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WAVEFORM ABSTRACT AND CONVEX FILTERS

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RESUME

Une méthode appelée "forme d'onde abstraite" laquelle peut parfaitement remplacer PCM, lorsqu'une transmission ou accumulation de signal de qualité est exigé etudie. Avant d'extraire, beaucoup d'échantillons fins sont pris afin d'assurer le lissage de la forme d'onde reconstitué à l'autre bout de chaîne. Après d'extraction, il ne reste plus qu'un vecteur à deux dimensions pour alimenter la chaîne D'où par cette méthode non seulement le bit-rate est réduit de facon massive mais le problème dè continuité de output PAM adjacent n'apparaît plus dans cette nouvelle méthode. Lorsque ce processus est utilisé comme un filtre passe-bas tous les zero-mean bruits sont enlevés complètement. Si queloues "formes d'onde" ressemble à des dents de scie ou le triangle, ce filtre va agir automatiquement comme le filtre de passe-partout afin de garder la bonne forme originale de la forme d'onde. Pour FFT, traiter ces sortes du pattern d'onde cause généralement augmmentation de la largeur de bande et davantage de délais. La seule exigence pour la forme d'onde abstraite est que le segment traîté de la forme d'onde doit être convexe. Pour l'environnement sans bruit, cette méthode est presque parfaite. Dans les environnement bruyants ceci resulte seulement le changement de niveau de la forme d'onde et le pattern d'onde est preservé. En utilisant quelques methodes conventionnelles ce mineur désavantage est remédiable. Excepté le changement de niveau quelquefois causé par le bruit de hasard, ceci theoriquement n'a pas de propagation. d'erreur. Mais FFT et d'autres méthodes ont ce problème. Enfin cette méthode est l'un et l'autre convient aux traitement de signaux digital et analogique. De plus pour le traitement de signal il est aussi utile dans le mathématique pour avoir l'approximation de quelques intégrations difficiles précisément comme l'exemple montre à la fin de cet exposé.

SUMMARY

This paper analyzes a new method called "waveform abstract", which can perfectly replace PCM when quality signal transmission and storage is required. Before the abstraction many fine samples are taken in order to guarantee the smoothness of the reconstructed waveform at the other end of the channel. After the abstraction there is only one 2-dimensional vector to feed the channel. Hence by this method not only is the bit-rate drastically reduced but the problem of discontinuity of the adjacent PAM output does not exist in this new approach. When this process is used as a lowpass filter, all the zero-mean noise is completely removed. If some waveforms have sawtooth or triangle patterns, this filter will automatically act as an all-pass filter in order to keep the original shape of the waveform. For FFT processing these kinds of wave shapes usually cause bandwidth increase and more delay. The only requirement for the waveform abstract is that the segment of a waveform being processed must be convex. For a noise-free environment, this method is almost perfect. In a noisy environment it only results in waveform-level-shift, and the wave shape is preserved. By using some conventional methods this minor disadvantage is remediable. Except for the level-shift sometimes caused by random noise, it has no error-propagation theoretically. But FFT and other methods do suffer from this problem. Finally this method is both suitable for digital and analog signal processing. In addition to the signal processing, it is also useful in mathematics to get the approximation of some difficult integrations precisely as the example shown at the end of the paper.

1. Introduction

In analog transmission systems the noise will be cumulated along the channel. The degradation caused by noise cumulation of an analog system is more rapid than that by bit error of digital one. The costs paid for improving perfomance by using digital systems are bandwidth increase and quantization noise. The maximum quantization noise is one half level step for most pulse coded systems except the gated PAM. If the bit-number in a code word is m, the maximum quantization error in terms of decibels is

$$\text{maximum quantization error} = -20(m+1)\log 2\text{db} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

For an eight-bit system the error of Eq. 1 is around -54db. In addition to the quantization error, the most serious problem is the difference between a real signal f(t) and its quantized PAM signal. This error is refered to in the literature as granular noise. For simplification let the area of this noise represent the magnitude of it. If f(t)=sint, the maximum slope is

$$\left.\frac{df(t)}{dt}\right|_{max} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Therefore the maximum error-power ratio becomes

$$\frac{\text{maximum differential power}}{\text{average power}} = \frac{2}{\Delta t}\int_0^{\Delta t} t^2 dt = \frac{2}{3}(\Delta t)^2 \quad \ldots\ldots\ldots(3)$$

If the sampling frequency is n-times faster than the signal frequency, then $\Delta t=2\pi/n$ and

$$\text{maximum error power ratio} = \frac{8}{3}(\pi/n)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The granular noise of Eq. (3) is then [14.2 - 20log n] . For n=12, higher rate than the Nyquist rate , the maximum error-power ratio will be -7.38db. Although this error can be smoothed by a high-order lowpass filter, it is obvious that the differential error is more serious than the quantization error. From the author's paper tilted "The Two Dimensional Abstracting Theorem and Its Properties"[1], the distortion of n=12 is below -120db, a very negligible noise compared to PCM systems. The purpose of this paper is to devise a method other than a sampling method which always invokes the help of an ideal lowpass filter. However, there is no such ideal lowpass filter in the world. A real filter has a transition band from the pass band to the stop band [2] . Therefore the Nyquist rate is impossible. Some unfavorable effects are also associated with high-order lowpass filters such as electromagnetic interference and ripples and spikes in the stop band, as well as in the pass band. An example of a second-order filter and its output response for a signal x(t) sampled at $f_s = 2f_0$ is shown in Fig. 1.

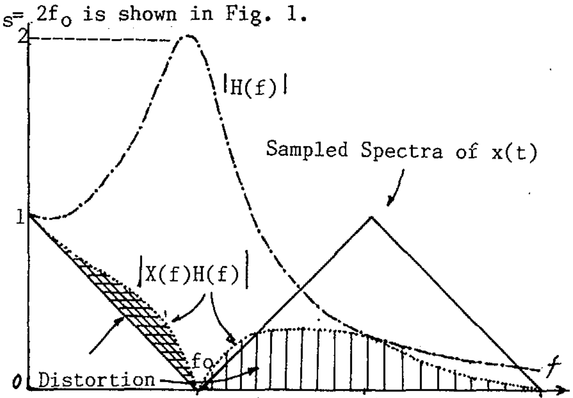

Fig.1: Spectra for signal x(t) sampled, $f_s = 2f_0$, and its total response after filtering, where the Q of the second-order lowpass filter is 2.

From Fig. 1 it is evident that an imperfect lowpass filter will grasp extra frequency bands as well as the base band of signal x(t). There a distorted portion of the base band is also clearly observed. Those higher frequency bands are the natural result of an impulse train convolved with x(t) in the sampling process. On the other hand, if a high-Q lowpass filter is replaced, there is a Q-times-high peak on the pass band. It can be concluded that the sampling process is so important in digital transmission but owing to the phenomena pointed above, the conventional digital system is on the horns of a dilemma. If a new system is compatiable with the conventional digital channel and has no such requirement of lowpass filter at the end of decoding, this will be a new event after Shannon's sampling theorem [3]. The basic difference of Shannon's sampling theorem and the author's abstracting can be state as follows:

(1) Shannon's sampling is mere a line of one instant of a time function, but the 2-D abstract is the area representation of a time function. It is equivalent to say that the 2-D abstract has an effect of infinite members of samples during a short time interval but only one sample for conventional smapling process.

(2) Shannon's sampling is a direct process and the digits in the channel represent samples, but for the abstracting, the digit stream in the channel for a specific time interval is only a vetor which carries the most important information about the waveform segment.

(3) The 2-D vector of waveform abstract can be generated either through analog means or by a fast sampler. The samples taken from the sampler are not for direct transmission but for the integration of a convex region.

(4) The decoding of the sampling process is a kind of complicate masking process but the decoding of the abstracting is a simple regenerative process which is not interfered by spurious bands.

(5) No error propogation and cumulation are in the abstracting theoretically, but both are unavoidable in the decoding of samples of PCM and PAM systems.

(6) The abstracting can be used as a tool to solve complicate mathematic problems. It is the best second-order approximation of functions among all known methods, such as Taylor's or Chebychev's. An example of new communication system implemented by the new principle is shown in Fig. 2.

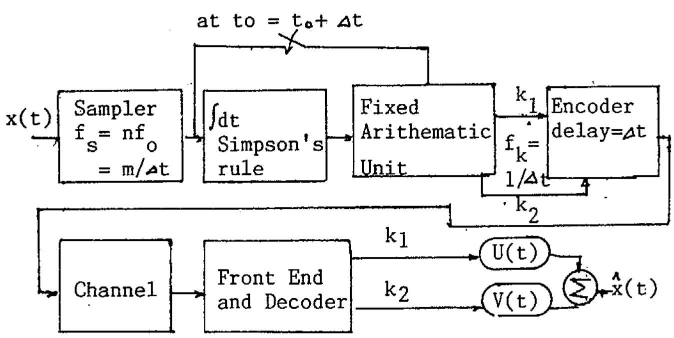

Fig.2: A New Communication System.

II. Waveform Abstract

If a waveform can be subdivided into several convex regions [4], any such region can be approximated by second-order functions precisely.

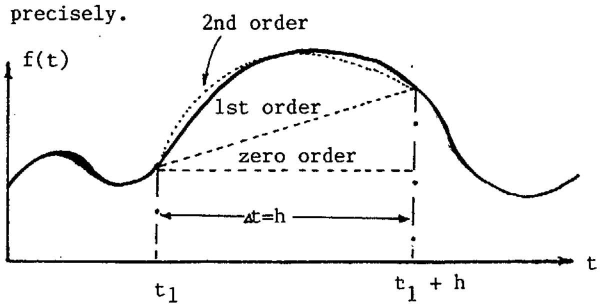

Fig. 3: The Methods of Waveform Representation

Inspecting Fig. 3 the second-order approximation is more favorable than the others. The problem is how to select the best method or better models for this second-order approximation. Usually the end points are known. There will be thousands of such parabolas met at the end points. Some famous mathematic expansions can be truncated to afford to the task for a very narrow region.

Either Taylor's or Chebychev's are not good for a wider region if they are truncated. The wider region implies lower bit rate or narrow bandwidth when the signal is transmitted. For bandwidth reduction of a and high resolution being maintained, this new method is one of the best solutions.

The general equation of such approximation is

$$f(t)= f(t) +K_1U(t) +K_2V(t) = g(u,v), \quad t1 \le t \le t1 +\Delta t \quad ......(5)$$

Where $f(t)$ is convex in the interval between t_1 and $(t_1 + \Delta t)$, and t_1 is an arbitrary point of time. This process will result in a delay of time, Δt. The $U(t)$ and $V(t)$ are periodic functions generated by the decoder of inverse abstract in the receiver. Hence,

$$U(t) = U(t+ \Delta t) \quad ....(6)$$

$$V(t) = V(t+ \Delta t) \quad ....(7)$$

As shown in Fig. 2 the function of the transmitter and channel is to send K_1 and K_2 to the decoder of a receiver. No predictors are necessary in either encoder and decoder. The decoder of this system is much simpler than DPCM, DM and RDM[5]. Since U(t) and V(t) are not specified yet, some criteria about them can be stated as follows:

(1) The periodic function of U(t) and V(t) must be physically realizable.

(2) The implementation of them must be as simple as possible.

(3) By them the reconstructed waveform has a good quality.

(4) If it is possible, U(t) and V(t) can be mutually generated by each other. This will ease the effort of synchronization and circuit design.

Since $t \cong (e^{bt}-e^{-bt})/2b$ for b is possitive and less than unity, the linear part of the waveform synthesis can also be expressed as a hyperbola. Hence U(t) and V(t) can be t, e^{-bt}, e^{bt}, t^2 and/ or their combinations. The exponential functions can be generated by diodes and RC circuits. The function t can also be generated by a single exponential function. The function t^2 is obtained by a current t passing through a capacitor C

Because

$$Vc = \frac{1}{C}\int_0^t i\ dy \qquad (8)$$

substitution of i=y and C=0.5 gives

$$Vc = t^2 \qquad (9)$$

The function t can also be generated by feeding a constant current of magnitude C in Eq.(8) . It gives

$$Vc = \int_0^t (1)\ dy = t \qquad (10)$$

In addition to the methods of generating U(t) and V(t), a microcomputer-based unconventional waveform generator, UCWG, is also available [6]. Because t and t^2 are easy to compare with other series. This paper will limit its discussion of U(t) and V(t) in terms of $(t-n\Delta t)$ and $(t-n\Delta t)^2$. Two important lemmas of the source reference will be concluded here without proof.

Lemma 1: If g(u,v) is exactly matched at the end-points of f(t) , and the area of both are the same, then there exists a closed interval $[t_1, t_1+h] = I$ such that

$$g(u,v) = f(t_1)+w_1(t-t_1)-w_2(t-t_1)^2, \quad t \in I, \qquad (11)$$

where

$$W_1 = \frac{1}{h}\left[-2f(t_1+h)-4f(t_1)+\frac{6}{h}\int_0^h f(x+t_1)dx\right], \qquad (12)$$

and

$$W_2 = \frac{6}{h^3}\int_0^h f(x+t_1)\ dx - \frac{3}{h^2}f(t_1) + (-1)\frac{3}{h^2}f(t_1+h) \qquad (13)$$

The dismatched portion of g(u,v) and f(t) is somewhat exagerated in Fig. 3. Though the negligible difference of g(u,v) and f(t) is exagerated in Fig 3, a good match can still be observed. If the variance of g(u,v) and f(t) is concerned, the following lemma is important.

Lemma 2: If g(u,v) is exactly matched at the end-points of f(t), at least one solution can make the difference square of g(u,v) and f(t) minimum. That is required g(u,v),

$$g(u,v) = f(t_1)+K_1(t-t_1)-K_2(t-t_1)^2, \quad t_1 \le t \le t+h \qquad (14)$$

such that

$$K_1 = \frac{30}{h^4}\int_0^h (hx-x^2)\ f(x+t_1)dx - \frac{1}{h}[1.5f(t_1+h)+3.5f(t_1)] \qquad (15)$$

and

$$K_2 = \frac{30}{h^5}\int_0^h (hx-x^2)f(x+t_1)dx - \frac{2.5}{h^2}[f(t_1)+f(t_1+h)] \qquad (16)$$

III. A Method of Determining the Fitness of the Reconstructed Waveform

Any segment of a reconstructive waveform can be represented by a Fourier series whether it is periodic or not. If

$$f(t)= a_o+ r_o+(a_1+r_1) \sin(wt+ \theta)+ (a_2 +r_2) \sin (2wt+2\theta)+ (a_3 +r_3)\sin(3wt+3\theta) +\ldots\ldots \qquad (17)$$

and the reconstructed waveform is

$$g(t)= a_o+a_1\sin(wt+ \theta)+a_2\sin (2w+2 \theta) a_3\sin(3wt+3 \theta)+\ldots \qquad (18)$$

The difference of f(t) and g(t) in the form of Fourier series will be a indicator to determine the fitness. For example, a sine wave can be represented by 12 connected pieces of reconstructed waveforms. By using Lemma 1 or Lemma 2, all the harmonies of the reconstructed wveform are below 10^{-9} order.

VI. The Convex Filter

In addition to the discussion above ,one of the important application of waveform abstract is to form a "convex filter". A convex filter cannot be classified into any category of conventional filters. Basically it has the properties of an ideal lowpass filter but it does not always present as a lowpass filter. For a convex waveform interfered by a zero-mean noise, if it is passing through abstracting process, the noise will be completely removed as an ideal lowpass filter in the following three cases.

Case I: The end-points are not interfered. Since the abstracting is a linear operation, the noise can have its own weighting factors $w_1(n)$ and $w_2(n)$ separately. Becuse the end-points are not interfered by the noise, therefore

$$n(t_1)= n(t_2)=0 \qquad (19)$$

where t_1 and t_2 are the end-points, and

$$\int_{t_1}^{t_2} n(t)dt=0 \qquad (20)$$

Then

$$w_1(n)= \frac{1}{h}[-2n(t_2)-4n(t_1)+\frac{6}{h}\int_{t_1}^{t_2} n(t)dt] =0 \qquad (21)$$

and

$$w_2(n)=\frac{6}{h^3}\int_{t_1}^{t_2} n(t)dt-\frac{3}{h^2} [n(t_1)+n(t_2)] =0 \qquad (22)$$

Case II: The noise has the form of

$$\sum_{m=1}^{\infty} \sin [mw(t-t_1)] .$$

for simplicity, let $t_1=0$, $t_2=h$, and $wh=2\pi$, then

$$W_1(n)=\sum_{m=1}^{\infty} \frac{1}{h}[-2\sin(2m\pi)-4\sin(0)] + \frac{6}{h^2}[\cos(2m\pi)-1] =0 \qquad (23)$$

and

$$W_2(n)=0 \qquad (24)$$

Case III: The signal has a form of $\cos(wt+ \theta)$.

Since it is a zero-mean noise, then

$$\cos(wt_1+\theta)= \cos(wt_2 +\theta) \qquad (25)$$

and

$$\int_{t_1}^{t_2}\cos(wt+\theta)dt=0 \qquad (26)$$

This means that two end-points are both shifted by a magnitude of $\cos(wt_1+\theta)$. It is only a DC component and the angle θ is random and unifomly distributed. So this level shift also has a zero-mean. This level shift can easily be blocked by a capacitor. From case II the convex filter acts as a band-stop filter or a comb filter. If the input waveform has a sawtooth shape, this waveform will be reconstructed of the same shape with a time delay h. It is known that from Fourier analysis, this waveform has very rich harmonics. At this moment the lowpass characteristics of the convex filter disappears and it plays the role of an all-pass filter and delay equalizer.

V. Properties and Applications

(1) Linear property

If

$$f(t)= g(t)+h(t) \qquad (27)$$

then

$$K_{1f} = K_{1g}+K_{1h} , \qquad (28)$$

$$K_{2f}= K_{2g}+K_{2h} , \qquad (29)$$

$$W_{1f}= W_{1g}+W_{1h} , \qquad (30)$$

and

$$W_{2f}= W_{2g}+W_{2h} \qquad (31)$$

(2) As an adaptive filter

When it is used as a filter it owns the following properties.

(a) An ideal lowpass filter,
(b) Band-reject filter,
(c) Comb filter,
(d) Delay line and,
(e) All-pass filter.

(3) Noise to DC conversion

The noise of frequencies of (n/h) is completely removed or turned to be DC.

(4) Applications

(a) For integrations

Any complicated function can be replaced by a precise second-order function by using the process of abstracting. For example, if

$$f(x)=\frac{4}{\pi}\int_0^1 (1-x^2)^{\frac{1}{2}} \cos x dx \qquad (32)$$

by lemma 2 the term $(1-x^2)^{\frac{1}{2}}$ can be simplified into the form of $(1+kx-x^2-kx^2)$. After some manipulation f(x) is readily obtained as

$$f(x)= 10(\frac{6}{\pi}-1.5)(\sin 1-\cos 1)+10(\frac{2.6}{\pi}-0.75)(\cos 1 - 1)=0.8776163... \qquad (33)$$

This approximation is more precise than that of Legendre-Gauss Quadrature when m=3. The answer of LGQ is 0.8856216 and the right answer ia about 0.880101714 [7].

(b) Quality signal storage and transmission

It is a new method of quality signal storage and transmission with less bit storage as well as less bit rate.

Conclusion:

The code of waveform abstract is an index of partial wveform, but the code of PCM only represents the lines in the waveform. Some research directions about this paper in the future are concluded here.

(1) A fast and accurate integrator.
(2) The non-uniform abstracting is more effective than the uniform one. The 3D abstraing can be used to pave the road.

For further bit reduction the Recursive Delta Modulation, RDM is more suitable to the others. The most important philosophy of the paper is that the regenerative process is the best way to eleminate noise or distortion.

Refereces

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