DIXIEME COLLOQUE SUR LE TRAITEMENT DU SIGNAL ET SES APPLICATIONS

NICE du 20 au 24 MAI 1985

MINIMUM EUCLIDEAN DISTANCE OF SOME BLOCK CODES AND BINARY CPFSK MODULATIONS

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RESUME

Avec cet article nous analyzerons l'intégration de la modulation et de la codage de canal.

La codage de canal sert a augmenter la distance Euclidienne entre les signaux modulés. On monstrera que la distance minimale Euclidienne est significativement augmenteé avec l'introduction de la codage de canal.
Nous déterminerons la configuration de la matrice de contrôle de parité avec différents codes linéaires binaires, pour obtenir le valeur maximal de la distance minimale Euclidienne.

Ensuite on déterminera la probabilité d'erreur d'un systéme de communication, qui integre les operations de modulation et de codage de canal.

SUMMARY

In this paper the integration of channel coding and modulation is analyzed. Channel coding is used to increase the Euclidean distance between modulated signals. It is shown that the minimum Euclidean distance is significantly increased by the channel coding.

Many different block codes are considered and the optimum configurations of the parity-check matrix of each code, which permits to achieve the higher minimum Euclidean distance, are determined.

The error probability of communication systems using integrated modulation and channel coding is determined.

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1. INTRODUCTION

An increasing attention has been devoted in the recent years, to the design of efficient modulation schemes for data transmission. The principal two parameters used to characterize the efficiency of a modulation scheme are the bandwidth occupance and the bit error probability.

Continuous-Phase-Frequency-Shift-Keying (CPFSK) modulation is one of the more attractive schemes for its good spectral properties. The error probability of a CPFSK scheme is minimized by using a Maximum-Likelihood demodulator, which chooses as the transmitted sequence that having the lower Euclidean distance from the received sequence. In this way, the bit error probability is determined by the Euclidean distance between the modulated signals. It is also well-known from the literature that, for high signal-to-noise ratio, the error probability is essentially determined by the minimum Euclidean distance.

When the desidered error probability cannot be achieved with the modulation scheme, a channel coding operation must be introduced. Channel coding creates a Hamming distance between codewords through the redundancy symbols.

The Euclidean and Hamming distances are different and they are used by the receiver in two distinct ways: the first infact, is used during the demodulation and the second during the decoding. For this reason, these two operations are always distinct and are optimized independently from each other.

Some examples can be found in the literature, in which the demodulation operation is employed in order to improve the channel decoding efficiency, through the reliability estimate of each received symbol $r_{i,j}$ [1] , [2] , [3] , [4] . The utilization of the reliability estimate improves the performance of the channel decoder.

Recently, Ungerboeck has described a method for the integration of modulation and channel coding [5]

In particular, Ungerboeck utilizes a channel coding scheme with expanded sets of multilevel phase signals in order to increase the Euclidean distance in the signal space. By using this type of channel encoding, a net improvement in the error probability can be obtained. In a similar way, Aulin and Sundberg have shown that convolutional codes can be used in order to improve the Euclidean distance of a modulated signal [6] .

In this paper the integration of channel coding and modulation in a communication system, in order to increase the Euclidean distance between modulated signals, is analyzed. In this way, a net improvement can be obtained in many cases with respect to the classical situation, in which modulation and channel coding are separate and indipendent. CPFSK modulation and block codes are considered. It is shown that the Euclidean distance of the integrated communication system depends from the configuration of the parity-check matrix H of the code. For a given code and a fixed value of the modulation index, some configurations of H are optimum, because the minimum Euclidean distance between the modulated signals assumes the maximum value. These configurations of H permit to achieve the minimum error probability.

Many different block codes, as Hamming codes, shortened Hamming codes and BCH codes are analyzed. For all these codes, the optimum configurations of the parity-check matrix have been determined as a function of the modulation index and of observation interval length.

By using these results on the Euclidean distance, the block error probability of communication systems using modulation and channel coding in an integrated way, is derived and compared with that obtained by a classical system. It is shown that in many cases, the block error probability P_b is lower than in the classical systems, also when the parity-check matrix configuration is not optimized. When H is optimum, a net reduction in P_b is obtained for all the signal-to-noise ratios.

2. INTEGRATION OF MODULATION AND CHANNEL CODING

If E denotes the energy of the signal and T the signaling time, a CPFSK modulated signal can be represented in the form [7] :

$$(1)\qquad s_k(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_o t + d_k \frac{\pi h t}{T} + x_k\right]$$

$$\text{for } KT \leqslant t \leqslant (K+1)T$$

being $f_o = \omega_o / 2\pi$ the carrier frequency, d_k the k-th informative symbol ($d_k = +1$ or -1), h the modulation index and x_k a phase term given by

$$(2)\qquad x_k = \left[x_{k-1} + (d_{k-1} - d_k)\,\pi h k\right] \mod 2\pi$$

being $x_o = 0$. The term x_k has been introduced in order to maintain the phase continuity at the end of the signaling intervals. For $h = 0.5$ the MSK modulation is obtained.

If $\underline{c}_i$ is a codeword of a code having length equal to n , then it can be associated with a vector in the signal space $\underline{s}_i(t)$ given by

$$(3)\qquad \underline{s}_i(t) = \left[s_{i,1}(t),\ s_{i,2}(t),\ \ldots, s_{i,n}(t)\right]$$

where $s_{i,j}(t)$ denotes the waveform used for the transmission of the j-th symbol of $\underline{c}_i$.

The Euclidean distance between the two signal sequences $\underline{s}_i(t)$ and $\underline{s}_j(t)$ associated with the two codewords $\underline{c}_i$ and $\underline{c}_j$ is [7] :

$$(4)\qquad D^2_{i,j} = \sum_{0}^{n-1}{}_{k} \int_{kT}^{(k+1)T} \left[s_{i,k}(t) - s_{j,k}(t)\right]^2 dt$$

By assuming $2\pi f_o T \gg 1$, the normalized minimum Euclidean distance d^2_{min} , in the case of the CPFSK modulation, can be written as :

$$(5)\qquad d^2_{min} = \min_{\substack{i,j\\ i\neq j}} \frac{1}{2E} \sum_{0}^{n-1}{}_{k}\, 2E \left\{ 1 - \int_{kT}^{(k+1)T} \frac{1}{T} \cdot \right.$$

$$\cdot \cos\left[(d_{i,k} - d_{j,k}) \frac{\pi h t}{T} + x_{i,k} - x_{j,k}\right] dt\Big\}$$

The parameter d^2_{min} characterizes the error probability of a communication system [7] and therefore is a quite important parameter. The Euclidean distance between two signal sequences depends on the modulation index h and the number of signaling intervals in which the two sequences differ.

The block-diagram of the communication system with Integrated Modulation and block Channel Coding (IMCC) is shown in Fig.1 .

At the transmitter side, an IMCC system has the same structure of a classical communication system.

At the receiver side the demodulation is performed every n time signaling intervals.

then the two codes have the same error correction capability and characteristics; such codes are called equivalent [8] . In particular, every code is equivalent to a code in a systematic form having a parity-check matrix of the form:

$$H = \left[P \ , \ I_{n-k}\right] \qquad (6)$$

being P a matrix (n-k) x k and I_{n-k} the (n-k) x (n-k) identity matrix .

On the contrary, in the IMCC systems the Euclidean distance depends on the configuration of H : by permuting rows or columns of H, the minimum Euclidean distance varies and therefore changes the performance of the system.

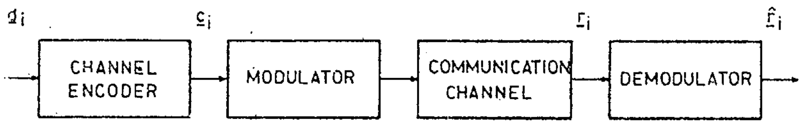

Fig. 1 General block-diagram of a communication system with integrated modulation and channel coding.

The signal $r_j(t)$ received at the j-th time signaling interval ($1 \leqslant j \leqslant n$) is stored to constitute the received signal sequence $\underline{r}(t)$. The demodulator compares $\underline{r}(t)$ with all the possible codewords $\underline{c}_i$ for $1 \leqslant i \leqslant 2^k$ and chooses that codewords $\underline{c}_i$, which corresponds to the signal sequence $\underline{s}_i(t)$ having the minimum Euclidean distance from the received vector $\underline{r}(t)$. No channel decoding operation is performed in this way at the receiver side. The Hamming distance, introduced by the channel coding operation, is utilized in an IMCC system only to increase the Euclidean distance between the transmitted signal sequences.

If a code has a minimum Hamming distance equal to d_H , then two codewords $\underline{c}_i$ and $\underline{c}_j$ differ at least in d_H positions. As a consequence, $\underline{s}_j(t)$ and $\underline{s}_i(t)$ differ, at least, in d_H time signaling intervals and therefore, the Euclidean distance is increased with respect to the case in which no channel coding operation is present. A net improvement is also achieved with respect to a classical communication system using channel decoding.
These advantages are obtained at the expense of the demodulation complexity. In a classical communication system the demodulation is performed symbol by symbol; in an IMCC system the demodulation is performed by comparing the received vector $\underline{r}(t)$ with all the 2^k possible signal sequences and for mean or high k the complexity can be very high.

Now we consider a block code (n,k), which is defined through a parity-check matrix H having dimensions (n-k).n.

If the parity-check matrices of two codes differ only for row or column permutations and combinations

The choice of the configuration of the parity - check matrix H is, therefore, quite important in order to optimize the performance of an IMCC system.

Unfortunately no theoretical approach was found by the authors and therefore the computation of the optimum H must be carried out by computer research.

3. RESULTS

In this section, the results concerning the Euclidean distance of IMCC systems for some block codes,are given. The codes considered herein are always in the systematic form. The Euclidean distance of an IMCC system depends on the configuration of the parity-check matrix H of the code. For all the codes considered in this paper a complete and exhaustive computer research was performed in order to determine the worst and the best configurations of H .

First, the Hamming codes (7,4) and (8,4) having a Hamming distance of 3 and 4, respectively are analyzed. The parity-check matrix, if considered in a systematic form, can assume for the two codes $N_c = 4!$ different configurations. In fact, matrix P in (6) can vary, while I_{n-k} is fixed once the code is assumed as systematic.

By considering all the possible configurations N_c of the matrix P the best and the worst matrix have been derived. These matrices depends on the modulation index h: a matrix which results optimum for a given h value can be non - optimum for other h values.

The best and the worst configuration of H for the analyzed codes are reported in Table 1 .

In this Table each column $\underline{h}_i$ of H is represented by a decimal number between 1 and $2^{n-k}-1$, whose binary representation is equal to $\underline{h}_i$. For semplicity, when two or more matrices give the higher or the lower d^2_{min} , only a matrix, randomly chosen, is reported in Table 1.

In Fig.5 the Euclidean distance for the BCH code (15,7) is represented. The configuration of the parity-check matrix considered,is shown in the same Fig.5 .

For such code, which has a Hamming distance equal to 5, the Euclidean distance is greatly increased with respect to Hamming codes.

CODE	MODULATION INDEX h	OPTIMUM MATRIX	CODE	MODULATION INDEX h	WORST MATRIX
(7,4)	0.1 ÷ 0.6 0.7 ÷ 0.9	(6,5,7,3,4,2,1) (6,3,7,5,4,2,1)	(7,4)	0.1 ÷ 0.5 0.6 0.7 ÷ 0.9	(5,3,7,6,4,2,1) (6,3,7,5,4,2,1) (5,3,7,6,4,2,1)
(8,4)	0.1 ÷ 0.5 0.6 0.7 0.8 ÷ 0.9	all matrices (11,13,15,7,9,5,3,1) (11,15,13,7,9,5,3,1) (11,15,7,13,9,5,3,1)	(8,4)	0.1 ÷ 0.5 0.6 ÷ 0.8 0.9	all matrices (7,15,11,13,9,5,3,1) (13,11,15,7,9,5,3,1)
(6,3)	0.1 ÷ 0.6 0.7 ÷ 0.8 0.9	(5,7,3,4,2,1) (3,5,7,4,2,1) (5,3,7,4,2,1)	(6,3)	0.1 ÷ 1.0	(5,7,6,4,2,1)
(5,2)	0.1 ÷ 0.3 0.4 ÷ 0.5 0.6 0.7 ÷ 0.9	(3,7,4,2,1) (6,3,4,2,1) (7,3,4,2,1) (3,5,4,2,1)	(5,2)	0.1 ÷ 0.5 0.6 0.7 0.8 ÷ 0.9	(6,5,4,2,1) (3,5,4,2,1) (3,6,4,2,1) (6,5,4,2,1)

Table 1. Optimum and worst configurations of the parity-check matrix for some Hamming codes.

As example, matrix (7) of the Hamming code (7,4) for h=0.5, gives $d^2_{min}=3$ and is one of the worst configurations,while matrix (8) gives $d^2_{min}=4$ and is one of the best configurations.

$$(7) \qquad H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$(8) \qquad H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In Fig. 2 the higher and the lower d^2_{min} obtained for different values of h are reported for the code (7,4). In this figure and in the following ones, curves b and a represents respectively the higher and the lower d^2_{min} , while curve c represents d^2_{min} for the uncoded CPFSK. The matrix configurations are reported in Table 1.

In Fig.3 the Euclidean distance for a system using a Hamming code (8,4) is reported.

Some shortened Hamming codes are also considered.

Shortened codes can be obtained by erasing some information symbols: the Hamming distance is not reduced by this operation [8] . In particular, codes (5,2) and (6,3) obtained from the Hamming code (7,4) have been analyzed. The minimum Euclidean distance is shown in Fig.4 for the code (5,2).

The evaluation of the optimum parity-check matrix for the code (15,7) requires a very high time computation. For this reason, the optimum H has been determined only for two modulation index values: h = 0.5 and h = 0.1 . The optimum and the worst matrix configurations are reported in Table 2 .

The error probability of a communication system using integrated modulation and channel coding is quite difficult to determine theoretically. Infact, the computation of the error probability requires the knowledge of the Euclidean distance between all the sequences. Moreover, a good approximation to the error probability for mean and high signal-to-noise ratios can be obtained by using the union bound [8] :

$$(9) \qquad P_b \leqslant Q\left(\sqrt{d^2_{min}\, E/N_o}\right)$$

where N_o is the noise spectral density.

In order to illustrate the gain obtained by using the integration of the modulation and channel coding operations, we present the results for a system using the Hamming code (7,4).

Fig.6 shows the error probability P_b of an IMCC system using a Hamming code (7,4) for the two configurations (7),(8). Curve a represents the error probability of a classical communication system using separate demodulation and decoding operations,curves c and b represent the upper bound on P_b,(9), for the parity-check matrix configurations (7) and (8) respectively.

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All these curves are obtained for a modulation index h = 0.5 .

As it can be seen from these results, an IMCC system offers a net improvement with respect to a classical communication system, in terms of P_b . The upper bound for the worst configuration of H is often lower than the error probability achievable with a classical communication system. Therefore, also in the worst case, a considerable gain is obtained by using an IMCC system. Moreover, the optimization of the matrix H is quite important, because it gives a further important improvement.

4. CONCLUSIONS

In this paper, the integration of channel coding and modulation operations in a communication system is described. Channel coding is used in order to increase the Euclidean distance in the signal space. It can be shown that the improvement in the Euclidean distance permits to achieve a reduction in the error probability, with respect to the case in which the two operations are performed in an independent and separate way.

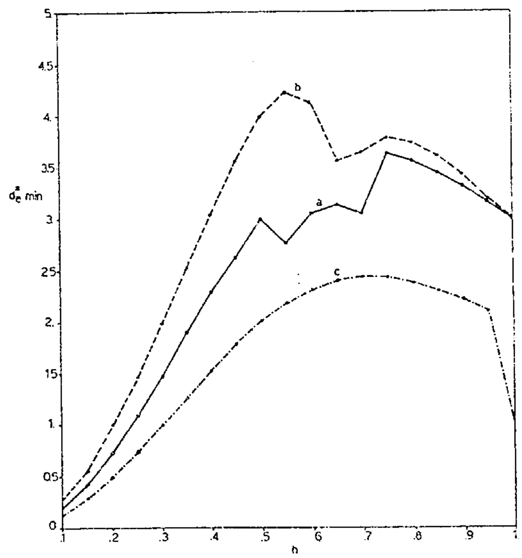

Fig.2 Minimum Euclidean distance for the Hamming codes (7,4) as function of h for best and the worst configuration of H.Binary CPFSK modulation is used.

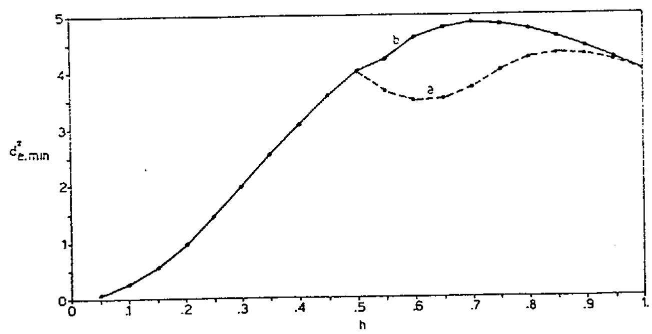

Fig.3 Minimum Euclidean distance for the Hamming code (8,4) as function of H,when binary CPFSK modulation is used.

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Fig.4 Minimum Euclidean distance for the code (5,2) as function of h, when binary CPFSK modulation is used

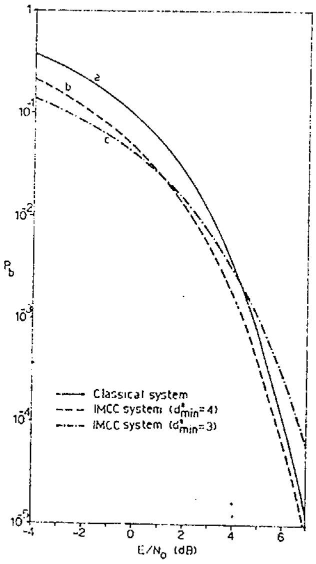

Fig.6 Error probability for a Hamming code (7,4) when an IMCC system and binary CPFSK modulation with h = 0.5 are used.

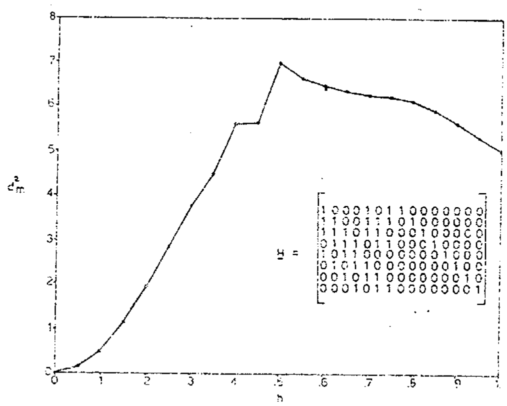

Fig.5 Minimum Euclidean distance for the BCH code (15,7) as function of h. Binary CPFSK modulation is used.

MODULATION INDEX h	OPTIMUM MATRIX	WORST MATRIX
0.5	(232,177,116,230,29,115,58,128,64,32,16,8,4,2,1)	(232,116,58,230,115,209,29,128,64,32,16,8,4,2,1)
0.1	(116,230,209,29,232,58,115,128,64,32,16,8,4,2,1)	(232,58,29,230,209,116,115,128,64,32,16,8,4,2,1)

Table 2. Optimum and worst configuration of the parity-check matrix for the BCH code (15,7).