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## NUMERICAL PROCESSING OF MULTIFREQUENCY MICROWAVE HOLOGRAPHY

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### RESUME

Parmi les différentes techniques radar l'holographie en micro-ondes est une méthode utile pour identifier la forme des réflecteurs de diffusion. L'usage de la reconstruction numérique des images planes au moyen de la transformation inverse de FRESNEL effectuée à une certaine fréquence assure une qualité des images grâce à laquelle il est possible d'identifier des objets arbitraires dont les hologrammes ont été mesurés en magnitude et phase dans le cas d'une fréquence fixe (holographie monochromatique). La simulation numérique tridimensionnelle de deuxième ordre (transformation discrète de FRESNEL-KIRCHHOFF) des hologrammes offre l'avantage de pouvoir étudier systématiquement les conditions de traitement d'images par exemple le théorème d'échantillonnage et les influence du bruit et de l'erreur de quantification. Par des critères appropriés il est possible de déterminer la distance des objets plans en faisant usage des procédés traditionnels d'optimisation numérique.

Pour résoudre le problème de l'ambiguïté lors de la détermination de la distance d'objets tridimensionaux il est nécessaire d'utiliser des signaux en fréquences multiples. La résolution axiale dépend directement de la bande de fréquences du signal traité. Pour les recherches décrites dans cette publication on a utilisé une gamme de fréquences de 6 à 15 GHz. La variation graduelle de fréquences permet l'utilisation directe des résultats de l'holographie monochromatique. L'enregistrement des hologrammes en fréquences multiples et la reconstruction d'images traitées par une transformation spéciale de FOURIER de fréquence/distance rendent possibles une séparation absolument exacte des parts d'un objet ainsi que leur relation avec les distances véritables. Avec la méthode choisie il est possible de construire un espace réel de l'objet divisé en "tranches" équidistantes dont la distance correspond à  $\Delta z = c/2B$  ( $c$ : vitesse de la lumière,  $B$ : bande de signal).

Les résultats démontrent en outre que l'influence du bruit ainsi que des erreurs de quantification même considérables sont tellement réduites par la transformation et le procédé de superposition que la reconstruction des parts de l'objet est pratiquement de même qualité que dans le cas de non perturbation.

### SUMMARY

Microwave holography is a useful tool in the various radar techniques in order to identify remote or inaccessible scattering objects. Plane computer reconstructions with the inverse FRESNEL-transform taken from measured amplitude and phase holograms at one frequency (monofrequency microwave holography) are of proper quality so that a reliable identification of arbitrary objects is possible. To investigate systematically the conditions for the imaging method, for instance the sampling theorem and the influence of noise and quantization error, a three-dimensional numerical simulation process using the discrete form of the FRESNEL-KIRCHHOFF's diffraction integral is used. It has turned out that a numerical range determination of plane objects is possible by formulation of suitable criterions where common optimization procedures can be applied.

To solve the problem of distance ambiguity for range determination of three-dimensional objects it is necessary to use multiple frequency signals in the image processing technique. The axial resolution at the multifrequency microwave holography depends directly on bandwidth of the signal used. For the investigations described in this paper a multiple frequency signal in the range of 6-14 GHz is utilized. The step by step alteration of the frequency enables the direct assignment of the results of the monofrequency holography. Multiple frequency recording of holograms and the corresponding image reconstructions evaluated in a special frequency/distance FOURIER-transform provide an absolutely exact separation of object parts and their relation to the true distances. With the chosen method a construction of a real object space divided into equidistant "slices" is possible where the distance of the slices is given by  $\Delta z = c/2B$  ( $c$ : velocity of light;  $B$ : bandwidth of the signal). Moreover, the results show that the influence of noise and even considerable stochastic errors is reduced by the transformation and superposition procedures so that the quality of the reconstructed object parts remains almost the same as in the undisturbed case.



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## 1. Introduction

Microwave holography<sup>1-7</sup> is a suitable technique for recognition of inaccessible or remote objects with regard to their geometrical shape. As has already been shown the monofrequency imaging process, derived from the FRESNEL-transform, is of acceptable quality for two- and three-dimensional metallic objects, if the necessary conditions for sampling theorems and spatial spectra are held.

For practical applications of the method usually the scattered field intensity and phase of illuminated objects are measured, while the reconstruction step is carried out on a digital computer employing standard signal processing techniques like the FFT. To perform complicated experiments and extensive investigations, where practical measurements are too expensive or effectively impossible, the computer simulation of holograms taken from two- and three-dimensional objects using the FRESNEL-KIRCHHOFF's diffraction formula was introduced<sup>1,5,6,7</sup>.

Furthermore systematic errors and stochastic disturbances of the hologram formation, such as noise caused by incomplete reflection characteristic of the illuminated object and by spurious noise sources in space have been investigated extensively<sup>7</sup>. It has turned out that the imaging process is rather insensible against stochastic errors as well as against quantization of the hologram data.

By formulation of suitable criteria a numerical range determination of plane single objects is possible under certain conditions<sup>7</sup>. These experiments have also shown the limitation of the monofrequency microwave holography:

- reliable unambiguous range determination of plane objects is only possible for the presence of just one object
- range determination of three-dimensional objects is impossible in most cases
- the axial resolution is rather bad, that means no assignment of reconstructed object parts to their real distances is possible

To solve these problems it is necessary to employ wide band signals as already known from the quasiholography techniques<sup>8</sup> and further time domain investigations<sup>9</sup>. The method described in this paper is based on the idea to introduce a FOURIER-transform from the frequency- to the distance domain<sup>10</sup>. The type of the wide-band signal thereby allows to utilize all the knowledge gained from the experiments in monofrequency holography.

First the multifrequency microwave holography will be applied to the problem of range determination for a single plane object and for three plane plates arranged in different distances.

The investigation of reconstructions of three-dimensional objects will show the influence of the signal bandwidth to the axial resolution of the process. By suitable choice of the employed signal it is possible to separate distinct object parts clearly even in case of complicated contours and to perform a real reconstruction space displaying the object parts in plane slices along the distance  $z$ .

Finally it will be shown that the procedure is largely insensitive against noise and quantization errors. The quality of the reconstructed objects remains good even in case of heavy disturbances.

## 2. Theory

The theory for the multifrequency microwave holography applied in this paper is based on an extension of the monofrequency holography to multiple frequency signals. In contrast to investigations in the time domain<sup>8,9</sup> the process is calculated in the domain of complex functions.

The most simple construction of a multifrequency signal can be achieved by altering the frequency of a transmitter step by step starting from a fixed frequency  $f_0$

$$f = f_0 + q\Delta f, \quad q = 0, 1, \dots, Q-1 \quad (1)$$

with  $Q$  being the number of samples with step size  $\Delta f$ . Hence the bandwidth of the signal is given by

$$B = (Q-1)\Delta f \quad (2)$$

The axial resolution  $\Delta z$  of the procedure depends directly on the bandwidth  $B$ <sup>10</sup>

$$\Delta z = \frac{c}{2B} \quad (3)$$

where  $c$  is the velocity of light.

The computer simulation of the diffracted wave at the point  $P_{ij}$  in the hologram plane (Fig. 1a) has to be calculated with the FRESNEL-KIRCHHOFF's diffraction formula in its discrete form<sup>6</sup> (4) separately for all  $Q$  frequencies, where the  $z$ -dimension of the object (Fig. 1b) is subdivided into  $P$  planes with the dimensions  $\Delta x \cdot M$  and  $\Delta y \cdot N$  and the complex hologram function  $U_{ijq}(z)$  is taken for  $I \cdot L$  points with the sample distance  $\Delta x_0, \Delta y_0$ .

$$U_{ijq}(z) = j \frac{1}{2\lambda} \sum_{p=1}^{P-1} \Delta x \Delta y \Delta z_p \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} O_{mnp} C_s \frac{e^{-jks_{mnp}}}{s_{mnp}} \quad (4)$$

$$\frac{e^{-jkr_{mnilp}}}{r_{mnilp}} \left\{ \cos(\vec{v}_{mnp}, \vec{r}_{mnilp}) - \cos(\vec{v}_{mnp}, \vec{s}_{mnp}) \right\}$$

with  $\lambda = \frac{c}{f}$  = wavelength

$$k = \frac{2\pi}{\lambda}$$

$O_{mnp} C_s$  = field distribution of the diffracting surfaces

$$s_{mnp} = \left\{ (x_{os} - x_w - \Delta x_p m)^2 + (y_{os} - y_w - \Delta y_p n)^2 + z_p^2 \right\}^{1/2}$$

$$r_{mnilp} = \left\{ (\Delta x_0 i - x_w - \Delta x_p m)^2 + (\Delta y_0 j - y_w - \Delta y_p n)^2 + z_p^2 \right\}^{1/2}$$

$$\vec{v}_{mnp} = \vec{n}_{mnp} v_{mnp} \quad \vec{n}_{mnp} = \text{direction of the area element } \Delta F$$

$$v_{mnp} = z_p \left\{ 1 + \left( \frac{\Delta x_p}{\Delta x} \right)^2 + \left( \frac{\Delta y_p}{\Delta y} \right)^2 \right\}^{1/2}$$

$$\cos(\vec{v}_{mnp}, \vec{r}_{mnilp}) = \frac{r_{mnilp}^2 + v_{mnp}^2 - D_1^2}{2r_{mnilp} v_{mnp}}$$

$$\cos(\vec{v}_{mnp}, \vec{s}_{mnp}) = \frac{s_{mnp}^2 + v_{mnp}^2 - D_2^2}{2s_{mnp} v_{mnp}}$$

$$D_1^2 = (\Delta x_0 i - (x_w + \Delta x m) - \frac{\Delta z_p}{\Delta x} z_p)^2 + (\Delta y_0 j - (y_w + \Delta y n) - \frac{\Delta z_p}{\Delta y} z_p)^2$$

$$D_2^2 = (x_{os} - (x_w + \Delta x m) - \frac{\Delta z_p}{\Delta x} z_p)^2 + (y_{os} - (y_w + \Delta y n) - \frac{\Delta z_p}{\Delta y} z_p)^2$$

The computer image reconstruction is carried out with the inverse discrete FRESNEL-transform<sup>6</sup>, which is of the form

$$O_{mnq}(z) = -j \frac{\Delta x_0 \Delta y_0}{\lambda z} s_{mn} e^{jks_{mn}} e^{jkz} e^{j\frac{k}{2z} \left\{ (x_w + \Delta x m)^2 + (y_w + \Delta y n)^2 \right\}} \quad (5)$$

$$\sum_{i=0}^{I-1} \sum_{j=0}^{L-1} U_{ijq} e^{j\frac{\pi}{\lambda z} \left\{ (\Delta x_0 i)^2 + (\Delta y_0 j)^2 \right\}} e^{-j\frac{2\pi}{\lambda z} \left\{ (x_w + \Delta x m) \Delta x_0 i + (y_w + \Delta y n) \Delta y_0 j \right\}}$$

and has to be calculated for all  $Q$  frequencies from the corresponding holograms.

For all former investigations<sup>1,5,6,7</sup> it was defined that the object function  $O_{mnp}$  is real;  $O_{mnp} = 1$  in the simulation procedure. The result of the reconstruction process  $O_{mnq}$  generally is complex due to the fact that the FRESNEL-transformation is an approximation of the simulation formula (4). Nevertheless for the recon-

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structured values we obtain  $|O_{mnq}| \approx 1, \angle O_{mnq} \approx 0$  for all frequencies if the reconstruction distance is in fact the true distance of the object. For all other reconstruction distances  $O_{mnq}$  is complex with different angles for each frequency. Consequently if we evaluate the results of reconstructions taken with different distances, e.g.  $z = z_0 + p\Delta z$  ( $p = 0, 1, \dots, Q - 1$ ), in form of a sum over all frequency samples

$$O_{mn}^i(z_0 + p\Delta z) = \sum_{q=0}^{Q-1} O_{mnq}(z_0 + p\Delta z) \quad (6)$$

we obtain a maximum of  $|O_{mn}^i|$  at the true distance  $z_p$  (Fig.1), whereby the object contours can be determined clearly.

For all other distance slices the reconstructions are of less quality and the value of the summ is decreased remarkably.

For the construction of an object space with e.g.  $Q$  slices built up with  $Q$  frequencies it is necessary to calculate  $Q^2$  reconstructions, which would be unacceptably time consuming. An alternative solution is the introduction of a FOURIER-transform from the frequency- to the distance domain<sup>10</sup>:

$$O_{mn}^*(z_0 + p\Delta z) = \sum_{q=0}^{Q-1} O_{mnq} e^{j \frac{4\pi}{c} q\Delta f p\Delta z} \quad (7)$$

The basic idea of the method is to calculate only  $Q$  reconstructions at the distance  $z_0$  and then to compose the distance slices at  $p \cdot \Delta z$  by merely forming the appropriately weighted sum over all  $Q$  samples. The disadvantage of this procedure - that objects or object parts located farther away from  $z_0$  are reconstructed not with the maximum quality - may be circumvented by limitation of the reconstruction range within e.g. one tenth of  $z_0$ . For extending the range, the procedure may be repeated with new start distances  $z_{\text{new}} = z_{\text{old}} + (1/10) z_{\text{old}}$ , as many times as necessary

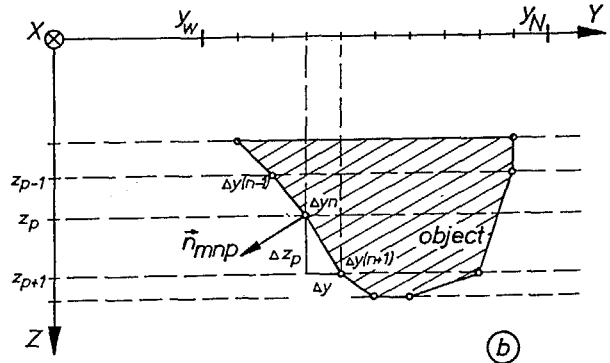
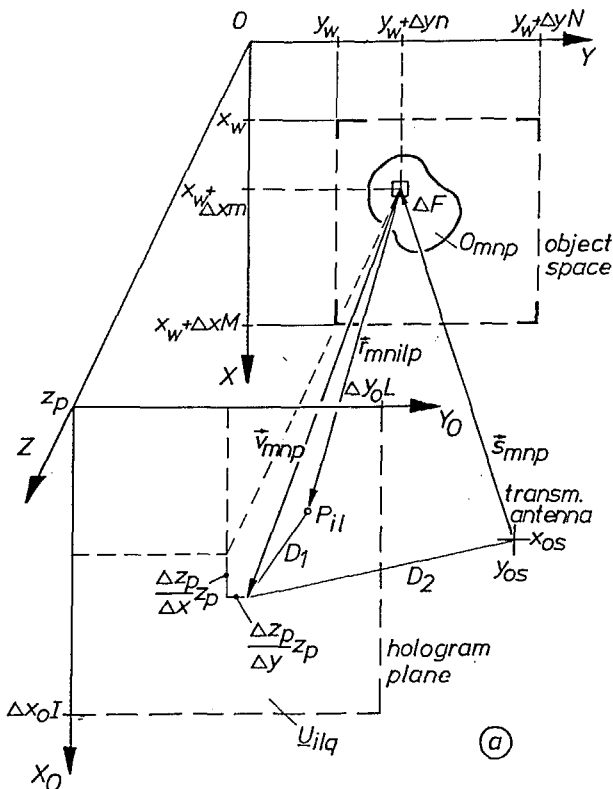


Fig. 1 Object and hologram plane  
a arrangement and related coordinates  
b subdivision of the three-dimensional object space into  $P$  planes perpendicular to the  $z$ -coordinate

3. Experimental results

Range determination

As already reported<sup>7</sup> the distance between plane objects and the hologram plane can be determined with the tools of monofrequency holography by formulation of suitable sharpness criterions. Unfortunately this method can be applied for the presence of plane objects parallel to the hologram field only and is ambiguous if there is more than one object in the investigated space. Because the method is based on the principle to find the minimum of an error function, moreover, many reconstructions have to be calculated along the  $z$ -axis, which has turned out to be time consuming. The axial resolution of the reconstruction procedure itself in monofrequency holography is generally by an order-of-magnitude worse than the lateral resolution. Fig. 2a shows that even for a simple plane plate the true distance cannot be assigned reliably if just one frequency is used.

The application of multifrequency holography employing the FOURIER-transform (eq.7) however provides a clear range determination with utilization of just 5 frequency samples (Fig. 2b).

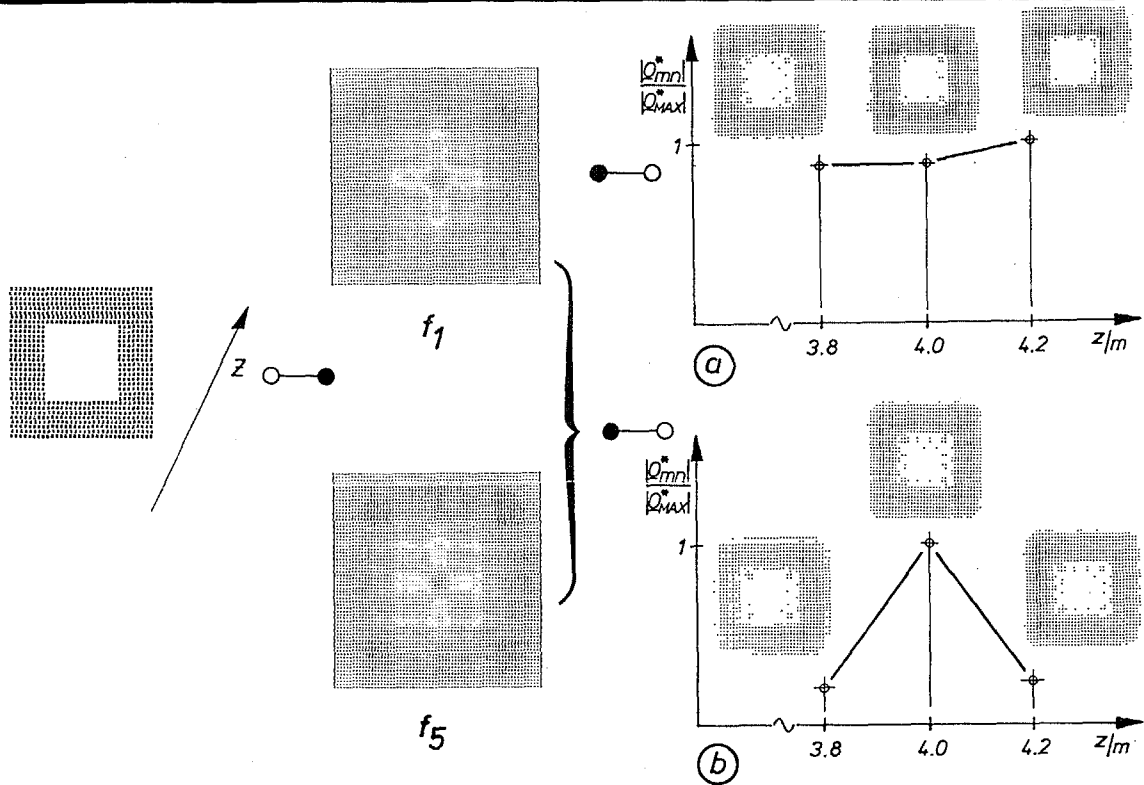
As a second example an arrangement of three plane plates located in different distances is chosen. Fig. 3a shows again that no unambiguous distance determination is possible with a monofrequency signal. The result of the multifrequency process (eq.7) is shown in Fig. 3b, the reconstructed arrangement is nearly an exact copy of the original model with a sufficient image quality. The sharpness of the reconstructed plates can be furthermore improved if the much more expensive step by step process (eq.6) is used (Fig. 3c).

Three-dimensional objects

The most interesting application of multifrequency holography is the recognition of three-dimensional objects. It has to be noticed that the depth information ( $z$ -direction) of objects as well as the contour information in  $x$ - and  $y$ -direction can be achieved only by sampling steps due to numerical processing. Furthermore it has turned out that just those object parts can be reconstructed which reflect energy into the hologram plane. These object parts are found out to be more or less parallel to the hologram plane and thus the reconstructed object is displayed in "slices" parallel to the hologram plane as a sufficient approximation. So the correct object description depends on the sampling width  $\Delta z$  and consequently on the signal bandwidth  $B$

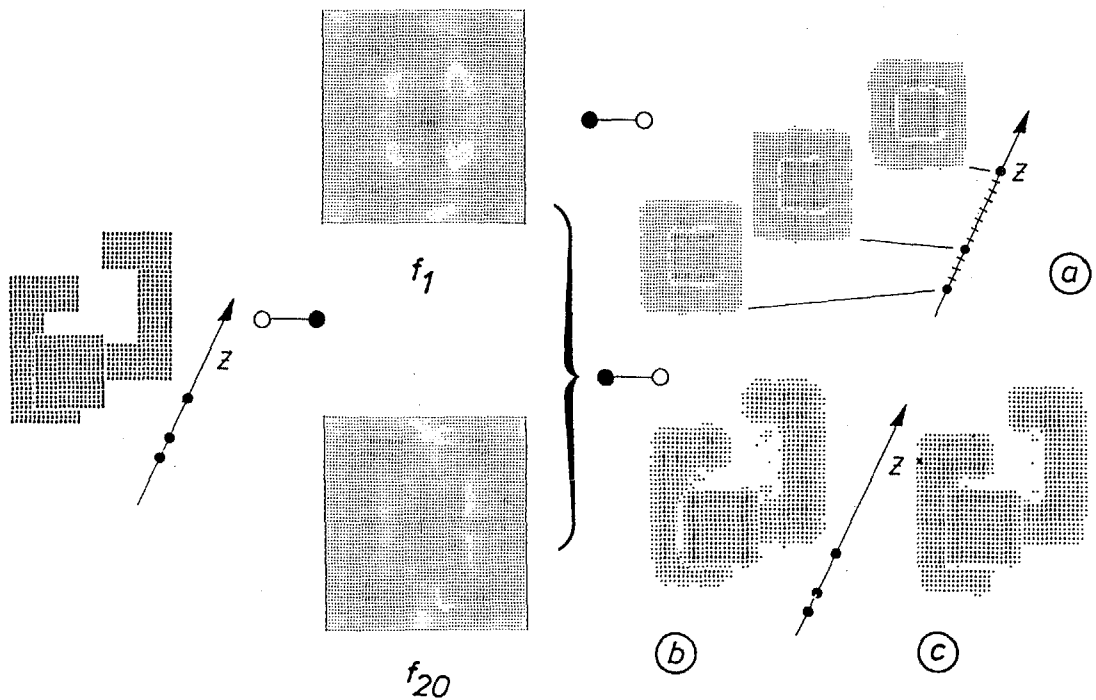


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**Fig. 2** Range determination for one object  
simulation data:  $x_M = y_N = 1$  m,  $M = N = 32$ ,  
 $x_W = 0.415$  m,  $y_W = 1.25$  m,  $z = 4$  m;  
 $x_{0I} = y_{0L} = 1.92$  m,  $I = L = 64$ ,  $x_{0S} = 0.83$  m,  
 $y_{0S} = 2.5$  m  
reconstruction data:  $x_M = y_N = 1.5$  m,  
 $M = N = 48$ ,  $x_W = 0.165$  m,  $y_W = 1.00$  m

- a reconstructions with three distances 3.8 m, 4 m, 4.2 m for one single frequency,  $f = 10$  GHz
- b reconstruction space built up with the FOURIER-transform,  $Q = 5$ ,  $\Delta f = 0.1875$  GHz,  $f_0 = 10$  GHz,  $z_0 = 3.6$  m,  $\Delta z = 0.2$  m



**Fig. 3** Range determination for three plane plates  
simulation data:  $x_M = y_N = 1$  m,  $M = N = 32$ ,  
 $x_W = 0.415$  m,  $y_W = 1.25$  m,  $z = 3.84$  m -  
4.32 m;  $x_{0I} = y_{0L} = 1.92$  m,  $I = L = 64$ ,  
 $x_{0S} = 0.83$  m,  $y_{0S} = 2.5$  m  
reconstruction data:  $x_M = y_N = 1.5$  m,  
 $M = N = 48$ ,  $x_W = 0.165$  m,  $y_W = 1.00$  m

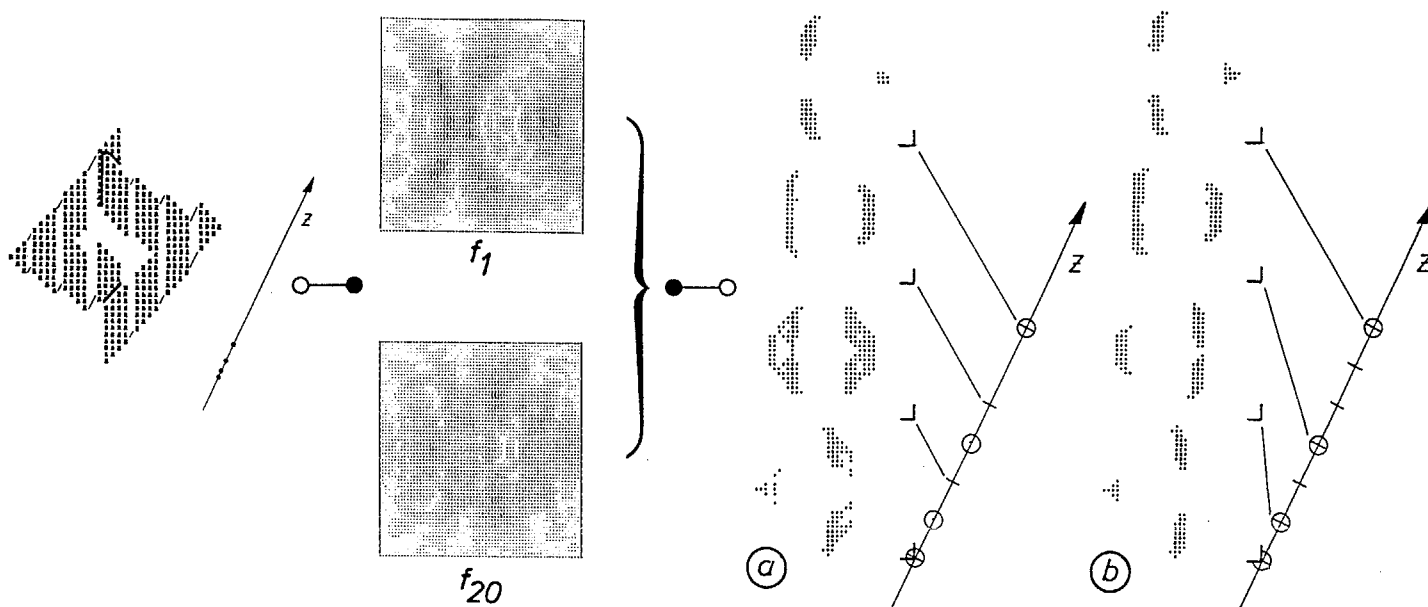
- a reconstructions at three distances 3.84 m, 4 m, 4.32 m for one single frequency,  $f = 10$  GHz
- b reconstruction space built up with the FOURIER-transform,  $Q = 20$ ,  $\Delta f = 0.1974$  GHz,  $f_0 = 8.00$  GHz,  $z_0 = 3.72$  m,  $\Delta z = 0.04$  m
- c reconstruction space built up step by step (eq. 6)  $z = z_0 + p\Delta z$ ;  $p = 0, 1, 2 \dots 19$ ; same data as in b



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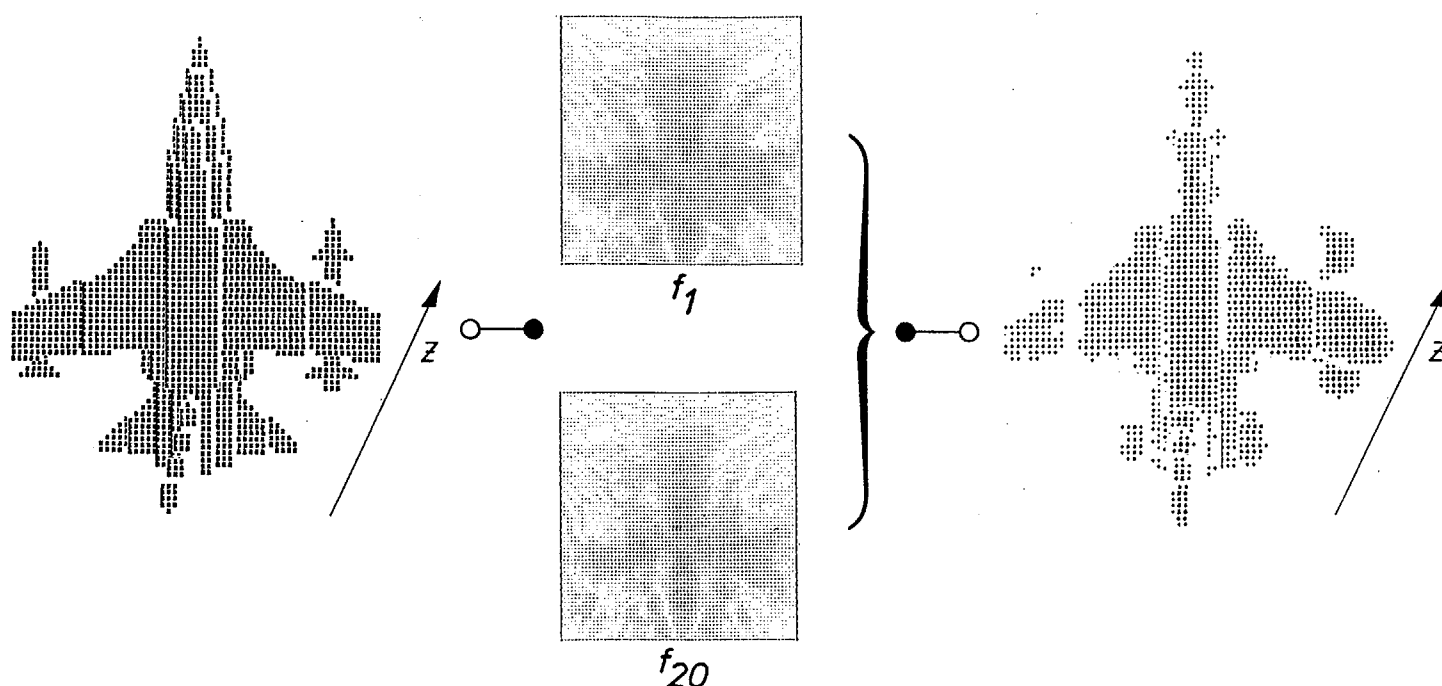
only which usually is chosen to be matched to the problem under investigation. The resolution experiment (Fig. 4a,b) shows the results of the imaging process for two different signal bandwidth. In both cases those object parts are reconstructed clearly which are assigned to the corresponding distance slices. In the reconstruction of Fig. 4a where  $\Delta z = 0,04$  m the object parts for which  $z = z_0 + 0,02$  m and  $z = z_0 + 0,06$  m cannot be recognized while in Fig. 4b with  $\Delta z = 0,02$  m all object parts can

be determined perfectly. The experiment furthermore demonstrates that there is no difference in quality between the reconstruction accuracy of small and large object parts as well as there is no difference whether the distance between two object parts is just  $\Delta z$ ,  $2\Delta z$  or  $3\Delta z$ . The "aircraft"-experiment (Fig. 5) conclusively shows definitely that also the recognition of a complicated three-dimensional object is possible with adequate image quality.



**Fig. 4** Resolution experiment, object "rhombus"  
simulation data:  $x_M = y_N = 1$  m,  $M = N = 32$ ,  
 $x_W = 0.415$  m,  $y_W = 1.25$  m,  $z = 4.00 - 4.12$  m;  
 $x_{0I} = y_{0L} = 1.92$  m,  $I = L = 64$ ,  
 $x_{0S} = 0.83$  m,  $y_{0S} = 2.5$  m  
reconstruction data:  $x_M = y_N = 1.125$  m,  
 $M = N = 36$ ,  $x_W = 0.3525$ ,  $y_W = 1.1875$  m

a reconstructions with the FOURIER-transform,  $Q = 20$ ,  $\Delta f = 0.1974$  GHz,  
 $f_0 = 12.5$  GHz,  $z_0 = 3.96$  m,  $\Delta z = 0.04$  m  
b reconstructions with the FOURIER-transform,  $Q = 20$ ,  $\Delta f = 0.3947$  GHz,  
 $f_0 = 12.5$  GHz,  $z_0 = 3.96$  m,  $\Delta z = 0.02$  m



**Fig. 5** Object "aircraft"  
simulation data:  $x_M = y_N = 1$  m,  $M = N = 70$ ,  
 $x_W = 0.415$  m,  $y_W = 1.25$  m,  $z = 4.00 - 4.28$  m;  
 $x_{0I} = y_{0L} = 1.92$  m,  $I = L = 64$ ,  
 $x_{0S} = 0.83$  m,  $y_{0S} = 2.5$  m

reconstruction data:  $x_M = y_N = 1$  m,  
 $M = N = 70$ ,  $x_W = 0.415$  m,  $y_W = 1.25$  m;  
 $Q = 20$ ,  $\Delta f = 0.3947$  GHz,  $f_0 = 26.5$  GHz,  
 $z_0 = 3.96$  m,  $\Delta z = 0.02$  m



**Noise influence**

All kind of random disturbances affecting the imaging process can be simulated by superposition of a complex noise signal in the hologram plane<sup>7</sup>

$$U'_{i1} = U_{i1} + c\xi_{i1}$$

where  $c$  is an amplitude factor and  $\xi$  a complex random variable which may be a function of location and time.

In the experiment shown in Fig. 6a  $Q$  amplitude- and phase noise patterns are generated and added to the  $Q$  holograms.

The image reconstructions demonstrate that despite the

heavy noise disturbance in the holograms ( $S/N = 3.0$ ) a clear object identification is possible. The noise influence is almost eliminated by the FRESNEL-transform procedure and due to the fact that all  $Q$  noise patterns are statistically independent and thus additionally equalized in the following FOURIER-transform.

**Quantization error**

The influence of magnitude- and phase quantization in the hologram function  $U_{i1}$  can be expressed as

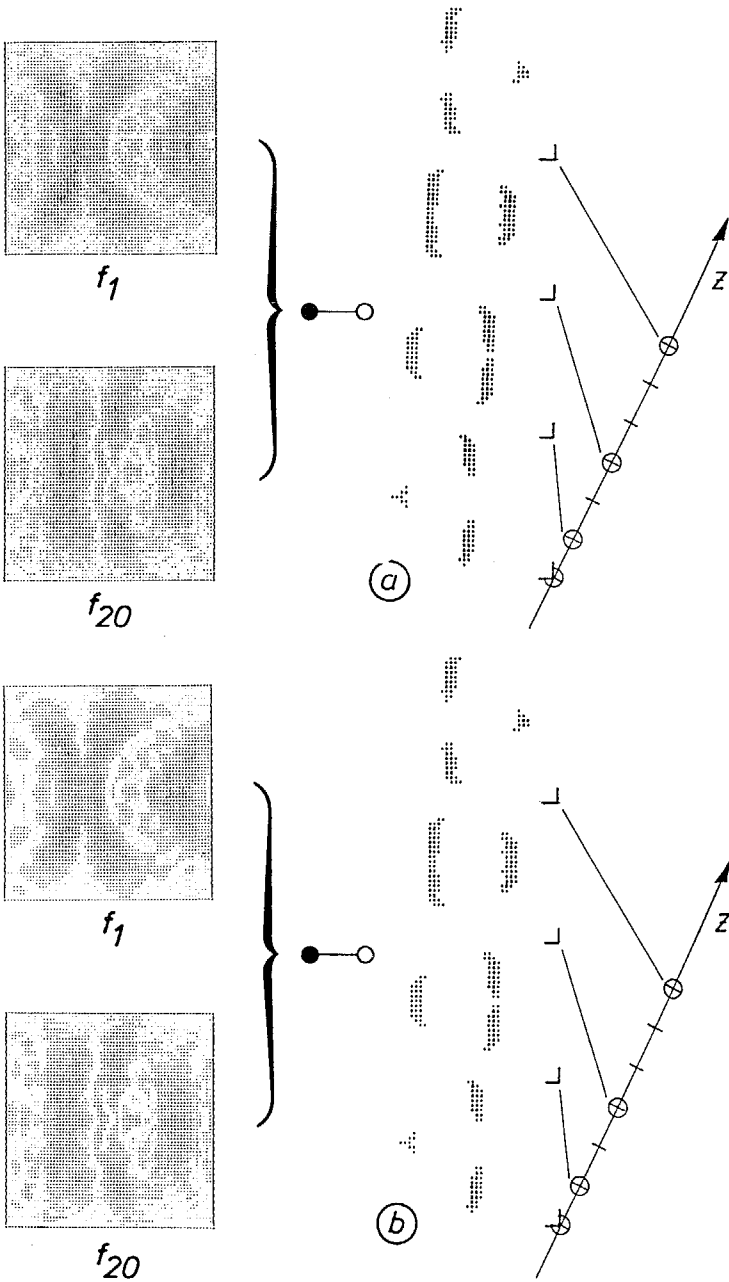
$$U'_{i1} = (U_{i1} + \Delta U_{i1}) e^{j(\varphi_{i1} + \Delta \varphi_{i1})}$$

where

- $\Delta U_{i1}$  = error of magnitude quantization
- $\Delta \varphi_{i1}$  = error of phase quantization

are stochastic variables.

Fig. 6b shows the influence of hologram amplitude- and phase quantization on the quality of the three-dimensional object reconstruction for 8 amplitude and 8 phase steps. Although the quantization is very rough the effect is largely eliminated as in Fig. 6a and the object can be recognized clearly.



**Fig. 6** Influence of noise and hologramm quantization, object "rhombus", data as in Fig. 4,  $Q = 20$ ,  $\Delta f = 0.3947$  GHz,  $f_0 = 12.5$  GHz,  $z_0 = 3.96$  m,  $\Delta z = 0.02$  m  
 a noise influence,  $S/N = |U|_{max}/c|\xi|_{max} = 3$   
 b quantization error, magnitude: 8 quantization steps phase : 8 quantization steps

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