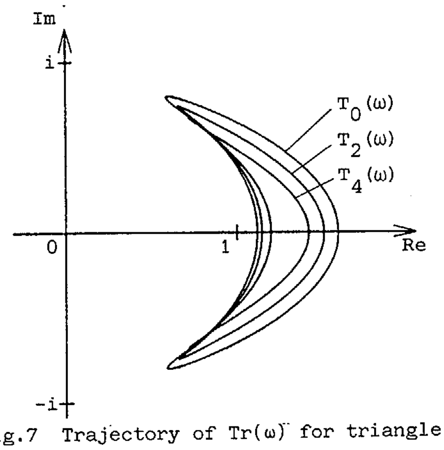

Fig.7 Trajectory of Tr(ω) for triangle p.d.f.

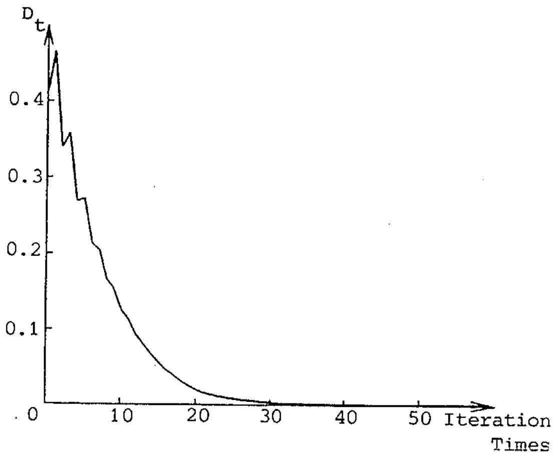

Fig.8 Evolution of D_t for triangle p.d.f. employing polarity detection Eq.(45)

References

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(2) A.Benveniste, M.Goursat, and G.Ruget, "Robust identification of a nonminimum phase system: Blind adjustment of a linear equalizer in data communication," IEEE Trans. Automat. Contr., vol. AC-25, pp.385-399, June 1980.

Appendix A

Let each of the sequence $\{a_k\}$ be independent and uniformly distributed on the interval [-V,V]. Assuming $t_0 > \sum_{k\neq 0} |t_k|$, then

$$\bar{a}_0 = \overline{a_0 \mathrm{sign}(\ a_k t_{-k})}$$

$$= \lim_{N\to\infty} \left(\frac{1}{2V}\right)^{2N+1} \int_{-V\le a_{-N},\cdots,a_N\le V} \cdots \int a_0 \mathrm{sign}(\ a_k t_{-k})\, da_{-N} \cdots da_N$$

$$= \lim_{N\to\infty} \left(\frac{1}{2V}\right)^{2N+1} \int \cdots \int \Big[\int_{\sum a_k t_{-k} \geq 0} a_0 da_0 - \int_{\sum a_k t_{-k} < 0} a_0 da_0 \Big] da_{-N} \cdots da_{-1} da_1 \cdots da_N$$

$$= \frac{V}{2}(1-\frac{1}{3}D_t)$$

Similarly,

$$\bar{a}_\ell = \overline{a_\ell \mathrm{sign}(\sum a_k t_{-k})} = \frac{Vt_{-\ell}}{3t_0} \quad .$$

Normalizing as $\bar{a}_0=1$, we have Eqs.(34) and (35). From these results, we obtain Eqs.(36) and (37) as follows

$$\bar{y}_0 = \sum \bar{a}_k t_{-k} = \frac{V}{2t_0}(1-\frac{1}{3}D_t)t_0 t_0 + \sum_{k\neq 0} \frac{V}{3t_0} t_{-k} t_{-k}$$

$$= \frac{Vt_0}{2}(1+\frac{1}{3}D_t)$$

and

$$\bar{y}_\ell = \sum a_{-\ell+k} t_{-k}$$

$$= \frac{V}{2t_0}(1-\frac{1}{3}D_t)t_0 t_{-\ell} + \sum_{k\neq\ell} \frac{Vt_{\ell-k}}{3t_0} t_{-k}$$

$$= \frac{Vt_{-\ell}}{6}(1-D_t) + \frac{V\phi_\ell}{3t_0} \quad .$$

Appendix B

Owing to the approximation as Eqs.(43) and (44), in order to show $||\Delta A_{r-1}|| > ||\Delta T_{r+1}||$, it is sufficient to prove the inequality,

$$\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} |A_r^{-1}(\omega)-1|^2 d\omega \quad < \quad \frac{T}{2}\int_{-\pi/T}^{\pi/T} |T_r^{-1}(\omega)-1|^2 d\omega \qquad \text{(B-1)}$$

or more simply,

$$|A_r^{-1}(\omega)-1| < |T_r^{-1}(\omega)-1| \quad (-\pi/T < \omega < \pi/T) \qquad \text{(B-2)}$$

On the other hand, from Eqs. (34) and (35), we have

$$A_r(\omega) -1 = \alpha(T_r(\omega) -1), \qquad \text{(B-3)}$$

where

$$\alpha = \frac{2}{3(1-\frac{1}{3}D_t)} \quad . \qquad \text{(B-4)}$$

It is easy to check $2/3 < \alpha < 1$ for $0 < D_t < 1$, under which we discuss the proof of (B-2). Since Eq. (B-2) is equivalent to

$$\frac{|A_r(\omega)-1|}{A_r(\omega)} < \frac{|T_r(\omega)-1|}{T_r(\omega)} \qquad \text{(B-5)}$$

then, from Eq. (B-3), the inequality to be proved becomes as follows

$$\alpha|T_r(\omega)| < |A_r(\omega)| \ , \qquad \text{(B-6)}$$

i.e.,

$$\alpha^2|T_r(\omega)|^2 < |A_r(\omega)|^2 . \qquad \text{(B-7)}$$

Taking ρ, θ

$$|T_r(\omega)|^2 = 1+\rho^2+2\rho\cos\theta, \qquad \text{(B-8)}$$

$$|A_r(\omega)|^2 = 1+\alpha^2\rho^2+2\alpha\rho\cos\theta, \qquad \text{(B-9)}$$

we have

$$|A_r(\omega)|^2 - \alpha^2|T_r(\omega)|^2$$

$$= 1-\alpha^2+2\alpha(1-\alpha)\rho\cos\theta \ . \qquad \text{(B-10)}$$

Since $\rho<1$ under $t_0 > \sum_{i\neq 0} |t_i|$, we obtain

$$|A_r(\omega)|^2 - \alpha^2|T_r(\omega)|^2$$

$$\geq 1-\alpha^2-2\alpha(1-\alpha)$$

$$= (1-\alpha)^2 > 0 \ , \qquad \text{(B-11)}$$

and the proof is completed.