DIXIEME COLLOQUE SUR LE TRAITEMENT DU
SIGNAL ET SES APPLICATIONS

NICE du 20 au 24 MAI 1985

WAVE PROPAGATION IN FLUID–FILLED PIPES

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RESUME

Le contrôle des ondes vibro-acoustiques dans les tuyaux remplis de fluide demande la connaissance des lois de propagations de ces ondes. Bien qu'une importante littérature existe à ce sujet, les méthodes d'intercorrélation ne sont pas efficaces pour la détermination des vitesses de propagations et des caractéristiques d'amplitude, à cause de la dispersion des ondes.

L'article montre qu'une simple analyse dans le domaine temporel ou fréquenciel ne convient pas à la détermination à la fois de la dispersion et des caractéristiques d'amplitudes. En revanche, une analyse dans le domaine spatio-temporel permet d'obtenir en même temps la relation de dispersion $k(\omega)$ ainsi que l'amplitude des ondes se propageant dans les 2 sens dans un guide d'onde.

En utilisant une telle technique de spectre de nombre d'ondes, on montre qu'il est possible de mesurer les coefficients de réflexion et et transmission d'une discontinuité dans un guide d'onde.

Des résultats expérimentaux ont été obtenus dans le cas des modes de flexion dans une barre sur laquelle était fixée une masse M de moment d'inertie I, qui seront comparés avec les calculs théoriques. De plus, on a pu mesurer les courbes de dispersion associées aux "modes" de flexions circonférentiels dans un tuyau vide, puis chargé par un fluide lourd.

SUMMARY

Control of undesirable transmission of noise and vibration along fluid filled pipes demands knowledge of the wave propagation characteristics. Becauses such waves are dispersive and wave reflection may occur at discontinuities, cross correlation method of experimental determination of wave propagation characteristics, and of reflection and transmission coefficients of discontinuities, is not effective.

The paper shows that a simple time or frequency analysis is not suitable for the determination of both dispersion and amplitude information.
On the other hand, it will be demonstrated that wave field analysis, involving frequency-wavenumber decomposition, is able to produce both the dispersion relation $k(\omega)$ and the amplitude of waves travelling in both senses in any type of waveguide.

Using a such a technique of wavenumber spectral analysis, it will be shown that it is possible to measure the complex reflection and transmission amplitude coefficients of a discontinuity in a waveguide.

With this technique, experimental results have been obtained for flexural waves in a rectangular cross-section beam, which is fitted with a mass-inertia as a discontinuity, and comparison with theoritical results will be presented. In addition, flexural waves in a piping system have been investigated, and by using a similar technique, it has been possible to measure experimentally the dispersion relations of various flexural modes of different circumferential orders of a piping system when in-vacuo, and also when filled with fluid.

1. INTRODUCTION

Pipes form essential components of many industrial, aerospace and marine vehicles systems, conveying mass energy and momentum from one part of a system to another.
In so doing, they frequently transport unwanted energy in the form of fluid and structural vibration between connected regions. In seeking to minimise vibration transmission a designer needs to understand the nature of the waves carried by a pipe, and the fluid which it contains, so that he can optimise the design of control elements incorporated into the system.
Within the audio frequency range, a number of dispersive form of waves, many involving cross-sectional deformation of a pipe may propagate [1].
In conjunction with theoretical analyses, a programm is in progress to develop experimental techniques to evaluate the propagation characteristics of high frequency dispersive waves in fluid filled pipes, and also the reflection and transmission behaviour of various forms of discontinuity in a pipeline, for which waves amplitude detection is necessary.
This paper presents some comments on the application of cross-correlation technique to the case of dispersive waves, and shows that simple time serie analysis of such a problem is not satisfactory.
Then, an alternative approach to dispersive propagation is proposed via a space and time analysis which leads to obtain both the dispersion relation and the magnitude information accurately.
The results reported herein were obtained by measurement on a simple uniform beam system carring bending wave. The beam cross-section dimension were 6mm x 50mm and it was 6.27 m in length. Then the technique was applied to a pipe the diameter of which was 160 mm and the thickness 5 mm and it was 6.50 m in length.
Results are presented for the "in-vaccuo" case and when the pipe is filled with hydraulic oil.

2. CORRELATION TECHNIQUE

The cross-correlation technique is widely used for propagation path identification purposes [2].
Although it is a highly successfull method in non dispersive propagation, a great deal of literature has already been published on its application to dispersive propagation, with apparently little practical success. Applying such a technique in the case of flexural waves in long thin bars, WINTER and BIES [3] used as input a wide band random signal and realised that dispersion considerably complicates the behaviour of the correlation function.
In a similar case, P.H. WHITE [4] assumed that a band limited random signal was the key to obtaining meaningful correlation in dispersive waves, but he reached the conclusion that in a multiple reflection process little information could be obtained by correlation, and that it is often impossible to distinguish between dispersive and non-dispersive propagation.
More recently, AOSHIMA and IGARASHI [5] applied a cross-correlation technique to the case of flexural waves travelling in a simple rectangular cross-section beam using a modulated 1/3 octave band noise. After squaring and low pass filtering the input (excitation) and the response signals, the cross-correlation peaks gave the group delay at the mid-band frequency.
However, despite the fact that they managed to determine the group velocity at certain frequencies and for short propagation paths, the correlation peaks were increasingly corrupted by the dispersion as the propagation paths became longer, as it is the case in multiple reflection at the end of a beam, or a structural discontinuity, and eventually, no magnitude information could be obtained.

BARGER [6] applied a broad band input to a fluid loaded structure but, rather than an accurate measurement tool, he only succeded in using cross-correlation as a diagnostic technique to determine by which path the wave traveled, either via the water or via flexural vibration in the structure.
The author of the present paper used cross-correlation in the case of a beam excited by a carrier wave modulated by a M-sequence.
Then, after heterodyning, the output signal was low-pass filtered and the resulting signal was cross-correlated with the M-sequence. The correlation represented on Fig. 1 shows two peaks corresponding respectively to the group delay of the incident wave and to the wave reflected at the free end of the beam, and for which the expected peak should be stape identical in magnitude and width to the first peak.

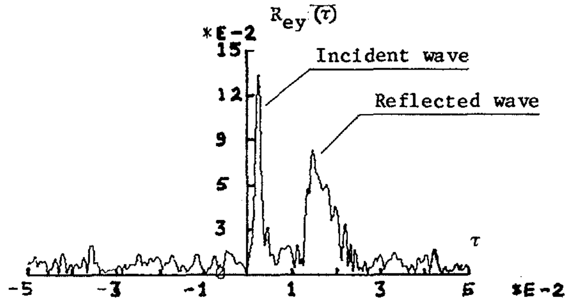

Figure 1 : Cross-correlation applied to flexural waves in a beam

Further investigations on cross-correlation technique show that the effect of the dispersion function enters in the expression of the correlation peaks which get more and more corrupted with propagation distance, get lower and wider and eventually vanish so that they cannot be relied upon when reasonable accuracy is required.

3. WAVE FIELD ANALYSIS

3.1. Two dimentional Fourier transform

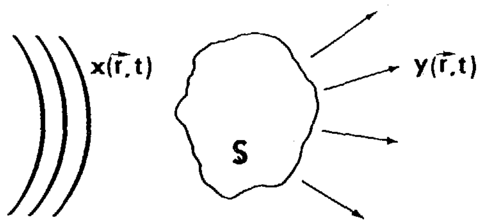

Figure 2 : Continuous response $y(\vec{r}, t)$ of S to a continuous excitation field $x(\vec{r}, t)$

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A system submitted to a continuous excitation field $x(\vec{r}, t)$ where $\vec{r}$ denotes the spatial position at which the excitation is applied (Fig. 2), responds with the response wave field $y(\vec{r}, t)$ such as excitation and response are related to each other by the double convolution.

$$y(\vec{r},t) = h(\vec{r},t) ** x(\vec{r},t) \qquad (1)$$

which takes the implicit form :

$$y(\vec{r},t) = \iint_{-\infty}^{+\infty} h(\vec{r}-\vec{r_1}, t-t_1) \cdot x(\vec{r_1}, t_1)\, d\vec{r_1}\, dt_1$$

Furthermore, by performing a two dimentional Fourier transform of a wave field $f(\vec{r}, t)$ as :

$$\mathcal{F}(\vec{k}, \omega) = \iint_{-\infty}^{+\infty} f(\vec{r},t)\, e^{-j(\omega t - \vec{k}\vec{r})}\, dt\, d\vec{r} \qquad (2)$$

the double convolution (1) becomes :

$$\mathcal{Y}(\vec{k}, \omega) = \mathcal{H}(\vec{k}, \omega) \cdot \mathcal{X}(\vec{k}, \omega) \qquad (3)$$

3.2. Wavenumber spectrum

Let us consider the transfer function $H(x, \omega)$ at a point x of a waveguide in which an incident and returning set of waves propagate along the x direction at frequency ω (Fig. 3).

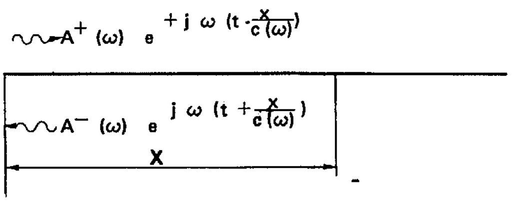


Figure 3 : Waveguide without discontinuity

If the waveguide is considered as responding to a white noise signal, $H(x, \overline{\omega})$ can be expressed as :

$$H(x,\omega) = \tilde{A}^+(\omega)\ e^{-j\omega\frac{x}{c(\omega)}} + \tilde{A}^-(\omega)\ e^{j\omega\frac{x}{c(\omega)}} \qquad (4)$$

The wavenumber transform of $H(X, \bar{\omega})$ consists of performing the Fourier integral :

$$\mathcal{H}(k,\omega) = \int_{-\infty}^{+\infty} H(X,\omega)\ e^{+jkX}\ dX \qquad (5)$$

Then applying (5) to (4) gives :

$$\mathcal{H}(k,\omega) = \tilde{A}^+(\omega)\ \delta\left(k - \frac{\omega}{c(\omega)}\right) + \tilde{A}^-(\omega)\ \delta\left(k + \frac{\omega}{c(\omega)}\right) \qquad (6)$$

The dispersion relation is therefore obtained by the location of the peaks of the Dirac Function, the complex magnitude of which are respectively A^+ for the positive travelling wave and A^- for the returning wave.

3.3. Determination of reflection and transmission coefficients

The wave guide considered on Fig. 4 is supposed being fitted with a discontinuity D associated with the reflection and transmission coefficients $\tilde{R}(\omega)$ and $\tilde{T}(\omega)$, and excited at one single point at end A.

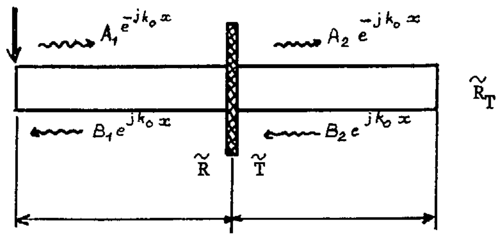


Figure 4 : Waveguide fitted with a discontinuity

At the frequency ω_0 (which corresponds to the wavenumber value k_0) the transfer function in both regions of the waveguide are :

$$H_1(r, \omega_0) = \tilde{A}_1\ e^{-jk_0 r} + \tilde{B}\ e^{jk_0 r}$$

$$H_2(r, \omega_0) = \tilde{A}_2\ e^{-jk_0 r} + \tilde{B}_2\ e^{jk_0 r}$$

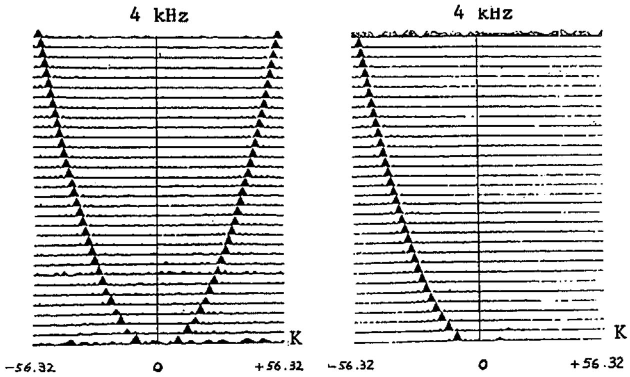
where $\tilde{A}_1$, $\tilde{A}_2$, $\tilde{B}_1$ and $\tilde{B}_2$ denote the complex amplitudes (normalised to the input spectrum component at ω_0). The analytic expression of the reflection and transmission coefficients have been deduced from the complex amplitudes [7].

$$\tilde{R} = \frac{\tilde{A}_1\tilde{B}_1 - \tilde{A}_2\tilde{B}_2}{\tilde{A}_1^2\ e^{-2jk_0 l_1} - \tilde{B}_2^2} \quad \text{and} \quad \tilde{T} = \frac{A_1 A_2\ e^{-jkl_1} - B_1 B_2\ e^{jk_0 l_1}}{\tilde{A}_1^2\ e^{-2jk_0 l_1} - \tilde{B}_2^2}$$

4. EXPERIMENTAL RESULTS

4.1. Flexural waves in a beam

Data were acquired by using a swept sine wave as excitation. The upper frequency was 4 kHz and results were obtained without an anechoic end (Fig. 5[a]) and with an anechoic end (Figure 5 [b]).



[a] Reflective end [b] Anechoic end

Figure 5 : Wavenumber spectra of flexural wave in a rectangular beam

Using the same conditions, a measurement was performed on the beam on which had been fitted a mass and moment of inertia (given in non-dimentional form, M = .36, J = 1.483.10^{-3}).
Both ends were anechoic and measurements were executed on both sides of the discontinuity. Figure 6 show superimposed and theoretical and the experimental curves of the reflection and transmission characteristics of such a discontinuity.

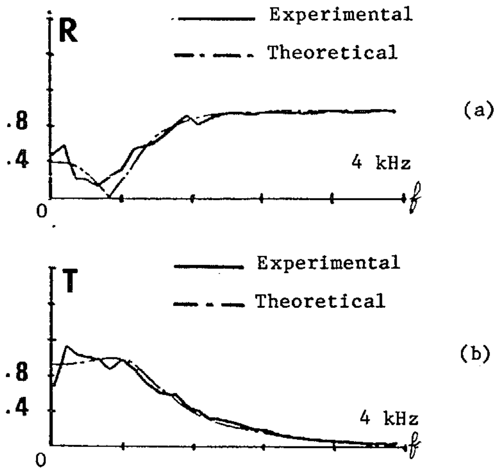


Figure 6 : Reflection coefficient (a) and transmission coefficient (b) of a mass-inertia fitted on a rectangular cross-section steel beam

The wavenumber transformation has been used very systematically on a dispersive system for which it has been possible to measure both the dispersion and the reflection/transmission characteristics.

4.2. **Flexural waves in a "in vaccuo" pipe**

When a pipe is submitted to flexural vibration, the deformation of its cross-section can be expressed in terms of an expansion into Fourier-serie, i.e. decomposed into circonferential modes (Fig. 7).
This representation has the advantage that each circonferential mode of order n corresponds to a wave-type which propagates according to its own dispersion relation [8].
Measurements were conducted on a 5 mm thick steal pipe, the diameter of which was 160 mm. A rig has been develop in order of exciting circunferential flexural modes individually by multiple excitation using an appropriate combination of vibrators.
Figure 8 shows a comparison between measured dispersion curves and theoretical curves calculated using the characteristic equations of motion of cylindrical shells [8] i the case of an empty pipe, for n = 2 and n = 3, and force frequency range up to 10 kHz.

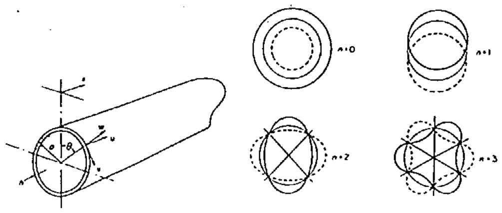


Figure 7 : Modal shapes

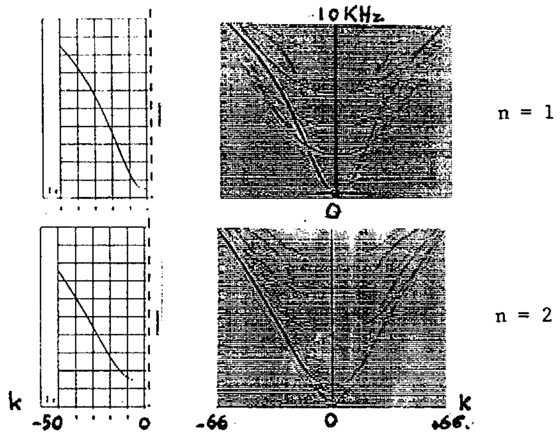


Figure 8 : Comparaison of theoretical dispersion curves of in-vaccuo pipe with k-spectrum

Figure 9 shows a similar comparison obtained in the case of a fluid filled pipe loaded with hydraulic oil. In this case, the behaviour of the shell is more complicated because of the acoustic modes in the fluid and at high frequencies, the n = 0 oder mode becomes clearly visible.

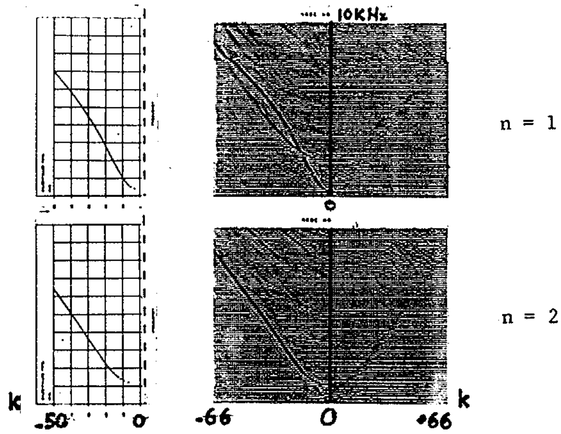


Figure 9 : Comparaison of theoretical dispersion curve of fluid filled pipe with k-spectrum

Conclusion

The interpretation of data given by wavenumber spectrum (k-spectrum) may probably involve a rather complicated patern recognition device in order to discriminate individual branches of circonferential flexural modes, and at this point, the obtention of reflection and transmission characteristics of a discontinuity fitted on a pipe has not been fully permitted.
However, it has been demonstrated the improvement of a space time analysis over simple time (frequency) analysis.

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AKNOWLEDGMENTS

The author would like to thank Dr F.J. FAHY for many stimulations and helpful discution, and the British Ministry of Defence for supporting this work.