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# COMPRESSION D'IMAGES DANS UN ESPACE DE PROJECTIONS
# IMAGE COMPRESSION IN THE PROJECTION DOMAIN

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## RESUME

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Nous presentons une methode de compressions de donnees exploitant l'existence de redondances angulaires presentes dans une image. En utilisant la transformation discrete de Radon, nous etablissons, entre les correlations spatiales et angulaires, une correspondance a partir de laquelle est engendree un espace de projections. Les correlations entre projections correspondant aux correlations angulaires de l'image sont eliminees en utilisant une technique de quantification vectorielle. Des taux de transmission de 1.0 a 20 bits/pixel ont ete obtenus avec des resultats d'une fidelite satisfaisante. Il est possible de developper de nouvelles compressions en utilisant les redondances inherentes intra-projections aux quelles peuvent etre associees les correlations spatiales d'une image pour un angle de projection donne.

## SUMMARY

ABSTRACT

An image data compression scheme that exploits azimuthal (angular) redundancy within an image is presented. Pictorial correlation is mapped into spatial and angular redundancy by applying the discrete Radon transformation (DRT) to an image space and generating a corresponding projection space. A vector quantization (VQ) technique is applied to the resulting projection domain for removal of inter-projection correlation which effectively corresponds to azimuthal correlation within the pre-transformed picture space. Bit rates within a 1.0-2.0 bits/pel range have been achieved for picture compression of acceptable fidelity and intelligibility. Further compression is possible by exploiting inherent intra-projection redundancy which may be associated with spatial correlation, within the image, at a given orientation (or projection angle).

## 1. Introduction

Image data compression is associated with the reduction of information subject to specified fidelity and intelligibility constraints. Many compression schemes have been proposed; however, most of the methods are classified into the two distinct categories of spatial predictive and transform domain techniques respectively [1]. Several image source coders have also been developed which utilize a combination of the features of the distinct coding methods.

Predictive techniques are designed to exploit spatial redundancy within image spaces by forming estimates of image samples over a prediction window and transmitting or storing, after appropriate quantization, the errors of estimation; these are used at the decoder to reproduce the image space faithfully. One dimensional predictive techniques are generally amenable to relative ease of design and speed of operation. Although two dimensional techniques have produced superior results in compression [2], greater computational complexity is inherent.

Many transform domain techniques abound [1,3] for picture coding. Typically, two dimensional unitary transforms are applied to sub-blocks of pictures such that most of the sub-image energy is confined to a small number of transform coefficients, to allow substantial discarding of low-energy samples and strategic zonal coding.

Some notable hybrid coding schemes, in which transform coding is applied along the image rows and predictive coding along the columns, have been developed by Habibi [4]. A similar approach is adopted in the designs of [5] where vector quantization is used for compression along columns.

A technique is here presented in which pictorial correlation is mapped into an equivalent form of spatial and angular dependency. The scheme implicitly provides an avenue for exploiting spatial dependency over and between different orientations within a given image space. This approach has been based on an appropriate theory of reconstruction of image planes from projections [6] which has found much use in applications such as medical imaging and radio astronomy. It is noted that Jain and Jain [7] have experimented with transform coding applied to medical image projection data to achieve large compression in reconstructed cross-sections.

Our fundamental approach is to generate projections, at angular intervals, on an image plane (size N1xN2) by employing the discrete Radon transform and then applying a VQ procedure to compress the resulting highly correlated projection domain. The projection domain is of the same dimensions as the image plane and the substantial redundancy within the domain arises from the low-pass filtering effect of projecting an image.

In this paper, we focus on the removal of inter-projection redundancy, where this redundancy actually dominates over intra-projection correlation for typical imagery. We therefore effectively take advantage of angular correlation within the image space by inter-projection coding. In applications that involve acquiring large amounts of data in projection space (e.g. medical imagery) before further processing, a projection domain compression (PDC) approach is immediately relevant.

## 2. The Radon Transform

The formal definition of the Radon transform applied to a continuous spatially limited plane, $f(x,y)$ (Fig. 1), may be expressed thus [6]

$$P(\theta,t) = R[\ f\ ] = \iint_D f(x,y)\,.\,\delta(x.\cos\theta + y.\sin\theta - t)\ dx.dy \tag{1}$$

where the parameters $\theta$ and $t$ indicate the orientations and positions of lines of integration. The line integration is forced by an impulsive 'aperture' function - the dirac delta kernel $\delta(.)$

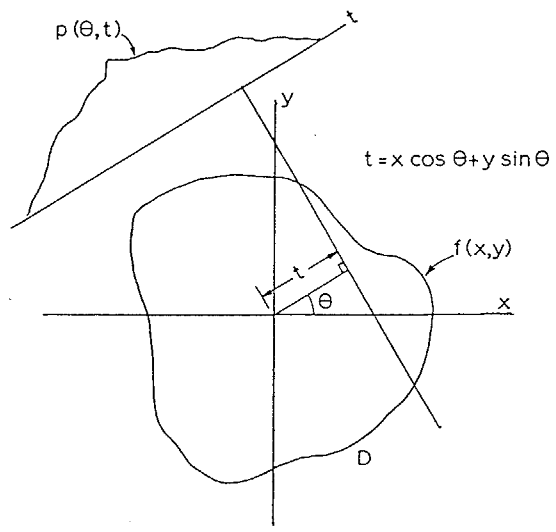


Fig.1 Projecting a space, f(x,y) to generate the projection p(Θ,t) at angle Θ

For discrete purposes, we are concerned with generating an NxN projection space given an image space of the same dimensions. We therefore overlay our spatially-discrete picture space, $f(u,v)$, with a circular plane, $f_c(i,j)$, with diameter equivalent to the diagonal length ($L_D$) of the space, $f(u,v)$, as illustrated in Fig. 2

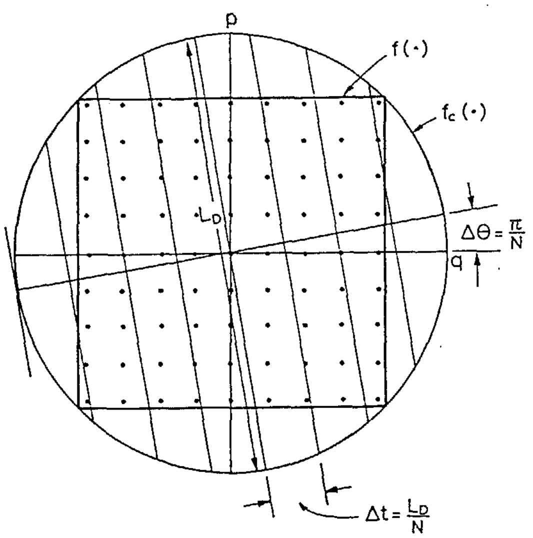


Fig. 2a Geometry for implementing the D.R.T. on an NxN image space, f(·),(N=8 in Fig)

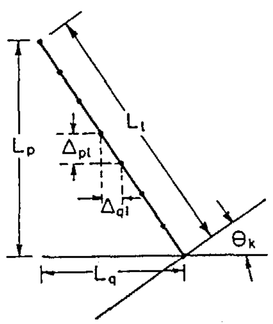


Fig. 2b Selection of points of integration along the $l^{th}$ line and $k^{th}$ orientation. $\theta_k$

The two planes, $f(u,v)$ and $f_c(i,j)$ are geometrically centred at the origin of a cartesian p-q space. The circular plane is a discretized space with a fixed sampling interval, $\Delta t = \Delta L_D/N$, between lines of integration. Points of integration are located along the lines by sampling in p-q space at intervals, $\Delta p_l = L_p/N$, $\Delta q_l = L_q/N$ where $(L_p^2 + L_q^2)^{\frac{1}{2}}$ is equal to the the length $L_l$ of the $l^{th}$ line of integration. To obtain projections at different orientations, $f_c(i,j)$ is rotated by constant angular displacements (with respect to the p-q space), $\Delta\phi = \pi/N$. Hence the discrete Radon transform has been modelled by

$$P(k,l) = \sum_{i,j}^{N-1} f_c\ (i\Delta p_l,\ j\Delta q_l).\delta(i\Delta.p_l \cos k\Delta\theta + j\Delta q_l \sin k\Delta\theta - l\Delta t) \quad (2)$$

where $k,l = 0,1,\dots,N-1$

and

$$f_c\ (i,j) = I\ [\ f(u,v)\ ] \qquad \text{for } 0 < u,v < N-1$$
$$= \mu_f \qquad 0 > u,v > N-1$$

The samples of the spatially discrete function, $f_c(i,j)$, are obtained by interpolating from and extending the actual picture space, $f(u,v)$, where $I[.]$ is the appropriate interpolation function . The rectangular picture space is extended to the circular space by setting $f_c(i,j)$ to the spatial mean, $\mu_f$, of the image for $0 > u,v > N-1$, where

$$\mu_f = \frac{1}{N^2} \sum_{u,v}^{N-1} f(u,v) \quad (3)$$

The uniform extension of $f(u,v)$ with its spatial mean, ensures that statistical variations of the image space are transformed into similar variations in the projection domain. The simple bilinear interpolating function [6] has been used for the function $I[.]$

We shall briefly describe the inverse Radon transform (IRT). The IRT may be defined as

$$f(x,y) = \int_0^{\pi} F^{-1}\ [\ |\omega|\ \times F\ [\ R\ (f)_\theta\ ]\ d\theta \quad (4)$$

where $F^{-1}$ is the continuous inverse Fourier transform (1D) operator. The argument of the operator is essentially a filtering operation on the Fourier transformed projection ($P(\theta,t) = R(f)_\theta$) at a given orientation, $\theta$. The filter function $|\omega|$ is the continuous spatial frequency variable of the Fourier transform. The integration over all distinct angles of projection $0 \leq \theta \leq \Pi$, is typically called a back-projection operation, which effectively associates all filtered projections with a point $(x,y)$ in the original image plane. A discretized version of (4) may be expressed as

$$f(i,j) \approx \sum_{k=0}^{N-1} F_N^{-1} \left\{ I\ [\ A(L) \times S(k,L)\ ] \right\} \quad (5)$$
for $L=0,1,\dots,N-1$

where $S(k,L)$ is the 1D N-point discrete Fourier transform of the $k^{th}$ projection (i.e. $S(k,L) = F_N[P(k,L)]$). $A(L)$ is a discrete filter function whose impulse response may be obtained by computing the central N Fourier series coefficients of a periodic and windowed version of the function $|\omega|$ (of equation (4)). The window is required for bandlimiting to limit aliasing in the sampled projection data. $I[.]$ , is a bilinear interpolating function that produces an interpolated form of filtered projections before back-projection. In reconstruction theory [7], this method is generally known as the convolution back-projection technique (CBPRT) where the convolution is effected between the filter function A(.) and the Radon transform on the $k^{th}$ orientation. Depending on the type of window employed to model A(.), different CBPRTs may be realised [8].

3. Theoretical basis for projection domain compression (PDC)

(i) The projection theorem: An important feature of the Radon transform may be described by

$$\sum_{l=0}^{N-1} P(k\Delta\theta, l\Delta t).w(l\Delta t) = \sum_{m,n}^{N-1} f(m\Delta x, n\Delta y) \cdot w(m\Delta x.\cos k\Delta\theta + n\Delta y.\sin k\Delta\theta) \quad (6)$$

which is basically a discrete version of the generalized projection theorem [7]; this infers that a specified operation, w(.), on the $k^{th}$ projection P(k,.), is equivalent to a related operation in the image space, f(m,n). Hence this theorem suggests that there is a substantial potential for implementing efficient compression schemes on pictorial data via the projection space.

(ii) Information preservation: Information preservation is a vital characteristic of the transformation of data. Andrews [9] has investigated information entropy transformations in the case of frequency transforms and has shown that entropy is preserved on transformation from a two dimensional continuous and/or complex amplitude space. Although an entropy measure of information does not take into account the fidelity and intelligibility constraints of image compression, it does provide a means of theoretically predicting the performance of source codecs. Theoretically, the entropy measure represents the lower bound of compression potential with an arbitrarily small distortion [10]; therefore entropy preservation indicates that it is possible to develop source codecs for bandwidth reduction or storage minimization in the spatial (image) domain that would perform as efficiently in a corresponding transform domain. We have also shown [11], in a similar manner to [10] that information is conserved on transforming an image to the projection space. Hence, to a large extent, the entropy considerations justify a projection domain coding scheme.

4. Implementation of coding scheme using VQ

Vector quantization is fundamentally a scheme for mapping a sequence of groups (or vectors) of continuous or discrete-amplitude samples into a set (or codebook) of predetermined vectors. VQ theory is well documented in the literature [12]. The dimensions of the codebook (M,K) specify the VQ where M is the number of vectors within the codebook and K is the vector size. A rate $R = \log_2 M$ bits/vector or r = R/K bits/sample may be specified. According to rate distortion theory [13] and Shannon's noiseless coding theorem [10], encoding information using vectors is a natural way of achieving lower bounds of compression, when the vector size becomes large.

For our simulations, we have designed the VQ using the iterative generalized Lloyd Algorithm, developed by Linde et al [14]. A 128x128 image (fig. 3), quantized to 8 bit/pel is the source for the projection domain compression. A DRT is applied to the source picture to generate a projection space of the same dimensions. This projection domain has been encoded using codebook sizes of (256,4), (256,8) and (512,8) respectively, corresponding to rates of 2.0, 1.0 and 1.125 bits/pel (figs. 4,5,6) respectively. The encoding has been designed to exploit inter-projection correlation corresponding to azimuthal redundancy.

Fig.3: Original Picture (128 x 128 pel) reconstructed from uncompressed projection space.

Fig. 4: PDC output using VQ codec at 2.0 bits/pel

Fig. 5: PDC output at 1.0 bit/pel

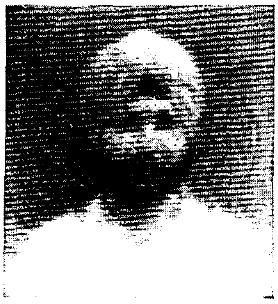

Fig. 6: PDC output at 1.125 bits/pel

For the lower bit rates, circular distortion is visible which is indicative of azimuthal compression. In fig. 4, some inherent smoothness persists in the picture; this may therefore suggest using the output of PDC (applied for inter-projection compression) as input into a conventional transform coder to achieve further compression. Alternatively, intra-projection redundancy may also be reduced in a more elaborate PDC scheme. As detailed designs [5] have shown, the VQ results may be substantially improved, for a given bit-rate, by using longer vectors in the encoding process. The PDC performance may also be improved with more sophisticated interpolation, however, an economic trade-off between complexity and compression quality is necessary to justify upgrading the interpolation technique.

## 5. Conclusion

A viable projection domain compression scheme, for pictorial data, is proposed. The emphasis has been on inter-projection compression, employing vector quantization, with a view to reducing equivalent angular redundancy in the image domain. Simulation results have shown attractive compression potential. By exploiting existing intra-projection, this image coding technique may be improved further. We believe PDC schemes are immediately relevant to medical imaging and other applications that involve acquiring large amounts of data in projection space before further processing.

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