DIXIEME COLLOQUE SUR LE TRAITEMENT DU
SIGNAL ET SES APPLICATIONS

NICE du 20 au 24 MAI 1985

A JUMP DETECTION ALGORITHM FOR PASSIVE SOURCE LOCATION

Jorge S. Salvador and José M. F. Moura

Complexo I, CAPS, Instituto Superior Técnico, Av. Rovisco Pais, 1096 Lisboa, PORTUGAL

RESUME

Le problème de Localisation Passive par effet Doppler, pour géométries déterministes, peut être formulé comme étant l'estimation d'un vecteur de paramètres à partir d'un ensemble d'observations non linéaires avec bruit. Ces paramètres définissent la position relative entre la source et le récepteur et peuvent être estimés d'après la methode du Maximum de Vraisemblance, ou le cas échéant, par filtrage de Kalman-Bucy. Bien qu'adapté à des applications de temps réel, la seconde méthode diverge fréquemment. Les observations sont fortement non linéaires puisqu'elles contiennent que de rares informations pendant de longues périodes de temps.

Cet article présente une nouvelle méthode pour la localisation passive avec trajectoires rectilignes, lors de problèmes de dispersion temporelle, ou ensembles linéaires de récepteurs lors de problèmes de dispersion spaciale. On approche le signal Doppler grâce à um signal à trois niveaux différents. On étudie les raports entre, d'un côté, les transitions entre niveaux, et de l'autre, le vecteur de paramètres et la concentration d'information dans les observations recues. Grâce à cette simplification, le problème d'estimation non linéaire de la localization passive est réduit à la détection de sauts de moyenne du signal reçu. Comme les niveaux et le nombre de transitions sont connus, le Teste du Rapport de Vraisemblance amène à une méthode de sommes cumulées que l'on peut réalizer d'une forme recursive et simple.

Pour que ce procédé soit valable, on présente un ensemble de simulations, montrant que l'approximation du signal de Doppler ne détériora pas trop la qualité des estimatives. Les estimatives des paramètres obtenues de cette façon sont étudies numériquement par des techniques de Monte Carlo et comparées à des résultats obtenus par le filtre de Kalman-Bucy. On conclut que l'algorithme de détection de sauts est beaucoup plus rapide et conduit à de meilleurs estimatives des paramètres inconnus.

SUMMARY

The problem of Doppler Passive Source Location, for deterministic geometries, can be formulated as the estimation of a vector of parameters from a set of noisy observations. These parameters define the relative position between the source and the receiver and may be estimated by the Maximum Likelihood Method [2] or, recursively, by Kalman-Bucy filtering [3],[4]. The second method, being well suited for real time applications, frequently diverges. The observations are highly nonlinear often containing little information for long periods of time.

This paper presents a new method for passive location for both rectilinear trajectories (temporal diversity problems) and rectilinear arrays of receptors, (spacial diversity problems). The Doppler signal is approximated by a signal with three distinct levels. It is shown how the transitions between levels are related to the vector of location parameters and to the concentration of the information in the received observations. With this simplification, the nonlinear estimation problem of passive location is reduced to the detection of the jumps of the mean of the received signal. As the levels and number of transitions are known, the Likelihood Ratio Test (LRT) leads, for this problem, to a cumulative sum method which is carried out, recursively, in a very simple way.

To validate this procedure, a set of simulations is presented, showing that the approximation of the Doppler signal does not deteriorate significantly the quality of the estimates. Comparison studies (by Monte Carlo Techniques) with the Extended Kalman-Bucy Filter (EKBF) are carried out, showing that the jump detection algorithm being faster leads to better estimates of the unknown parameters.

1. INTRODUCTION

In this paper, two situations for passive location are considered: 1 - a temporal diversity problem where the trajectory is a straight line and the target has a constant velocity v. 2 - a spacial diversity problem with a rectilinear array. Since the velocity estimation presents no dificulties (it is the mean value of the Doppler signal for far away targets) a normalized Doppler

$$h(x_0,s) = \frac{x_{01}+s}{\sqrt{(x_{01}+s)^2+x_{02}^2}}$$

will be used.

The vector $x_0=[x_{01},x_{02}]^T$ defines the Source/Receiver relative position (Figure 1).

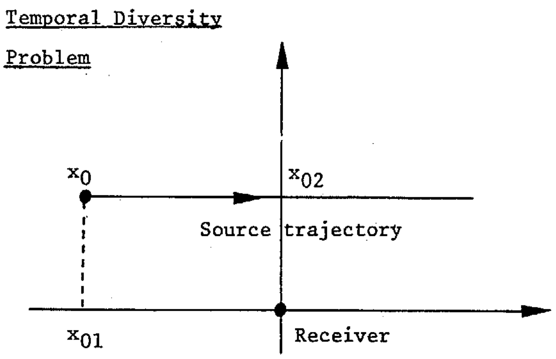

Figure 1 - Source/receiver geometry

The problem of passive source location can be formulated as the estimation of the source location vector x_o from a set of noisy observations

$$z(s)=h(x_0,s)+w(s) \qquad s\in[0,L]$$

where w(s) is assumed to be white Gaussian noise with spectral height r and $h(x_o,s)$ is the normalized Doppler signal previously defined.

2. THE FISHER INFORMATION MATRIX

The Fisher Information Matrix [6] defined as

$$J(x_0,L) = (1/r)\int_0^L \left(\frac{\partial h(x_0,s)}{\partial x_0}\right)\left(\frac{\partial h(x_0,s)}{\partial x_0}\right)^T ds$$

is analitically evaluated in [4]. Its asymptotic value (L-->oo) is

$$J_{11} = \frac{1}{r\,x_{02}}\left[\frac{3}{8}\left(\frac{\pi}{2}+\arctan(\xi)\right) + \frac{\xi}{4(1+\xi^2)^2} + \frac{3\xi}{8(1+\xi^2)}\right]$$

$$J_{12} = J_{21} = \frac{1}{r\,x_{02}}\ \frac{-1}{4(1+\xi^2)^2}$$

$$J_{22} = \frac{1}{r\,x_{02}}\left[\frac{1}{8}\left(\frac{\pi}{2}+\arctan(\xi)\right) - \frac{\xi}{4(1+\xi^2)^2} + \frac{\xi}{8(1+\xi^2)}\right]$$

where $\xi=-x_{01}/x_{02}$.

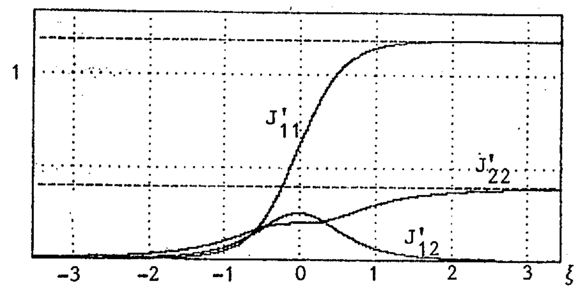

Figure 2 - Normalized Fisher information matrix $J'=r\,x_{02}\,J$

Figure 2 shows that most of the information is concentrated in the interval of length 4 x_{02} centered at the point of closest approach $(0,x_{02})$.

3. A SIMPLIFIED MODEL FOR THE DOPPLER SIGNAL

The estimation of the vector of parameters x_o from the set of noisy observations can be conceptually decomposed into the estimation of the zero cross instant for the Doppler signal, which is $-x_{01}$, and the estimation of the derivative $m=1/x_{02}$ at this point (Figure 3).

A simplified model is proposed for the Doppler signal (Figure 4) which is adapted to the techniques of jump detection, and enables the evaluation of the zero cross instant and of the Doppler derivative from the instants of jump s_1,s_2.

After a discretization with step H, the simplified model for the observations is

$$z(k) = y(k)+w(k) \qquad k=1,\dots n$$

where the perturbation w(k) is white Gaussian noise with variance $\sigma^2=r/H$,

$$y(k) = \begin{cases} -1 & 0 \le k < r_1 \\ 0 & r_1 \le k < r_2 \\ 1 & r_2 \le k \le n \end{cases}$$

and,

$$r_i = s_i/H \qquad (i=1,2)$$

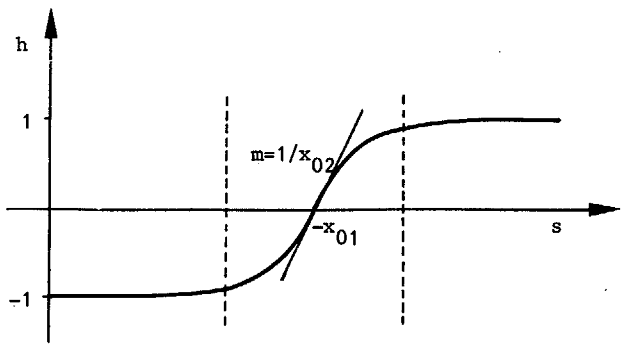

Figure 3 - Doppler signal: geometric meaning of x0

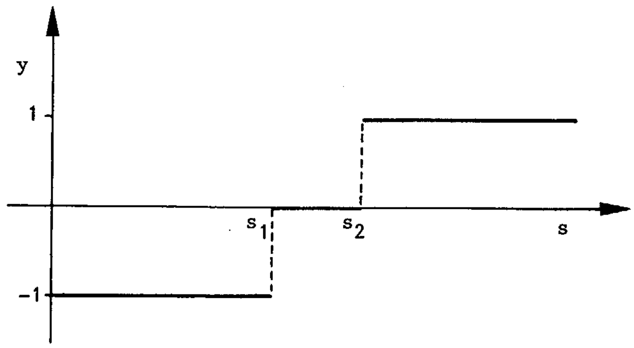

Figure 4 - Simplified model for the Doppler signal

4. THE LIKELIHOOD RATIO TEST (LRT)

Two hypothesis will be tested. Under H0 only white Gaussian noise is observed. Under H1 the observations consist of y(k) corrupted by noise. That is,

$$\begin{aligned} &H0: \; z(k) = w(k) && k=1,\dots,n \\ &H1: \; z(k) = y(k)+w(k) && K=1,\dots,n \\ & && (1<r_1,r_2<n) \end{aligned}$$

The likelihood ratio is

$$\Lambda(r_1,r_2) = \frac{p(Z/H1)}{p(Z/H0)}$$

with Z = {z(k):k=1,...n}. The conditional probability densities, p(Z/H0) and p(Z/H1), can be evaluated easily and, following closely [1] it gives

$$\ln \Lambda(r_1,r_2) = (1/\sigma^2)\ \{\ 2\sum_{k=1}^{n} z(k) - \sum_{k=1}^{r_1}(z(k)+1/2) - \sum_{k=1}^{r_2}(z(k)-1/2)\ \}$$

Substituting the unknown parameters r_1,r_2 by its maximum likelihood estimates the test is presented as

$$g_n(1/2)+g_n(-1/2) \underset{H0}{\overset{H1}{\gtrless}} \lambda$$

where λ is a threshold and

$$g_n(a) = (1/\sigma^2)\ \{\ \sum_{k=1}^{n}(z(k)-a) - \min_{1<r<n} \sum_{k=1}^{r}(z(k)-a)\ \}$$

Let

$$N_i(n) = n - \hat{r}_i(n) +1 \qquad (i=1,2)$$

be the number of terms that have contributed to the cumulative sum $g_n(a)$ from the last time it was zero. The estimate $\hat{r}_i(n)$ of r_i, (i=1,2), from the first n observations is expressed as

$$\hat{r}_i(n) = n - N_i(n) +1 \qquad (i=1,2)$$

This means that the estimates for r_1, r_2 can be updated recursively since [1]

$$N_i(n+1) = N_i(n)\ u(g_n(a)) + 1$$

where u(x) is the step function

$$u(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

The function $g_n(a)$ verifies the recursion

$$g_{n+1}(a) = (g_n(a)+z(n)-a)^+$$

with $(x)^+ = x\ u(x)$. This method gives a very simple recursive algorithm.

5. APPLICATION OF THE LIKELIHOOD RATIO TEST TO PASSIVE SOURCE LOCATION

If the signal y(s) is substituted by the true Doppler h(xo,s), the instants of jump in the absence of noise will be

$$s_1 = \arg\min_s \int_0^s (h(x_0,t)+1/2)\ dt$$

$$s_2 = \arg\min_s \int_0^s (h(x_0,t)-1/2)\ dt$$

The instants of jump are, therefore, defined by the set of equations (Figure 5)

$$h(x_0,s_1) = -1/2$$

$$h(x_0,s_1) = 1/2$$

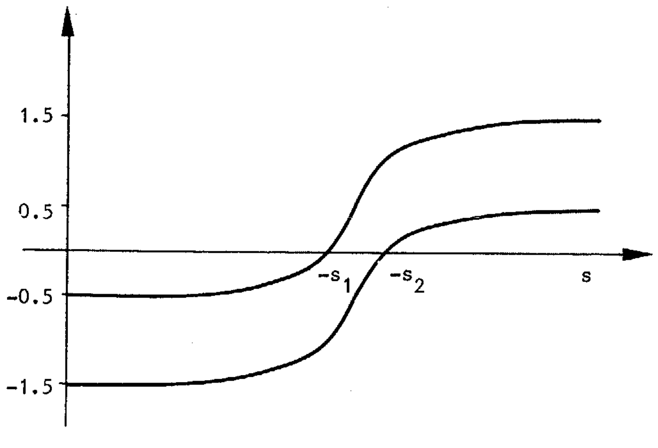

Figure 5 - Geometric meaning of the instants of jump

'he solution is

$$x_{01} = -(s_1+s_2)/2$$

$$x_{02} = \sqrt{3}\ (s_2-s_1)/2$$

if it is assumed that x0 must belong to the upper half plane. The estimates for s_1, s_2 are obtained from the estimates of r_1, r_2 by

$$\hat{s}_i = H\ \hat{r}_i \qquad (i=1,2)$$

6. NUMERICAL SIMULATIONS

A set of numerical simulations was carried out assuming that x0 = (-8,2) and L=16. This insures that all the significant information about the parameter is available. The noise power level is r=0.01 and the discretization step H=0.01

Figure 6 shows the result of 100 Monte Carlo runs and the 3σ elipsoid. The elipsoid was constructed from the Cramer-Rao bound [6] (evaluating the Fisher Information Matrix with ξ=4) being justified by numerical tests of gaussianity and biaseness [4]. This figure is repeated in Figure 7 with another scale so that it can be compared with the results of the Extended Kalman-Bucy Filter (EKBF) initialized with x(0)=(-5,1), P(0)=16 I. The EKBF results are displayed in Figure 8.

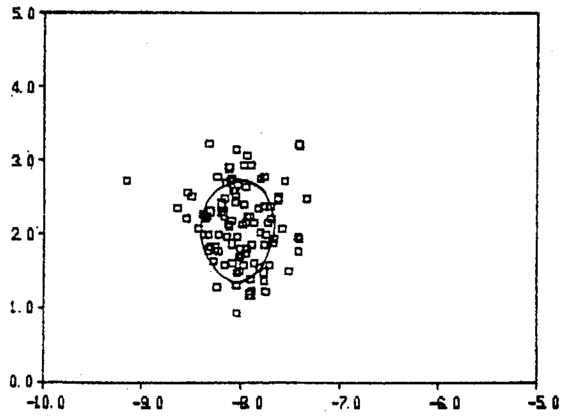

Figure 6 - Passive location by jump detection algorithm

The calculations with the jump detection algorithm took less than 4 min of CPU against the 6 hours needed by the EKBF in a MV8000 computer. It should be noted that the EKBF was implemented with a general purpose routine which had not been optimized for the particular problem of parameter estimation.

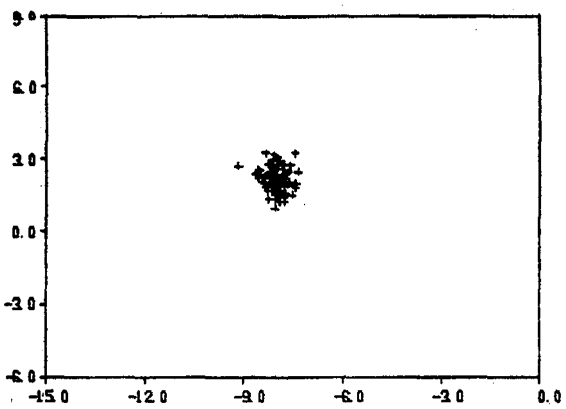

Figure 7 - Passive location by jump detection algorithm

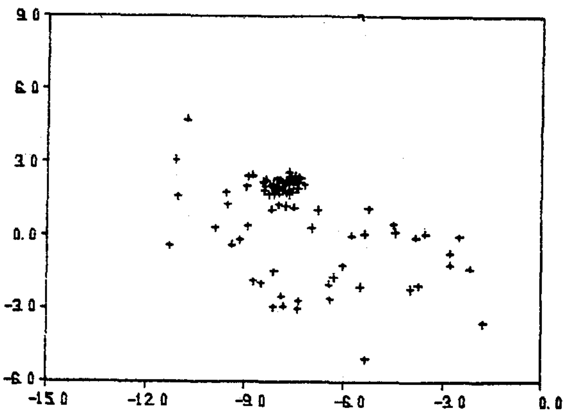

Figure 8 - Passive location by Kalman-Bucy filtering

7. CONCLUSIONS

This paper has shown that the method of abrupt changes detection can be successfuly used in passive location problems. The algorithm is simple, being an efficient way of processing the Doppler signal if the trajectory (or the array) is linear and if the observations span most of the relevant information.

REFERENCES

[1] - M. BASSEVILLE "Détection Séquentielle de Sauts de Moyenne" in Développement et Utilization de Nouveaux Outils Mathématiques pour l'Automatique et l'Analyse de Systèmes:Détection de Ruptures dans les Modèles Dynamiques de Signaux et Systèmes", CNRS, Paris, March, 1984

[2] - J.M.F. MOURA - A.B. BAGGEROER "Passive Systems Theory with Narrow Band and Linear Constraints: Part I - Spacial Diversity" IEEE Oceanic Engineering, Vol OE-3, N 1, 1978

[3] - J.M.F. MOURA "Range and Bearing Determination: Maximum Likelihood and Kalman-Bucy Techniques" in "Adaptive Methods in Underwater Acoustics", Ed. Heinz G. Urban, 1984

[4] - J.S. SALVADOR "Técnicas Recursivas em Localização Passiva por Efeito de Doppler", Thesis, Dept. Eng. Electr. and Comp., Instituto Superior Técnico, Portugal, July, 1984

[5] - A.S. WILLSKI "Detection of Abrupt Changes in Dynamical Systems" in Développement et Utilization de Nouveaux Outils Mathématiques pour l'Automatique et l'Analyse de Systèmes Détection de Ruptures dans les Modèles Dynamiques de Signaux et Sistèmes", CNRS, Paris, March, 1984

[6] - H.L. VAN TREES "Detection, Estimation and Modulation Theory", Part I, John Wiley Sons, 1969