COHERENT AND INCOHERENT MULTIPLE BEAMFORMING

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RESUME

L'idée fondamentale à la base d'un beamformer multiple est l'estimation simultanée de la direction, de l'amplitude ou de la puissance d'un ensemble de sources. A cet effet il faut considérer dans l'algorithme l'interaction totale de ces sources différentes. La méthode d'estimation du maximum de vraisemblance, qui garantit un rendement optimal, est une approche particulièrement appropriée à une telle technique. Nous espérons atteindre la séparation de sources densement rapprochées, sans perdre le robuste comportement du beamformer conventionnel. Afin de construire un beamformer multiple, nous allons utiliser dans ce travail deux modèles différents, qui sont basés sur un traitement cohérent et incohérent des échantillons temporels.

SUMMARY

The basic idea behind a multiple beamformer is to estimate simultaneously the direction, amplitude and/or power of a set of sources. For this purpose the complete interaction of the different sources must be considered within the algorithm. A suitable approach to such a technique is connected to the maximum-likelihood parameter estimation method, which ensures optimal performance. What we expect to gain is to resolve closely-spaced sources without losing the robust behaviour of the conventional beamformer. In this paper two different models will be used to construct a multiple beamformer. These are based on coherent and incoherent treatment of the time samples.

INTRODUCTION

Beamforming in general can be understood as a procedure for combining spatially distributed measurements either in a predefined or in an adaptive way to increase the sensitivity of the receiving system in the directions of interest.

The usual assumption in beamforming is that the signal arrives from a single source. In this paper two methods are described that allow us to form multiple beams in order to steer the receiver simultaneously to more than one sound source. For this it is necessary to include the complete interaction of the different sources in the beamformer [1, 2].

What we expect to gain is to resolve closely-spaced sources without losing too much of the robust behaviour of the conventional beamformer.

It is known from statistical theory that the Cramer-Rao lower bound is a good measure for the robustness of the signal processor. If there is a method that achieves this Cramer-Rao lower bound, then it is the maximum-likelihood parameter estimation technique.

The paper is divided into three parts:

First, the background behind the multiple beamformer is specified.

Second, two different models suitable for the multiple beamformer are presented. These are based on coherent and incoherent treatment of the time samples.

Third, the performance of these two processors with respect to detection and resolution capabilities is analyzed. This analysis is based on simulated data.

BASIC ASSUMPTIONS

The assumptions on which the validity of this analysis is based, must be set out first:

1. The received sound field consists only of a small number of point sources. Spatially coloured ambient noise is excluded to simplify the analysis.

2. The point sources are assumed to be in the extreme far field, so that they can be treated as stationary.

3. The data are prefiltered with a narrowband filterbank and can be represented by complex values.

4. The measurement errors and the signal amplitudes are approximately gaussian.

OUTLINE OF THE PROCEDURE

The general outline of the procedure for the multiple beamformer can be sketched as follows:

- Estimation of the number of sources
  e.g. using the Eigenvalue test of Bienvenu and Kopp [3].

- Coarse bearing estimation with a high-resolution technique
  e.g. Pisarenko's method, extended Prony technique [1, 4].

- Multiple beamformer containing fine bearing and amplitude/power estimation.

- Test of success
  e.g. conventional beamforming on the "signal free data" [1].

The element of interest in this procedure is the fine bearing and amplitude/power estimation defining the multiple beamformer.

Corresponding to the assumptions made, the narrowband signals are modelled as gaussian variables. In this paper two asymptotic cases are treated:

- The sources are deterministic; that is, they are gaussian variables with non-zero mean and zero variance.

- The sources are purely random; that is, they are gaussian variables with zero mean and non-zero variance.

Two different algorithms have been tailored to these cases to allow easy understanding of the performance one can expect from the multiple beamformer concept.

- Coherent mode:
  The algorithm is sensitive to the deterministic part of the signals and combines the data coherently in time.

- Incoherent mode:
  The algorithm senses the power rather than the amplitude of signals, since they are random in time.

These restrictions, however, are not dominant, as it is possible to construct mixed and more versatile models to cover all situations of interest.

PRESENTATION OF THE ALGORITHMS

In the next section the two algorithms are presented.

COHERENT MODE

Let the measurements be given by the narrowband hydrophone amplitudes

$$y_{n,t} \tag{1}$$

where n is the hydrophone index
t is the time index.

Let the signal model be defined by

$$z_{n,t} = \psi^*_{n,t}\, a \tag{2}$$

where a is the source amplitude vector
$\psi_{n,t}$ is the sound field measurement transfer vector
n,t as above.

With the assumption of gaussian measurement errors the log likelihood is given by

$$L = \text{const} - 2 \log \sigma - \sum_{n,t} \left| \frac{y_{n,t} - z_{n,t}}{\sigma} \right|^2 \tag{3}$$

The multiple beamformer is then described by the maximum-likelihood solution

$$\begin{aligned} a &= \phi^{-1} b \\ \sigma^2 &= \sum_{n,t} \left| y_{n,t} \right|^2 - b^* \phi^{-1} b \end{aligned} \tag{4}$$

where $b = \sum_{n,t} y_{n,t} \psi_{n,t}$

$\phi = \sum_{n,t} \psi_{n,t} \psi^*_{n,t}$

INCOHERENT MODE

Let the measurement in this mode be given by the time-averaged spatial cross correlation matrix

$$R = \langle y_t\, y_t^* \rangle_T \tag{5}$$

where $\langle\ \rangle_T$ indicates the time average over T samples
y_t is the hydrophone measurement vector at time t.

Let the signal model be defined by

$$Q = \sum_k A_k S_k S_k^* \tag{6}$$

where k is the source index
A_k is the source power
S_k is the source bearing vector.

The log likelihood is now defined by

$$L = \text{const} - 2 \log \sigma - \sum_{n,m} \left| \frac{R_{n,m} - Q_{n,m}}{\sigma} \right|^2 \tag{7}$$

where n,m correspond to hydrophone indices.

The multiple beamformer is found again through the maximum-likelikood solution:

$$\begin{aligned} A &= G^{-1} P \\ \sigma^2 &= \sum_{n,m} \left| R_{n,m} \right|^2 - P^T G^{-1} P \,, \end{aligned} \tag{8}$$

where $P_k = S_k^* R S_k$

$G_{kj} = \left| S_k^* S_j \right|^2$

k,j are source indices.

To complete the multiple beamformer a fine-bearing estimator has to be included in the algorithm. By inspection of the log likelihood we can see that, having estimated amplitude or power, the residual log likelihood is given by

$$L_r = \text{const} - 2 \log \sigma \ , \qquad (9)$$

which depends only on the unknown bearing values of the different sources. The optimal fine-bearing estimate can therefore be found by maximizing this residual log likelihood. Unfortunately, there is no simple way of obtaining the solution of the corresponding maximum likelihood equation since it is highly nonlinear. However, any appropriate iterative technique could be used to find the desired solution. Here we have used an extended Newton procedure [1]. Consequently, in most cases, where the starting point is within the convergence area, a stable solution is found after two or three iteration steps.

In concluding the presentation of the multiple beamformer it is interesting to note that in the single source case all two algorithms are close to the conventional beamformer:

- coherent mode = Fourier transform of the data
- incoherent mode = Wiener approach to the conventional beamformer

RESULTS

Since the multiple beamformer can be understood as an extension of the conventional beamformer, this paper presents a direct comparison between

- conventional beamformer
- multiple beamformer coherent mode
- multiple beamformer incoherent mode.

To simplify the analysis only simulations fitting the model assumptions are reported. Deterministic signals are simulated for the coherent mode and zero-mean random signals are simulated for the incoherent mode and for the conventional beamformer. We therefore concentrate on the limiting performance of the multiple-beamformer concept. The simulations were carried out for 32 hydrophones and 40 time samples.

SINGLE-SOURCE PERFORMANCE

First the single-source case is treated, where we expect that the performance of multiple and conventional beamformers are equivalent.

AMPLITUDE/POWER ESTIMATION

- Coherent mode:
  The estimation error is found to be independent of the signal-to-noise ratio and close to the Cramer-Rao lower bound [1, 2]. Due to the enormous processing gain given by

  gain = the number of hydrophones multiplied by the number of independent time samples,

  the estimation error can be very small.

- Incoherent mode:
  The standard deviation of the estimation error is found to be proportional to the signal power. A more detailed analysis shows that the imperfect estimation of the cross correlation matrix has to be associated with this performance limit.

- Conventional beamformer:
  The estimation error is equivalent to that of the incoherent mode.

BEARING ESTIMATION

The results of the comparison between the different techniques concerning the bearing estimation have been compiled in Fig 1. This figure plots the standard deviation of the bearing estimation for a single source against the input signal-to-noise ratio. The standard deviation is given in dB relative to the classical resolution limit given by half the mainlobe width.

- Coherent mode:
  As to be expected, the standard deviation of the bearing estimation error is inversely proportional to the signal-to-noise ratio and close to the Cramer-Rao lower bound [1, 2].

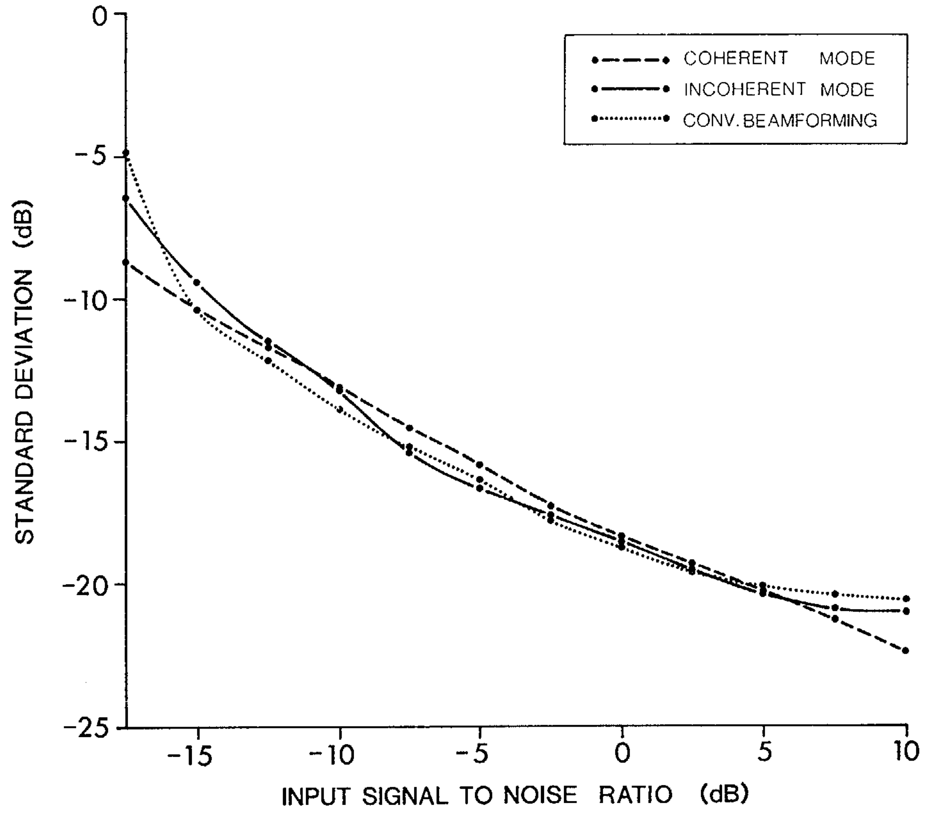

FIG. 1 PERFORMANCE OF BEARING ESTIMATION (single source)

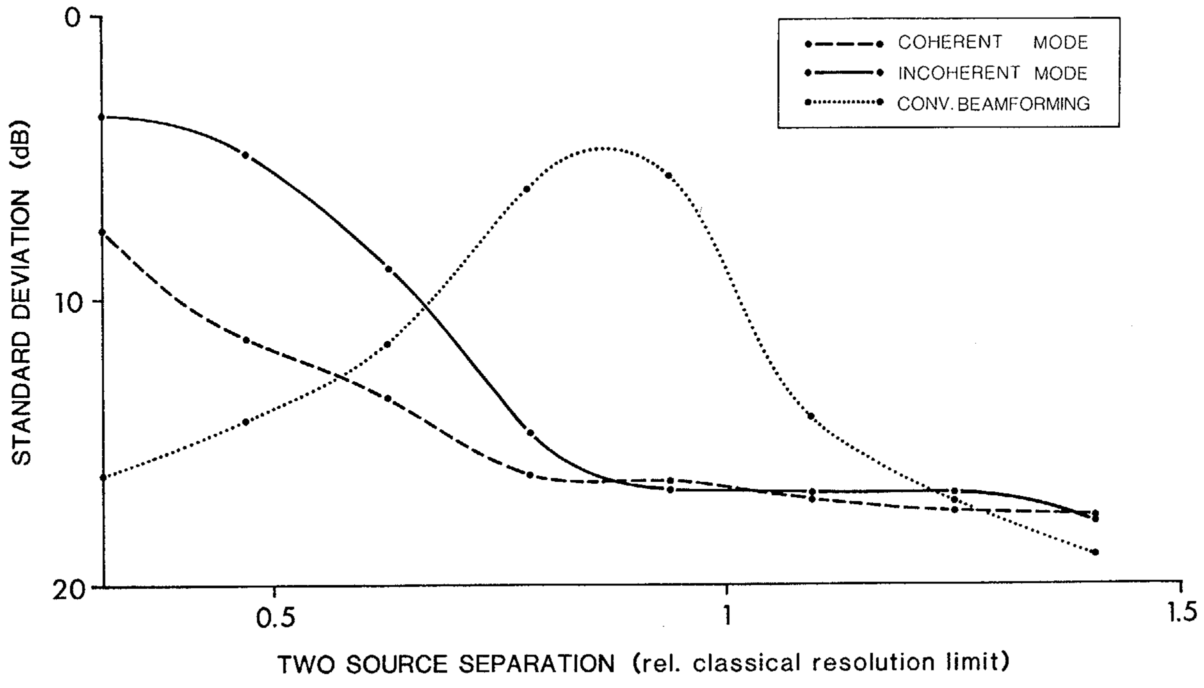

FIG. 2 PERFORMANCE OF BEARING ESTIMATION (two sources)

- Incoherent mode:
  For signal-to-noise ratios between 0 and -15 dB the estimation error is close to that of the coherent mode. However, for very low signal-to-noise ratios the incoherent mode performes worse than the coherent mode of the multiple beamformer. At very high signal-to-noise ratio the standard deviation is limited by the resolution of a 1024 point FFT used within the algorithm.

- Conventional beamformer:
  For signal-to-noise ratios greater than -15 dB the standard deviation of the conventional beamformer is close to the coherent mode of the multiple beamformer. For very low signal-to-noise ratios the conventional beamformer must obviously fail, while the multiple beamformer in its coherent mode continues to perform optimally down to -30 dB [1, 2]. At very high signal-to-noise ratios the standard deviation is limited again by the use of a 1024 point FFT.

TWO-SOURCE RESOLUTION PERFORMANCE

We now look at the two-source resolution case. Here two equal strength sources are simulated at different spacing. The amplitude or the power, respectively, have been assumed to be at 0 dB signal-to-noise ratio.

RESOLUTION LIMIT

Resolution of the two sources can be observed above the following limits, measured in units of half the main lobe width:

- coherent mode 1/3
- incoherent mode 2/3
- conventional 1

These values correspond to a detection probability of 50%.

BEARING ESTIMATION

Figure 2 shows the standard deviation of the bearing estimation for two sources at different separation. The separation unit as well as the reference value for the dB scale of the standard deviation is chosen to be the classical resolution limit (half the mainlobe width). The advantages of the multiple beamformer with respect to the conventional method can be seen clearly also for this criterion. Since at low separations of the sources, say below 0.75, the conventional beamformer has no resolution capability, the decreasing standard deviation cannot be associated with the resolution performance.

CONCLUSIONS

It has been shown that under the given assumptions, the multiple beamfomer concept is capable of improving resolution performance with respect to the conventional beamformer. In particular, it can be observed that the increased resolution is not achieved at the cost of decreased robustness of the estimation procedure. However, careful modelling of the reality is necessary and increased computational effort must be accepted to obtain optimal results.

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