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NORMAL MODE PROPAGATION AND HIGH RESOLUTION METHODS

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RESUME

C'est plutot common, l'application du modélisation de propagation d'onde plane pour les sonar passif. Pour un matrice interspectrale, un vecteur-source normé dependera seulment sur la direction. De l'autre côté un chenal stratifie (l'ocean et ses délimitation) produit pluseur (M) modes. Le résultat est un vecteur-source avec M+1 paramètres. Ici un develope un méthode pour traiter ce modélisation. Un sous-produit de ce méthode peut etre un mesure de la qualité du modélisation.

SUMMARY

It is quite common in passive sonar systems to apply a plane wavefront propagation model. With respect to the spectral cross correlation matrix (SCCM), a normalized point source vector will depend only on bearing (and elevation in a more general case). On the other hand a layered channel such as the ocean and its boundaries gives rise to several (M) modes. This leads to a source vector with M+1 parameters. A method is developed to handle this model. A by-product of the method might be a measure for model quality.

INTRODUCTION

The consequence of a plane wavefront propagation model is that the contribution from a point source to the SCCM has maximum three parameters: the power at the array, the bearing, and the elevation. The normalized source vector will depend only on bearing and elevation. A filter vector matched to this model is usually called the steering vector.

In the real stratified ocean, this model is generally not correct. It might be expedient for modestly sized arrays, but as soon as ambition for higher resolution leads to both larger arrays and high resolution methods, one might be heading for problems. Improved propagation models have shown that the layered propagation channel formed by the ocean and its boundaries gives rise to several modes, all with different horizontal wavenumbers <1>.

Using this model, the new normalized source vector is a linear combination of M vectors, one for each mode. The individual mode vectors depend only on bearing and local environmental conditions, and are therefore assumed to be known (except for bearing). On the other hand, the combining weights depend on the total channel from source to receiver (including their positions), and are assumed to be unknown parameters.

A generalization of a method by Bienvenu and Kopp <2> is developed to handle this model. Our results of simulations so far indicate no improvement in bearing resolution relative to a plane wavefront model. But, fundamentally, discrete sources can be discriminated only when they have different source vectors. For the plane wavefront model, the only discrimination parameters are bearing and elevation. In the normal mode case, because of the richness of parameters, the discriminating capacity is highly improved. Even sources at the same bearing are discriminated as long as they have sufficiently different source vectors.

The N eigenvectors of the SCCM will span an N-dimensional space. This space is divided into two spaces, one containing the source vectors, and one orthogonal to the source vectors. Finally, the sum of the outer products of the source vectors should equal the SCCM. If we are unable to find source vectors satisfying all these conditions, it might be an indication that the sound propagation model should be modified.

In addition to <2>, this paper is based on ideas expressed by Mermoz in <3>, <4>, and <5>. Some background for this paper is also in <6>.

MODELS AND ASSUMPTIONS

Our passive sonar receiver system is a completely general array with arbitrary geometry consisting of N sensors (hydrophones). Furthermore the data are quadrature bandpass filtered, so that a delay τ can be substituted by a phase factor $\exp(-j2\pi f\tau)$.

A point source in the far field combined with the normal mode propagation model, will produce the following data vector on the array:

$$X = s \sum_{m=1}^{M} a_m D_m(\beta) = s \underset{\sim}{D}(\beta) A \qquad (1)$$

where s is a zero mean random variable, and a_m is the complex amplitude of the m'th mode, normalized such that the mode amplitude vector A has unit length:

$$A = \left[a_1 \; \dots \; a_M\right]^T \qquad (2)$$

$$A^* A = 1 \qquad (3)$$

D_m is the m'th mode vector <1>:

$$D_m = \begin{bmatrix} u_m(z_1)\, e^{-jk_m[x_1\cos(\beta)+y_1\sin(\beta)]} \\ \vdots \\ u_m(z_N)\, e^{-jk_m[x_N\cos(\beta)+y_N\sin(\beta)]} \end{bmatrix} \qquad (4)$$

and $\underset{\sim}{D}$ is the mode matrix:

$$\underset{\sim}{D} = \left[D_1 \; \dots \; D_M\right] \qquad (5)$$

For the m'th mode, k_m is the horizontal wavenumber, and $u_m(z)$ is the mode amplitude (eigen)function. x_i, y_i, and z_i are the coordinates of the i'th sensor and β is the bearing of the point source relative to positive x-axis.

In a spatially coherent situation, the expected SCCM will be:

$$\underset{\sim}{R} = E\{X X^*\} = \sigma^2 \underset{\sim}{D} A A^* \underset{\sim}{D}^* = F F^* \qquad (6)$$

where σ is the standard deviation of s, and F is:

$$F = \sigma \underset{\sim}{D} A = \sigma \sum_{m=1}^{M} a_m D_m \tag{7}$$

The parameter β is dropped for convenience.

Finally, let there be T point sources contributing to the SCCM:

$$\underset{\sim}{R} = \sum_{i=1}^{T} \sigma_i^2 \underset{\sim}{D}_i A_i A_i^* \underset{\sim}{D}_i^* = \sum_{i=1}^{T} F_i F_i^* = \underset{\sim}{F} \underset{\sim}{F}^* \tag{8}$$

where we have introduced the source matrix $\underset{\sim}{F}$:

$$\underset{\sim}{F} = [F_1 \ldots F_T] \tag{9}$$

Our task is to estimate the unknown parameters of an estimated SCCM, assuming T<N and M≤N-T. Before doing that, we will develop some mathematical properties of our model SCCM.

MATHEMATICAL PROPERTIES

The matrix $\underset{\sim}{R}$ has orthonormal eigenvectors and real nonnegative eigenvalues: E_n, λ_n, $n \in \{1 \ldots N\}$. The eigenvalues are indexed in decreasing order.

Assuming T<N, it follows that:

$$\lambda_n = 0 \qquad n \in \{T+1 \ldots N\} \tag{10}$$

As a consequence:

$$\underset{\sim}{R} = \underset{\sim}{E}_{\parallel} \underset{\sim}{\Lambda}_{\parallel} \underset{\sim}{E}_{\parallel}^* \tag{11}$$

where we have introduced:

$$\underset{\sim}{E}_{\parallel} = [E_1 \ldots E_T] \tag{12}$$

$$\underset{\sim}{\Lambda}_{\parallel} = \mathrm{diag}(\lambda_1 \ldots \lambda_T) \tag{13}$$

$\underset{\sim}{E}_{\parallel}$ spans a T-dimensional subspace $S_{\parallel}$. The source vectors F_i have to be linear combinations of these eigenvectors:

$$F_i = \sum_{n=1}^{T} z_{n,i} \lambda_n^{1/2} E_n = \underset{\sim}{E}_{\parallel} \underset{\sim}{\Lambda}_{\parallel}^{1/2} Z_i \tag{14}$$

and, even more compactly:

$$\underset{\sim}{F} = \underset{\sim}{E}_{\parallel} \underset{\sim}{\Lambda}_{\parallel}^{1/2} \underset{\sim}{Z} \tag{15}$$

where we have introduced:

$$Z_i = [z_{1,i} \ldots z_{T,i}]^T \tag{16}$$

$$\underset{\sim}{Z} = [Z_1 \ldots Z_T] \tag{17}$$

Inserting (15) into (8) yields:

$$\underset{\sim}{R} = \underset{\sim}{E}_{\parallel} \underset{\sim}{\Lambda}_{\parallel}^{1/2} \underset{\sim}{Z} \underset{\sim}{Z}^* \underset{\sim}{\Lambda}_{\parallel}^{1/2} \underset{\sim}{E}_{\parallel}^* = \underset{\sim}{E}_{\parallel} \underset{\sim}{\Lambda}_{\parallel} \underset{\sim}{E}_{\parallel}^* \tag{18}$$

Premultiplying by $\underset{\sim}{\Lambda}_{\parallel}^{-1/2} \underset{\sim}{E}_{\parallel}^*$ and postmultiplying by the conjugate transpose yield:

$$\underset{\sim}{Z} \underset{\sim}{Z}^* = \underset{\sim}{I} \tag{19}$$

As the T source vectors defining F are independent, Z is nonsingular and has an inverse. From (19):

$$\underset{\sim}{Z}^{-1} = \underset{\sim}{Z}^* \tag{20}$$

$\underset{\sim}{Z}$ is seen to be an unitary matrix.

The remaining eigenvectors will define a subspace $S_{\perp}$ orthogonal to the source vectors F_i:

$$\underset{\sim}{E}_{\perp}^* F_i = \underset{\sim}{E}_{\perp}^* \underset{\sim}{D}_i A_i = 0 \qquad i \in \{1 \ldots T\} \tag{21}$$

in obvious notation.

ESTIMATION

Let $\hat{\underset{\sim}{R}}$ be an estimate of an SCCM of rank T. We want to match it to our model matrix $\underset{\sim}{R}$ given by (8). The parameters to be estimated are A_i, β_i, and σ_i, $i \in \{1 \ldots T\}$.

In the ideal case, the parameters of a source should satisfy (21). In practice, however, it might be reasonable to try to minimize the norm:

$$J(\beta, A) = A^* \underset{\sim}{D}^*(\beta) \hat{\underset{\sim}{E}}_{\perp} \hat{\underset{\sim}{E}}_{\perp}^* \underset{\sim}{D}(\beta) A = A^* \underset{\sim}{Q} A \tag{22}$$

while obeying (3). The matrix $\hat{\underset{\sim}{E}}_{\perp}$ consists of eigenvectors of $\hat{\underset{\sim}{R}}$, corresponding to $\underset{\sim}{E}_{\perp}$ for $\underset{\sim}{R}$. $\underset{\sim}{D}(\beta)$ is the model part, taken from (5). As a function of β, the minimum of J(β,A(β)) is ε, the smallest eigenvalue of $\underset{\sim}{Q}$, and A(β) will be the corresponding eigenvector.

If we plot J(β) obtained this way versus β, all local minima indicate possible point source parameter solutions β and A(β). The closer J(β) is to zero, the better match to the model.

The last parameter σ can be obtained by combining (7) and (15), using estimated quantities:

$$\hat{F} = \hat{\sigma} \underset{\sim}{D}(\hat{\beta}) \hat{A} = \hat{\underset{\sim}{E}}_{\parallel} \hat{\underset{\sim}{\Lambda}}_{\parallel}^{1/2} \hat{Z} \tag{23}$$

Premultiplying (23) by $\hat{\underset{\sim}{\Lambda}}_{\parallel}^{-1/2} \hat{\underset{\sim}{E}}_{\parallel}^*$:

$$Z = \hat{\underset{\sim}{\Lambda}}_{\parallel}^{-1/2} \hat{\underset{\sim}{E}}_{\parallel}^* \underset{\sim}{D}(\hat{\beta}) \hat{A} \hat{\sigma} \tag{24}$$

From (20) follows the fact that Z has unit length, with the obvious result:

$$\hat{\sigma} = [\hat{A}^* D^*(\beta) \hat{E}_{\parallel} \hat{\Lambda}_{\parallel}^{-1} \hat{E}_{\parallel}^* D(\beta) \hat{A}]^{-1/2} \tag{25}$$

What happens if there is more than one source on a given bearing β_0? Because the T_0 sources cannot be at the same point, they will certainly have different mode amplitude vectors A_i. In the ideal case, this means that for the bearing β_0 there will be T_0 different A vectors making (21) equal to zero. In other words, there should be T_0 eigenvectors of Q in (22), producing small eigenvalues. The corresponding vectors A_i can no longer be uniquely estimated, but are linear combinations of these T_0 eigenvectors. One should therefore plot several of the smallest eigenvaues of Q to make sure one will determine the bearing of all the sources.

Another problem should also be addressed. There might be situations in which our model matrix D in (5) does not have full rank M. As an example, D has rank 1 at broadside with a horizontal linear array. In such a case Q will have M-1 zero eigenvalues at broadside even if there are no sources there.

If D has rank K, the problem can be solved by substituting D in (22) by a matrix G of rank K, spanning the same subspace as D. G could be produced from D by a Gram-Schmidt procedure, or consist of the K eigenvectors of $D D^*$ with nonzero eigenvalues. In any case one will have:

$$D = G U \tag{26}$$

$$U = G^* D \tag{27}$$

G can be expressed uniquely by D only if D has full rank M.

Returning to the estimation, G should now substitute D in (22). Otherwise the procedure will be as before. However, the resulting eigenvectors H of Q will no longer represent the mode amplitude vectors A. A can only be uniquely estimated insofar as D has full rank M. In that case:

$$\hat{A} = U^{-1} H \tag{28}$$

Some preliminary simulations have confirmed all the described estimation properties, but so far a matrix completely matched to the model matrix has been used.

MODEL QUALITY

Let us assume we have been able to obtain T vectors $\hat{F}_i$ satisfying (21):

$$E_{\perp}^* \hat{F} = 0 \tag{29}$$

Is it guaranteed that this $\hat{F}$ matrix will satisfy (6)? No! To see this, let us construct a matrix with the structure of (15):

$$\hat{F} = E_{\parallel} \Lambda_{\parallel}^{1/2} \hat{Z} \tag{30}$$

and choose $\hat{Z}$ so that it is not unitary:

$$\hat{Z} \hat{Z}^* \neq I \tag{31}$$

Then it follows from (11), (30), and (31):

$$R = E_{\parallel} \Lambda_{\parallel} E_{\parallel}^* \neq E_{\parallel} \Lambda_{\parallel}^{1/2} \hat{Z} \hat{Z}^* \Lambda_{\parallel}^{1/2} E_{\parallel}^* = \hat{F} \hat{F}^* \tag{32}$$

But at the same time (29) is satisfied:

$$E_{\perp}^* \hat{F} = E_{\perp}^* E_{\parallel} \Lambda_{\parallel}^{1/2} \hat{Z} = 0 \tag{33}$$

using the orthogonality of the eigenvectors.

The general observation is that (15) and (19) together are sufficient conditions to be satisfied by the modelled source vectors F_i. On the other hand, (29) is not a sufficient condition. It is sufficient to satisfy (15), but not (19).

From the above discussion, two questions immediately arise:

- How can source vector estimates $\hat{F}_i$ based only on (29) be any good?
- As (15) and (19) together constitute a sufficient condition, while (29) alone is not, why base the estimation primarily on (29), instead of on (15) and (19)?

A conjecture is that (29) is sufficient if the correct source vector model has been used. In our case, if the model described by (6) and (7) is correct, and really desribes how a point source contributes to the SCCM, then estimates obtained from (29) alone will also satisfy not only (15), but also (19).

If this is correct, an interesting consequence is that the degree to which the estimated matrix Z (derived from the estimated source vectors) will satisfy (19), is a measure for the quality of the model <3>.

The answer to the second question is that it is simpler to use (29), instead of (15) and (19), in order to obtain estimates, because (29) decouples the estimation of the individual source vectors. (19) is seen to be a highly coupled condition, and difficult to include explicitly in the estimation procedure.

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