ON THE RESOLUTION PROPERTIES OF A PARTICULAR TOMOGRAPHIC TECHNIQUE FOR INDUSTRIAL TESTING USING ULTRASOUNDS OR MICROWAVES

M. Bramanti - A. Tonazzini

Istituto di Elaborazione dell'Informazione del CNR - Via S. Maria, 46 - 56100 PISA, ITALY

RESUME

SUMMARY

Résumé

Dans ce travail on étudie les propriétés d'une particulière technique de tomographie cohérente basée sur le traitement du champ rétrodiffusé par le corps qui doit être examiné.

On assume que le corps, homogène et avec les mêmes carathéristiques électromagnétiques du milieu environnant, contient deux points irradiateurs à distance D entre eux; en se rapportant à cette condition expérimentale, on étudie le pouvoir de résolution du méthode tomographic examiné, dans le cas où on emploie des micro-ondes ou des ultra-sons pour explorer le corps.

Après avoir défini un convenable Facteur de Séparation S, on dérive, pour différentes valeurs de D, des mappes dans lesquelles S dépend des valeurs de la fréquence centrale et de la largeur de la bande des fréquences employées dans la mesure.

Summary

In this paper the resolution properties are studied of a particular coherent tomographic technique based on the processing of the backscattered field from the body under test.

It is assumed that the body itself, homogeneous and matched to the external environment, contains inside two point scatterers at distance D; with reference to this experimental condition the resolution power of the examined tomographic method are studied in the case where microwaves or ultrasounds are used as exploring radiation. By defining a suitable Separation Factor S, maps of S are derived for various D vs. the midband frequency value and the bandwidth value used in the measure.

ON THE RESOLUTION PROPERTIES OF A PARTICULAR TOMOGRAPHIC TECHNIQUE FOR INDUSTRIAL TESTING USING ULTRASOUNDS OR MICROWAVES

1. Introduction

The interest for non-invasive non destructive testing methods was always present in materials technology. A common problem is, for example, the detection and location of small unwanted inclusions inside the material bulk, air bubbles e.g., caused by faults in the manifacturing process.

The methods used for these purposes are generally based on the interaction between an acoustic or electromagnetic wave and the sample of material under test; the presence of flaws modify the above said interaction allowing the detection and, in some cases, the location of the flaws themselves.

A great variety of measuring methods and techniques was proposed and experimented for the above said purposes; two performances are in all cases of paramount importance: the sensibility, i.e. the capacity of detecting very small flaws in the material, the resolution, i.e. the capacity of distinguishing between flaws located very close.

Let us here consider the resolution performances. By referring to methods derived from radar or sonar techniques, resolution is a function of the geometrical sizes of the material volume interacting with the exploring wave; it follows that an increase of the resolution requires an improvement of focalization properties of the radiating system with increased difficulties in designing and constructing.

It is interesting to note that the above said direct relation between resolution and directivity of radiating system does not exist in the case that tomographic techniques are applied for the above said testing purposes.

As well known, three different tomographic techniques have been proposed and experimented: x-ray tomography, ultrasound tomography and microwave tomography. By using these three different exploring radiations maps of different physical properties of the tested material are obtained: density for x-rays, wave impedance for ultrasounds or microwaves.

This paper deals with a particular tomographic technique (see fig. 1) using coherent radiation (ultrasounds or microwaves) [1] where the body B to be explored is irradiated by a C.W. source with a sufficiently flat radiation diagram so that the tridimensional scattering strenght function inside volume V may be substituted by a two dimensional one $g(x,y)$; body B rotates and the distance between the radiation source and B is such that the incident wave may be considered plane and uniform. For each angular position Θ_i of B the backscattered field is received and processed in order to obtain the $g(x,y)$ function. As it will be shown in the next paragraph by means of a complete exploration of B ($0 \leq \Theta_i \leq 2\pi$) it is thus possible to obtain the two-dimensional Fourier transform of $g(x,y)$ on a whole circumference with center in the origin of Fourier plane and radius $2/\lambda$,

where λ is the wavelength of the used exploring radiation.

The aim of this paper is obtaining a more precise evaluation of the performances of the method itself where used for detecting flows located inside an homogeneous medium.

Particularly the relation between the resolving power of the method vs. the frequency values of the exploring wave will be studied.

2. The examined tomographic technique [1]

With reference to fig. 2 the received signal due to the backscattering of points at distance v from u axis may be expressed as (apart from a not important multipling factor)

$$P(v,\Theta) = \int_{-\infty}^{+\infty} g_1(u,v)\exp[-j(4\pi/\lambda)v]\,du \qquad (2\text{-}1)$$

where

$$\begin{cases} u = x\cos\Theta + y\sin\Theta \\ v = -x\sin\Theta + y\cos\Theta \end{cases} \qquad (2\text{-}2)$$

and $g_1(u,v) = g(x,y)$

Therefore the whole received signal, at the angular position Θ, is

$$G_1(\Theta) = \int_{-\infty}^{+\infty} P(v,\Theta)\,dv = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g_1(u,v)\exp[-j(4\pi/\lambda)v]\,du\,dv =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\exp[-j(4\pi/\lambda)(-x\sin\Theta + y\cos\Theta)]\,dx\,dy \qquad (2\text{-}3)$$

By putting

$$f_x = \frac{2\sin\Theta}{\lambda}$$

$$f_y = \frac{-2\cos\Theta}{\lambda} \qquad (2\text{-}4)$$

eq. (2-3) becomes

$$G_1(\Theta) = G(f_x, f_y) =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\exp[j2\pi(f_x x + f_y y)]\,dx\,dy \qquad (2\text{-}5)$$

Eq. (2-5) shows that the received signal from the direction Θ may be considered as the two-dimensional Fourier Transform of the scattering strength density function $g(x,y)$ evaluated for the values of the Fourier variables f_x and f_y expressed by eqs. (2-4); it is interesting to note that such equations define a circumference

ON THE RESOLUTION PROPERTIES OF A PARTICULAR TOMOGRAPHIC
TECHNIQUE FOR INDUSTRIAL TESTING USING ULTRASOUNDS OR MICROWAVES

in the Fourier plane described by the equation

$$f_x^2 + f_y^2 = (\frac{2}{\lambda})^2$$

By means of a complete exploration of B ($0 \leq \Theta \leq 2\pi$) it is thus possible to obtain the value of G on the whole circumference with center in the origin of Fourier plane and radius $2/\lambda$, where λ is the wavelength of the used exploring radiation.

It follows, at least from a theoretical point of view, that, by using a suitable number of complete measures at different frequencies, function G may be sampled at any predetermined set of points so that the required scattering strength density function g(x,y) may be computed with any assigned approximation degree.

In practical cases, while no particular problem generally arises for the respect of positioning body B at any Θ, it is impossible to use such a great number of frequency values that a computation of $G(f_x,f_y)$ function is allowed for any distance from the origin. Many questions are to be enquired from this respect, the principal perhaps being where to evaluate function $G(f_x,f_y)$ in order to reconstruct g(x,y) with an assigned accuracy.

This is very difficult problem because its solution generally requires some assumptions on B which is just the body to be explored.

3. The resolution performances of the examined technique

In this paragraph the simple case will be considered where body B consists of two point scatterers in an homogeneous medium matched to external environment.

Let us assume that the two scatterers have equal scattering strength and are on x axis with abscissa values $-D/2$ and $+D/2$; the g(x,y) function may be written as

$$g(x,y) = A\delta(x+D/2,0) + A\delta(x-D/2,0) \qquad (3-1)$$

where $\delta(x,y)$ is the two-dimensional Dirac impulse.

Let us also assume that the experimentally determined function $\hat{G}(f_x,f_y)$ is

$$\hat{G}(f_x,f_y) = G(f_x,f_y) \, H(f_x,f_y) \qquad (3-2)$$

where
$G(f_x,f_y)$ = Fourier Transform of g(x,y) and

$$H(f_x,f_y) = rect(\frac{DR}{\sqrt{f_x^2+f_y^2}-Rm}) =$$

$$= \begin{cases} 1 & \text{if} \quad Rm-DR/2 \leq \sqrt{f_x^2+f_y^2} \leq Rm+DR/2 \\ \\ 0 & \text{elsewhere} \end{cases} \qquad (3-3)\cdot$$

with

$$Rm = \frac{2}{c} \, (f_1+f_2)/2$$

and

$$DR = \frac{2}{c} \, (f_2-f_1)$$

(c. is the phase velocity of the exploring wave for the medium surrounding the scatterers).
It means that complete observations ($0 \leq \Theta \leq 2\pi$) of the body B are available for frequency values continuously varying between f_1 and f_2. On the basis of equations (3-1) and (3-2) the reconstructed scattering strength function $\hat{g}(x,y)$ is

$$\hat{g}(x,y)=g(x,y)*h(x,y) \qquad (3-4)$$

where

$$h(x,y) = \mathscr{F}^{-1}\left[H(f_x,f_y)\right] =$$

$$= R_2 \frac{J_1(2\pi R_2\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} - R_1 \frac{J_1(2\pi R_1\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}$$

and J_1 is the Bessel function of first kind, order one and R_1, R_2 are related to f_1 and f_2 values by the expressions

$$R_1 = 2/c \, f_1 \qquad (3-5)$$

$$R_2 = 2/c \, f_2$$

The function $\hat{g}(x,y)$ is a revolution surface around z-axis and its shape depends on DR and Rm values. A typical graph of $|\hat{g}(x,y)|$ vs. $r = \sqrt{x^2 + y^2}$ is shown in fig. 3.

With reference to this figure let us define "Separation Factor" S the quantity

$$S = \frac{D - (\Delta x_1 + \Delta x_2)}{D} \qquad (3-6)$$

ON THE RESOLUTION PROPERTIES OF A PARTICULAR TOMOGRAPHIC TECHNIQUE FOR INDUSTRIAL TESTING USING ULTRASOUNDS OR MICROWAVES

We shall say the two point scatterers to be "distinguishable" if S value is not zero, to be "non-distinguishable" if S = 0 or not defined.

In this paper, by means of a computer simulation, the resolution power of the above described technique is studied; the obtained results are presented by mapping the S value in the (DR, Rm) plane.

In order to choose the Rm range considered in the mapping reference was made to the cases where microwaves or ultrasounds are used for testing the material.

In the case of microwave exploration an experimental condition was assumed where the used frequency range is $1 \div 10$ GHz so that it results ($c - 3 \times 10^8$ m/s)

$$\frac{20}{3} \leq Rm \leq \frac{200}{3} \qquad \text{(MKS units)}$$

For each Rm, DR goes from zero to its maximum allowed value.

For ultrasounds an experimental condition was considered where the frequency range is $100 KHz \div 1MHz$ and the material has a phase velocity equal to 2000 m/s; in this case it results

$$100 \leq Rm \leq 1000 \text{ (Mks units)}$$

For each Rm, DR, as in the previous case, goes from zero to its maximum allowed value.

The results of the simulation are shown in fig. 4 and in fig. 5 relative to microwaves and ultrasounds, respectively.

In both cases for the examined distance values, the following considerations are allowed:

i) for an assigned distance D between the two scatterers a maximum value for Rm exists (Rm*) such that the separation factor S is zero for any DR. It also exists a minimum value for Rm (Rm**) such that the separation factor S is different from zero for any DR. For Rm* < Rm < Rm** a suitable range for DR always exists where S is different from zero.

In every case the S value increases as Rm increases for an assigned DR and S value also increases, for an assigned Rm, as DR increases. It is very interesting to note that such value Rm* increases as D diminishes.

ii) For an assigned DR the transition from regions with low S values (S < 0.2) to regions with high S values (S > 0.5) is very abrupt, that is a "threshold behaviour" exists for S vs. Rm.

iii) For Rm values enough higher than the "threshold region", S factor is not greatly dependent on DR.

iv) The "threshold region" begins at lower values of Rm as DR increases.

For a microwave testing in the above said frequency range the minimum distance D between the two scatterers for which S values greater than zero may be obtained is about 1 centimeter; for ultrasounds with frequency in the range $100 KHz \div 1MHz$ and for the assumed phase velocity in the material (plastic material, for example) S values different from zero may be obtained also at distances D less than 1 mm.

Conclusions

In this paper the resolution power of a particular coherent tomographic technique using ultrasounds or microwaves was investigated.

Particularly reference was made to the case where the body under test is homogeneous, matched to the external environment and contains inside only two point scatterers at distance D. For this condition a separation factor S was defined and its value was mapped vs. the mid frequency and bandwidth value of the used exploring radiation.

The obtained results allow to derive some useful criteria, for the choice of the exploring frequency values in order to maximize the resolution performances both in the case of microwaves as in the case of ultrasounds.

For a more complete evaluation of the considered testing method also its localization accuracy should be investigated. This study is in progress and the first obtained results show that a further limitation arises for the working range in the (Rm,DR) plane.

References

1. Mensa D.L., Halevy S., Wade G., "Coherent doppler tomography for microwave imaging", Proc. of the IEEE, vol. 71, no. 2, pp. 254-261.

2. Bramanti M., Tonazzini A., Salerno E., "Some theoretical aspects of a backscattering based tomographic imaging technique", Internal Report of IEI - CNR, no. B84-25.
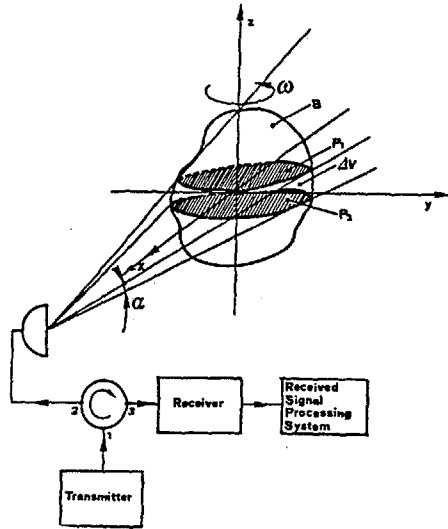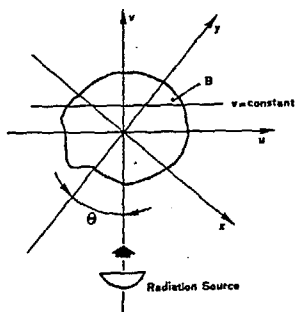
Fig. 1 - Diagram of the examined tomographic technique.

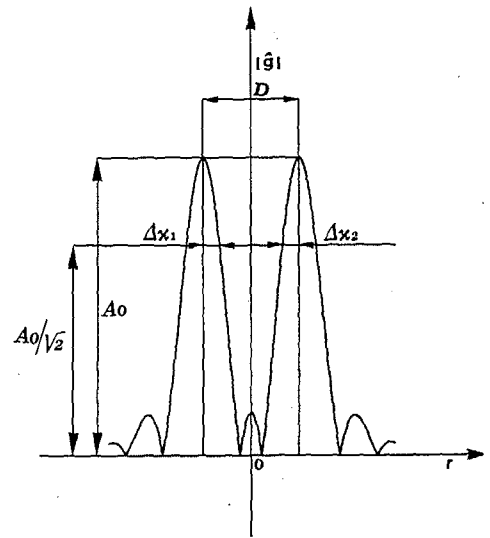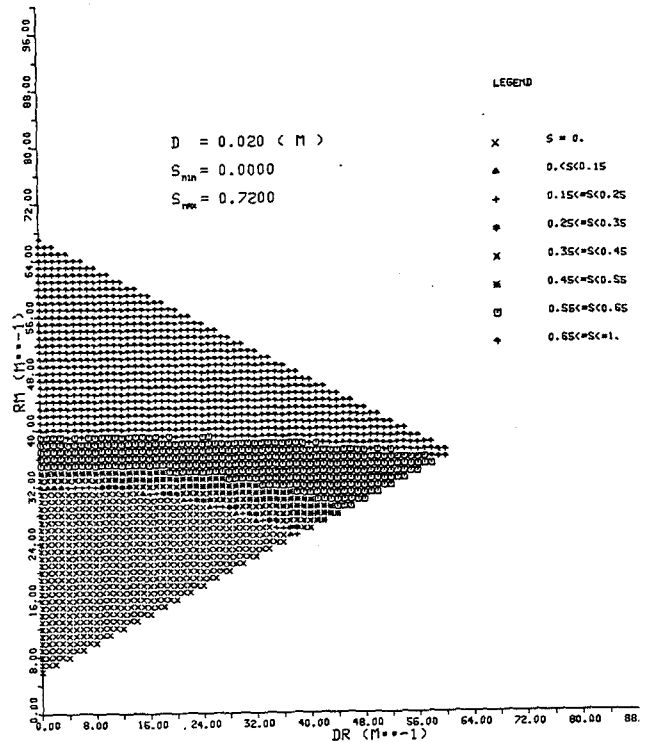Fig. 2 - The geometry assumed for the measuring method.

Fig. 3 - Typical section of the reconstructed scattering strength density function $|\hat{g}|$ for two equal strength point scatterers.

ON THE RESOLUTION PROPERTIES OF A PARTICULAR TOMOGRAPHIC
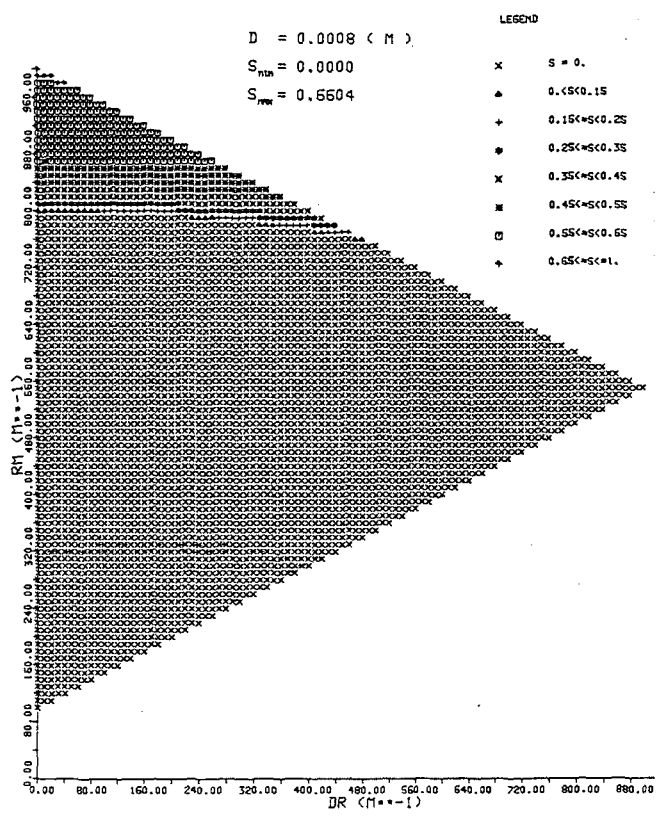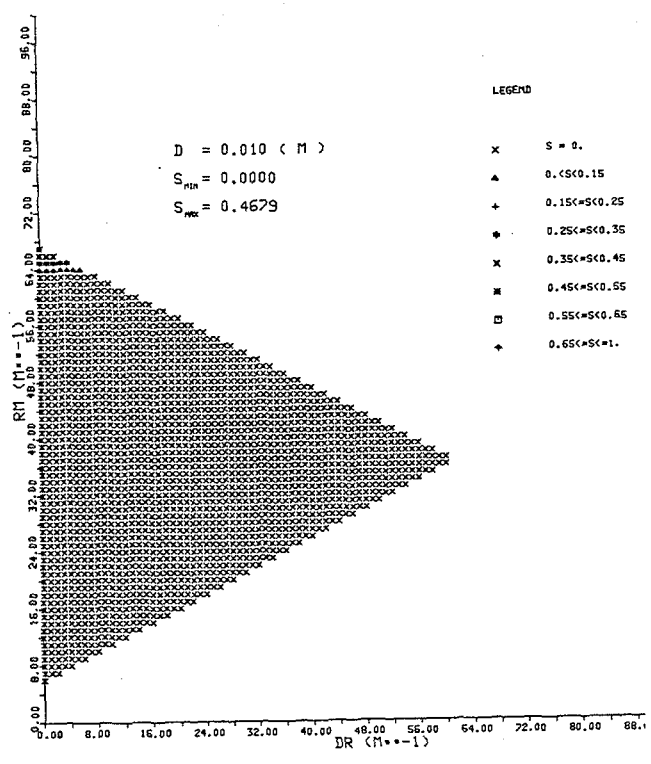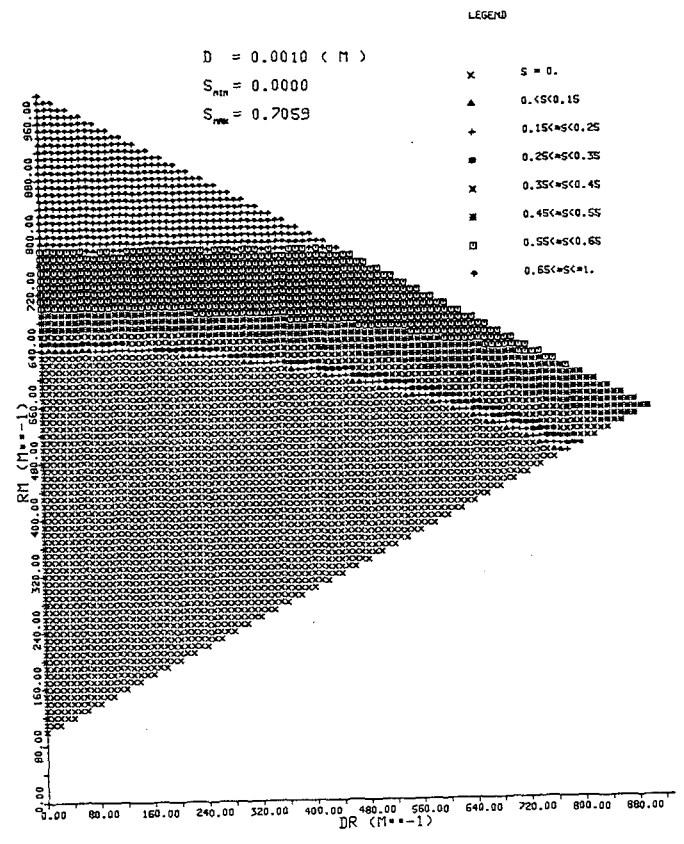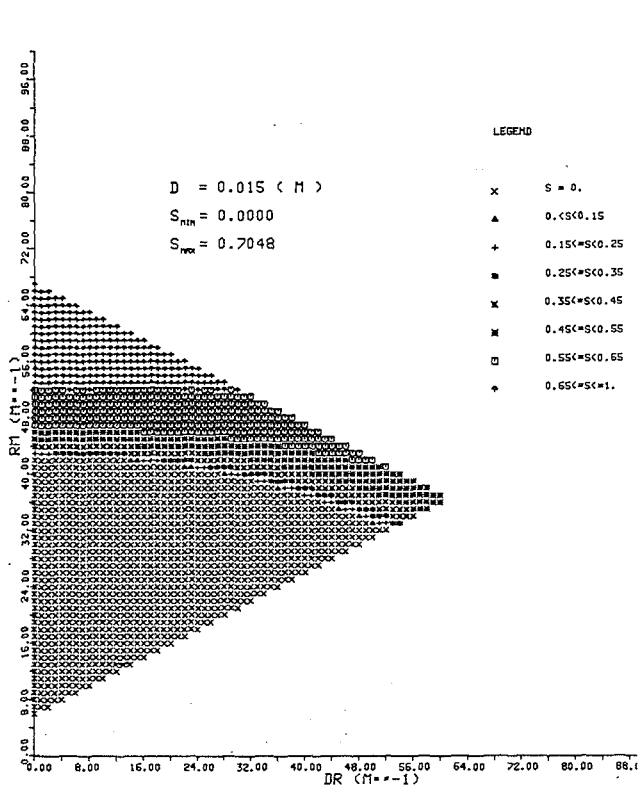TECHNIQUE FOR INDUSTRIAL TESTING USING ULTRASOUNDS OR MICROWAVES

Fig. 4 – Maps of the Separation Factor S vs. DR
and Rm for three different values of
distance D between the two scatterers;
reference is made to microwave explora-
tion in the frequency range 1 ÷ 10 GHz.

Fig. 5 – Maps of the Separation Factor S vs. DR
and Rm for two different values of
distance D between the two scatterers;
reference is made to ultrasound explora-
tion in the frequency range 0.1 ÷ 1MHz.