ITERATIVE DECONVOLUTION ALGORITHMS FOR HIGH RESOLUTION SPECTRAL ANALYSIS AND BEAMFORMING

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RESUME

Des méthodes de déconvolution itératives peuvent transformer un spectre de Fourier en spectres à haute résolution analogues à ceux obtenus par des méthodes haute résolution du type MEM (maximum d'entropie). Une telle méthode (WB2) avait été présentée au GRETSI il y a deux ans. La présente communication décrit deux nouvelles variantes de la méthode. La première (WB3) est une version plus performante de WB2 qui utilise le critère de maximum d'entropie pour résoudre les ambiguités de la déconvolution. La seconde méthode, appelée SBE (smooth background estimator), utilise un critère mixte de maximum d'entropie et de minimum d'entropie croisée avec une partie du spectre supposée connue a-priori, mais au lieu de requérir un spectre de référence, elle calcule en même temps que le spectre total le spectre bruit de fond seul, supposé lisse, qui sert ensuite de référence pour le critère de minimum d'entropie croisée. Les descriptions des deux méthodes sont suivies d'exemples d'application à l'analyse spectrale et à la formation de voies.

SUMMARY

Iterative deconvolution techniques can transform a Fourier spectrum into spectra similar to the ones obtained through MEM (maximum entropy method) or other high resolution methods. One such technique (WB2) was presented in the last GRETSI conference. This paper presents two new iterative techniques. The first (WB3) is an improved version of WB2, and as the latter, uses the maximum entropy criterion to select one solution in the infinite set of possible spectra. The second method can make use of a-priori knowledge of components of the spectrum, by minimizing the cross-entropy between the spectrum and the a-priori reference, but instead of requiring an a-priori spectrum like the Minimum Cross Entropy method, it estimates, together with the spectrum, a background noise spectrum which is used as the a-priori reference for the cross-entropy minimization, this last method is called SBE (smooth background estimator). Descriptions of the two techniques are followed by examples of applications to spectral analysis and beamforming.

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INTRODUCTION

The Ambient Noise Group at SACLANT ASW Research Centre is involved in measuring the directionality of underwater ambient noise at low frequencies (100 to 1500 Hz). Below 500 Hz, shipping usually dominates the noise which presents significant horizontal and vertical anisotropy.

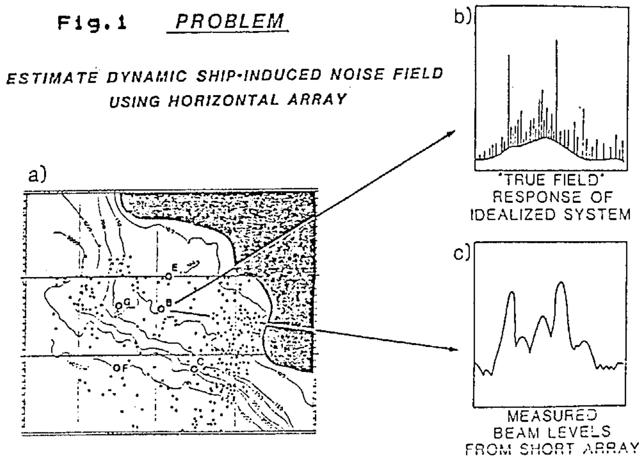
This directionality is measured with linear arrays, horizontal and vertical, of finite dimensions. The large width of the conventional beams severely limits directionality measurements at low frequencies, and high-resolution techniques must be used to overcome the limitations of the short apertures. Unfortunately, most high resolution techniques (Maximum Entropy Method, Maximum Likelihood Method, Eigenvalue Decomposition, etc..) were designed to solve another problem: they perform well in detection and localization of sources in the noise field, but they do not estimate very well the ambient noise field itself in the directions where its level is low, specially in presence of loud sources in other directions. Their performance degrades also very fast if one or more of the high level sources move during the observation. In ambient noise measurements, the levels of the quiet sectors are more important than the locations of the high-level sources.

The ANG group has, over the last few years, developped a family of non-linear iterative deconvolution techniques, to estimate the ambient noise directionality from the conventional beam levels: one such technique, WB2 (Wide Band estimator) was presented at the last GRETSI <1>, and has also been described in other papers <2,3>. The present paper describes two other techniques of this family, the first of which being an improved version of WB2, called WB3. The second algorithm overcomes a limitation of WB2 and WB3: the maximum entropy constrain causes the estimate to oscillate in the vicinity of the loud sources, in a manner not unlike the side-lobes of a conventional unshaded beamformer. These oscillations are reduced by smoothing the background noise around (and excluding) the high level sources, and the technique is therefore named SBE, for Smooth Background Estimator.

THE PROBLEM AND WB2

Figure 1 illustrates the rationale for the WB2 and WB3 techniques. Fig. 1a shows the positions on a map of 280 ships, as observed from an aerial surveillance during a noise measurement exercice: many of these can be expected to contribute to the noise at position B, where the measurement was taking place. A perfect, infinite array would perceive a "true" field as depicted in Fig. 1b. The real array, being of limited aperture, shows a blurred image of the field as in Fig. 1c. The regeneration of the "true" field from the measured beam levels is an ill-defined inverse problem and extra constrains, or criteria must be applied in order to select one unique solution amongst the infinity of possible ones.

Fig.1 PROBLEM



The reconstruction method is shown in Figure 2. Fig. 2a and 2b are identical to Fig. 1b and 1c. The measured conventional beam levels, in decibels, are labelled M (M is a vector, of dimension N, if N is the number of conventional beams from the conventional beamformer).

The field estimate F (Fig. 2c) is selected so that it is compatible with the measured values, and has maximum entropy (in other words, of all the "compatible" fields, the selected one is the smoothest, the one that does not have a structure to which the conventional measurement would be "blind").

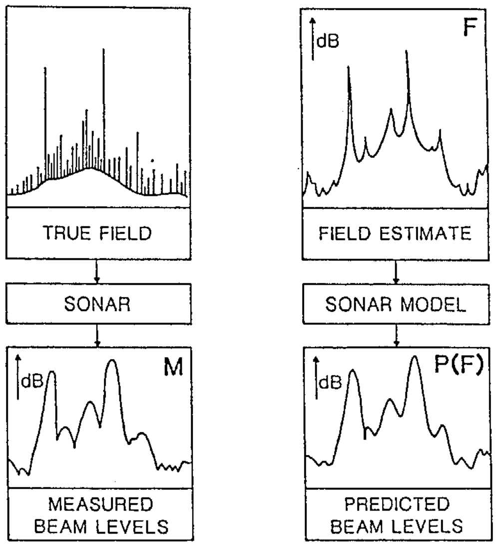


Fig.2

OPTIMALITY CRITERIA for F

(1) MAXIMUM ENTROPY

$\sum_i F_i$ MAXIMUM UNDER THE CONSTRAIN OF:

(2) DECIBEL LEAST MEAN SQUARE ERROR (dBLMS)

$\sum_n (M_n - P_n(F))^2$ MINIMUM

The acoustic field is modelled as a collection of I independant plane waves, equidistant in wavenumber. The I plane waves are considered statistically locally independant: the output of one conventional beam is calculated by summing the energy contributions of all plane waves seen by the beam, without interference between the sources contributing to the same beam.

The sonar response, P(F) (represented in Fig. 2d), to this field estimate is calculated or "predicted" by convolving F with measured or computed beam patterns of the array. The field F is said to be compatible with the measured sonar output M if the distance M-P(F) is small or minimum. This family of techniques uses a "distance" expressed in decibels, whereas most other high-resolution techniques are using a power least-squares approach, through the use of hydrophone complex cross-spectral matrixes, which are a linear power measurement. This "decibel match" concept forms the strength of the techniques, and the readers will find in references <1,2> explanations for this unusual error quantifier.

To summarize the criteria, the field estimate F should be a collection of I plane waves having maximum entropy, i.e.

$$\sum_{I} F_i \quad \text{maximum} \qquad \text{(exp. 1)}$$

where F_i is the level of the field in direction i, subject to the constrain of minimum mean-squares error in decibels, i.e.

$$\sum_{N} (M_n - P_n(F))^2 \quad \text{minimum} \qquad \text{(exp. 2)}$$

in which M_n is the measured level of beam n and $P_n(F)$ is defined as

$$P_n(F) = 10 \text{ Log } \left(\sum_{I} 10^{\frac{F_i + R_{n,i}}{10}} \right) \qquad \text{(equ. 3)}$$

where $R_{n,i}$ is the beam pattern of beam n, defined as the response (in dB) of beam n to a plane wave of level 0 dB in direction i.

The apparently complex Eq. 3 is just a power sum of the plane wave components of F, weighted by the beam pattern, as seen by beam n. The set of constraints of exp.1 and 2 is not simple to solve exactly, as Exp.2 is a non-linear function of F. We do not know at present of any analytical method to find the exact solution, but we will be satisfied with an approximate solution provided that it has a small mean-squares error and an entropy close to maximum.

WB2 and WB3 are two iterative techniques which yield approximate solutions with small mean-squares error and large entropy. They do so by starting with a field F_o having a large entropy and their iteration steps are designed to reduce the mean-squares error $|M-P(F)|^2$, without decreasing the entropy too much (i.e. without introducing arbitrary structure in the field estimate). The iteration stops when the residual mean squares error is small enough, or stops decreasing, or after a preset number of iterations.

Before explaining the details of the algorithms, the example in Fig. 3, obtained from WB2, is used to clarify what is meant by small residual mean-squares error and large entropy. The true field is shown in dashed lines; it consists of an incoherent noise field (horizontal dashed line), plus four plane waves (vertical dashed lines) with signal-to-noise ratios of -10, 0, +10 and +30 dB respectively (the signal-to-noise ratio is defined for one hydrophone). The hann-weighted 40-elements array, used with a 64 points FFT beamformer, yields 64 measured beam values (the points labelled FFT). The WB2 estimate is represented by a continuous line. The residual error vector M-P(F) is plotted as crosses along the 0 dB line. This residue would be null for the exact solution to expressions 1 and 2, as the conventional beam level measurements are free from any estimation error: in this theoretical example the residual error for the WB2 solution is not null, but its value is very small indeed. The mean-squares error is about 0.25 dB, and the largest error is slightly above 2 dB.

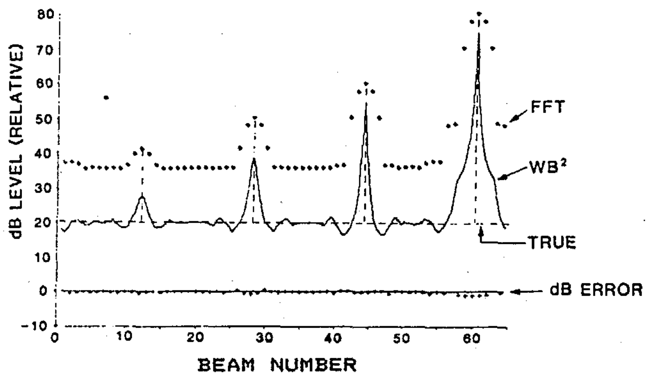


Fig.3 WB2 results on a simulation

The "lack of entropy" of the solution is more difficult to estimate. One could be tempted to say that, as the background is flat and the conventional beamformer output does not oscillate, the maximum entropy estimate should also be flat, and that the oscillations of the WB2 estimate around the high level sources represent a lack of entropy. This is not true, as the maximum entropy constraint is not trying to give a solution with a smooth background, but rather a solution which has to be as smooth as possible, including the sources. That overall smoothness constraint causes the field to oscillate around the high level sources (these oscillations also exist with Burg's MEM technique, when a large number of poles is estimated: the MEM users limit the number of poles to avoid undesirable oscillations, thus reducing the resolution and the accuracy of their estimate in the quiet sectors). The sharp low amplitude wiggles are errors due to the WB2 implementation, which uses integer arithmetics, as well as to the many approximations in the algorithm, but their amplitudes are reasonably small.

WB3: ALGORITHM AND RESULTS

The WB2 algorithm suffers from two defects: it can start with a first estimate having a false structure (deep narrow "holes") if the conventional beam power level measurements have not been integrated long enough and show fast, large amplitude variations in level from beam to beam , as shown on Figure 4. The first estimate of WB2 for this set of data strictly follows these variations, assuming that the beam-to-beam oscillations in the measured vector M reflect a "structure" of the field, when in fact they are merely measurement errors, due to lack of integration or sidelobes of the conventional beams.

ITERATIVE DECONVOLUTION ALGORITHMS FOR HIGH RESOLUTION SPECTRAL ANALYSIS AND BEAMFORMING

The other possible source of inaccuracy in WB2 is the "rough" way in which the correction to the field is derived from the beam level errors.

WB3 is an attempt to cure these two problems: it uses the same flowchart as WB2 <1,2>, but the first estimate is different, smoothing the possible spurious oscillations of the conventional beam level measurements, and in the same time narrowing the peaks corresponding to the high level sources. In short, the first estimate used by WB3 has more entropy and is closer to the final solution than the first estimate of WB2.

The first estimate is obtained as:

$$F_{o,i} = \underset{N}{\text{minimum}}(M_n - R_{n,i} - 10\text{Log}(BW_n)) \qquad \text{(equ. 4)}$$

The obtention of this first estimate is illustrated on Fig. 4, the crosses represent the measured conventional beam levels, the dotted line is the first estimate for WB2. the continuous line is the first estimate for WB3. The fast oscillations in beam levels are filtered out better by the WB3 first estimate, and one can also clearly see the sharpening of the high level sources. The data shown on this figure correspond to a real measurement, not a simulation.

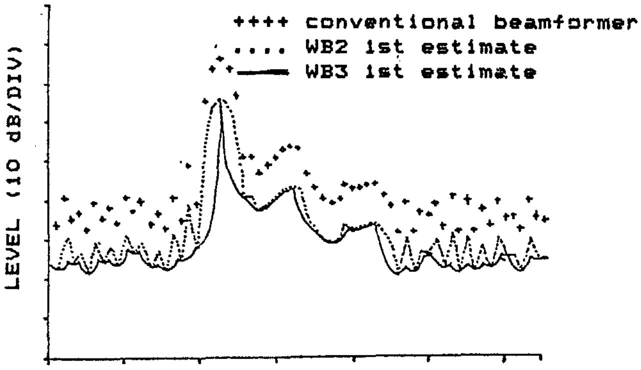


Fig. 4 WB3 first estimate is smoother than WB2 first estimate

In the iteration loop, the corrections are smoother, they are obtained in a way that could be described as a modified gradient search to minimize the mean-squares error, the modifications of the gradient search have the purpose of "keeping the entropy high". The formula for the iteration is:

$$F_{k+1,i} = F_{k,i} + \frac{\sum_N (M_n - P_n(F_k)) \times (IMP_{n,i})^c}{\sum_N (IMP_{n,i})^c} \times W_i \qquad \text{(equ. 5)}$$

with:

$$IMP_{n,i} = 10^{\frac{F_{k,i}+R_{n,i}-P_n(F_k)}{10}} \qquad \text{(equ. 6)}$$

and:

$$W_i = \frac{\text{minimum}(TIMP_i, AVIMP_i)}{AVIMP_i} \qquad \text{(equ. 7)}$$

$IMP_{n,i}$ is proportional to the derivative of the prediction of the beam level P_n relative to the field element $F_{k,i}$, and measures the dependance of beam n to the field element F_i: if $IMP_{n,i}$ is small, then a change of F_i will not affect P_n, and the error (M_n-P_n) should not contribute to the corrections to F_i. If $IMP_{n,i}$ is big then the correction to F_i will have a strong effect on P_n and F_i should be corrected in order to reduce the error (M_n-P_n) most effectively.

$TIMP_i$ is the total importance of F_i, defined as:

$$TIMP_i = \sum_N IMP_{n,i} \qquad \text{(equ. 8)}$$

$AVIMP_i$ is defined as $TIMP_i$ for a flat field (constant F_i).

The exponent c in Equation 5 can vary from 2 to 6, values smaller than 2 slow down the convergence of the iteration, c=2 provides less oscillations in the background level, c=6 gives slightly sharper resolution on high level sources, with more oscillations in the background. For all the examples shown in this report, c=2, as for ambient noise measurements it is more important to have less oscillations in the background noise level.

Figure 5 shows one result from WB3 on a test field similar to the one in Figure 3, but where the single sources have been replaced by "pairs", to show the high-resolution capabilities of WB3. The residual oscillation in the estimate of the noise pedestal are still present, although lower in amplitude than on the WB2 estimate (Fig. 8 of <1>).

These residual oscillations of the pedestal estimate could be annoying by biasing the background noise estimation for ambient noise measurements, and also in hiding low level sources that could lay in these areas, with the same consequences than sidelobes have for conventional beamformer. But their reduction will be obtained only by modifying the maximum entropy constrain, for example by forcing only part of the spectrum to be smooth, (i.e. to have maximum entropy), after exclusion of the detected high level sources.

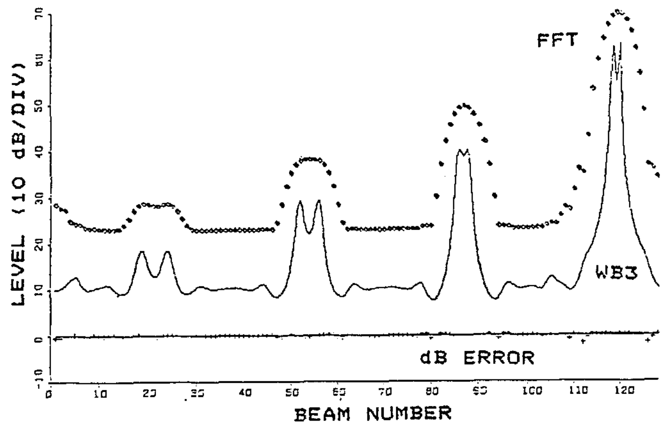


Fig. 5 WB3 estimate on a simulation: 4 pairs on white noise

ITERATIVE DECONVOLUTION ALGORITHMS FOR HIGH RESOLUTION SPECTRAL ANALYSIS AND BEAMFORMING

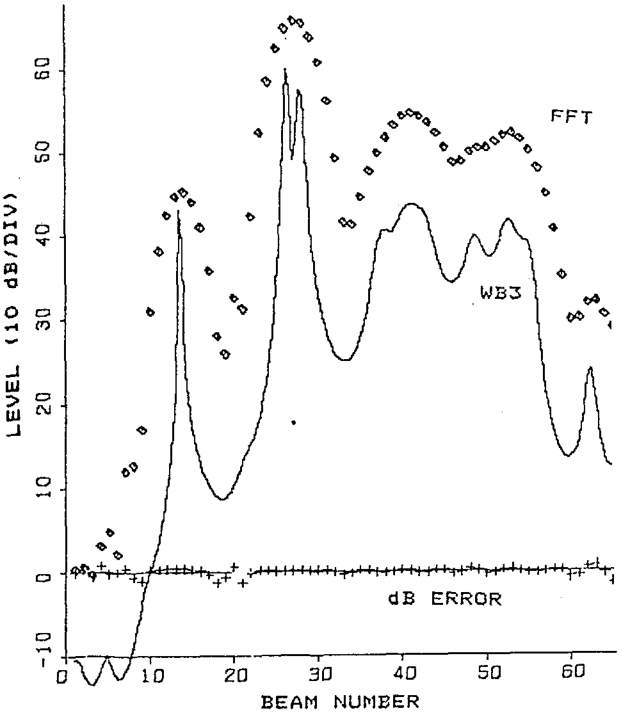


Fig. 6 WB3 estimate on Kay and Marple data

Figure 6 shows the WB3 estimate for the Kay and Marple data set <4>, to be compared to Figure 13 of <1>. The WB3 estimate for the smooth part of the spectrum (to the right) is smoother than the WB2 estimate.

At this point, the authors felt that a substantial improvement of these deconvolution techniques could be obtained only through modification of the set of constrains to be solved: the background, or pedestal, has no physical reason to oscillate because of a nearby high-level source, and, if the estimate does oscillate, these oscillations are an error. The constrains were modified to include that reasoning, and the modified technique was called SBE as it attempts to smooth the estimate for the background in the vicinity of the high-level sources.

SBE: ALGORITHM AND RESULTS

The Smooth Background Estimator algorithm considers the ambient noise field as a sum of two different components: the contribution from all the detected high-level sources, and the background field (including the lower level sources). This arbitrary discrimination is justified by the fact that in the WB3 estimate, only the high-level sources create "side-lobes" problems.

The flowchart for SBE is similar to the one of WB3, and so is the first estimate: but for each step of the iteration, SBE estimates two fields: one is the ambient noise field, the other is a smooth background noise field. This last is obtained through nonlinear filtering of the total field minus the contributions from all the detected high level sources. The correction to the field is then computed from the errors as in WB3, but when $TIMP_i$ is small, an extra correction is applied to force the new field towards the smooth background.

In practice, the smooth background is obtained in the following steps:

First, the high-level sources are detected, according to their importance, by examination of TIMP: wherever a peak in TIMP is higher than 3 x AVIMP, there is a high-level source. The angular extend of the source is calculated, and the energy from the source is removed by clipping the field in the sector of the source, to its values on the edges of this sector. A clipped TIMP function is obtained in the same way.

The clipped field is then filtered by convolution with a "constant importance filter": for each cell i of the field, the background value is a mean value of the clipped field weighted by the clipped TIMP: the mean is calculated over a variable width area, so that the sum of the weights TIMP under the area is kept constant. This "constant importance filter" insures that in the sectors where the importance TIMP is high enough, the structure of the background is preserved, while in the sectors where TIMP is small (i.e. where the measurement carry little information about the field), the field is smoothed as much as possible.

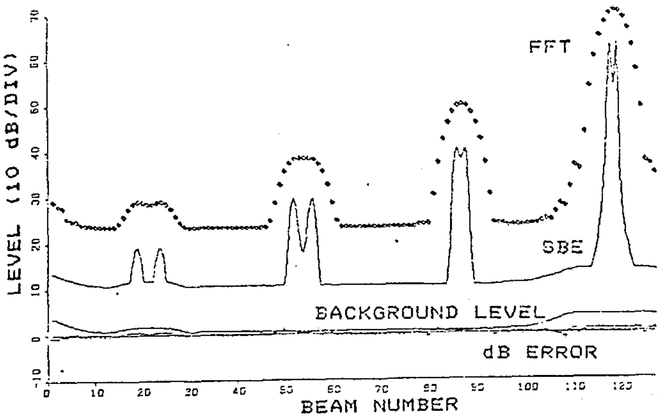


Fig. 7 SBE estimate on a simulation: 4 pairs on white noise

Figure 7 shows the SBE estimate corresponding to the example in figure 5. The background field is represented with an offset of -10 dB and is visible as the lower curve. The estimate for the total field does not oscillate any more like the one in figure 5, and the pairs of sources are also resolved better.

Figure 8 represents the estimate of SBE for the KAY and MARPLE data <4>. The lower curve represents again the smooth component of the spectrum, with a -10 dB offset. The higher curve, represents the SBE estimate to the total spectrum, and shows a substantial improvement over the WB3 estimate: The resolution of the three line components is better, and the image of the coloured noise component is smoother.

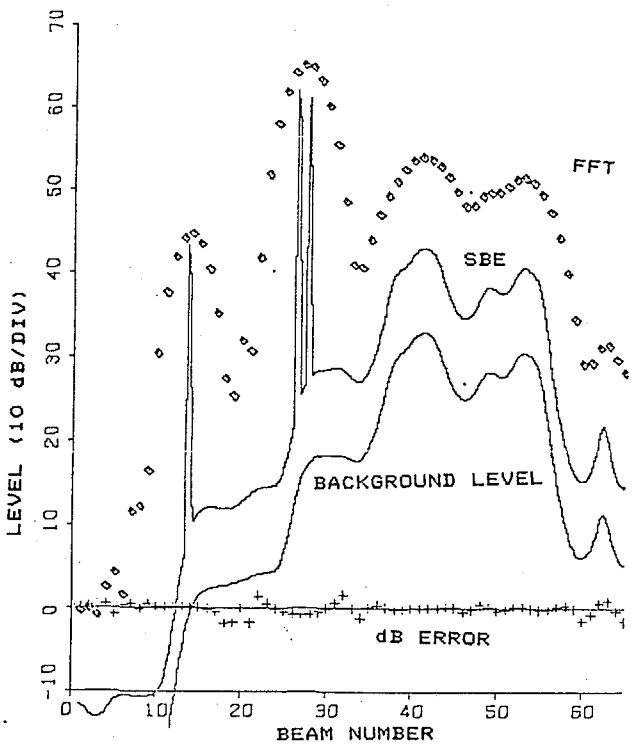


Fig. 8 SBE estimate on Kay and Marple data

COST OF THE ALGORITHMS

The cost of computing the WB3 or SBE estimates is difficult to compare with that of the other high-resolution methods, because of the radically different operations involved. WB3 and SBE require more elementary operations, and a larger storage than the other techniques for small arrays. The operations are vectorized, and most of them are standart in libraries of software for array processors. For large arrays, these techniques become very attractive, as their demand in storage grows only proportional to the array size, the calculus growing like the product NxI, if N is the number of conventional beams (i.e. twice the number of array channels), and I the definition (or number of points) required for the WB3 or SBE output.
Most other techniques have storage growing as N^2, and calculus as N^3. For problems with more than one dimension, WB3 and SBE can also be used, without any theoretical difficulty: the multidimensional field is estimated on a finer grid, from multidimentional FFT, or any other conventional imaging technique on a coarse grid. The techniques could, for example, be applied to problems like deblurring optical pictures, if the impulse response due to the blurring agent is known.

To give a rough idea of the complexity involved, WB3 or SBE would compute a 512 points field estimate from a set of 128 conventional beams in about 20 (WB3) or 30 (SBE) seconds (15 iterations), on a minicomputer hewlett-packard 1000 F (compiled FORTRAN, no use of Vector Instruction Set).

WB3 has also been coded in interpreted BASIC on a COMMODORE C64 home computer, and it took about 2 hours to calculate the estimate for the Kay and Marple data as shown on Fig 6 (20 minutes for the 16 FFT, and about 20 minutes per iteration loop). The BASIC code for the algorithm is only 45 lines long. The program needs a work space of less than 30 Kilobytes.

For real-time applications, it would also be possible to speed-up the calculus of SBE, as the background noise could be estimated for each new acquisition only, instead of for every iteration loop as is done now.

CONCLUSIONS

Two new deconvolution techniques have been presented, which show substantial improvement upon the original technique presented at the last GRETSI <1>. The WB3 and SBE techniques can easily be implemented on small systems, and provide resolution capabilities similar to the ones obtainable through more complex algorithms using cross-correlation or cross-spectral matrixes. They can improve the resolution of any imaging system, without requiring any particular geometry for the imaging aperture (beams and arrays can have any shape or sampling, as long as the conventional beam shapes can be calculated or simulated). They can easily be modified to include a-priori information available to the user, or to solve more complex problems, like the ambiguity resolution problem presented in another paper at this conference <5>.

REFERENCES

<1>BERROU J.L, BOULON P. Une nouvelle methode haute-resolution rapide et robuste, 9eme GRETSI, mai 1983, pp 1015-1021

<2>BERROU J.L, WAGSTAFF R.A. IEEE ASSP WORKSHOP II on Spectral Analysis, TAMPA (FA) nov 1983 pp139-145

<3>WAGSTAFF R.A, BERROU J.L. A fast and simple nonlinear technique for beamforming and spectral analysis, JASA 75(4) april 1984 pp1133-1141

<4>KAY S.M. and MARPLE S.L.Jr Spectrum analysis - a modern perspective. Proceeding of the IEEE,69, 1981 pp 1380-1419.

<5>BERROU J.L, WAGSTAFF R.A. Une méthode itérative pour mesurer l'anisotropie horizontale du bruit ambiant sous-marin. 10eme GRETSI 1985