OPTIMUM NONCOHERENT SPACE-DIVERSITY DETECTION OF WEAK SIGNALS IN NON-GAUSSIAN NOISE *

Luciano IZZO   Luigi PAURA

Dipartimento di Ingegneria Elettronica, Università di Napoli
via Claudio, 21   I-80125 Napoli, Italy

RESUME

Le problème ici étudié rentre dans le cadre de la synthèse des algorithmes asymptotiquement optimaux de détection des signaux noyés dans des bruits non gaussiens. Plus précisément on utilise la diversité spatiale pour la détection non cohérente d'un signal faible à bande étroite affecté de fluctuations d'amplitude ("fading") lentes.

La performance asymptotique (i.e. dans l'hypothèse d'un nombre élevé d'échantillons dans l'intervalle d'observation) de la structure envisagée est evaluée et comparée avec celle d'un détecteur sous-optimal récemment proposé par les auteurs dans le but de semplifier l'implémentation.

SUMMARY

The theory of asymptotically optimum detection in non-Gaussian noise is considered to synthesize a space-diversity structure for the noncoherent detection of a bandpass signal subject to slow amplitude fluctuations.

The large-sample performance of the proposed detection structure is evaluated and numerical results are presented and discussed. Moreover, a comparison between the asymptotically optimum detector and a suboptimum structure, operating according to a simple scheme recently proposed by the authors, is made.

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1. INTRODUCTION

The Gaussian noise assumption has been largely adopted to synthesize and analyze optimum receiver structures for both coherent and noncoherent reception. Such an assumption, although greatly simplifies the implementation and the analysis of the receiver structures, is not always justified because there is a variety of natural and man-made electromagnetic sources that exhibits highly non-Gaussian characteristics |1-3|. Such non-Gaussian background noise can drastically degrade |4-6| the performance of the conventional systems (i.e., those optimized with respect to the Gaussian noise).

The weak signal assumption allows one to simplify the problem of the reception in non-Gaussian noise environments. Such an assumption, on the other hand, is reasonable because usually the detection algorithms must assure near-optimum behaviour just when it is necessary, i.e., when the signal is weak compared to the arbitrary noise added by the channel.

The locally optimum detectors (LOD's), which approach optimum performance as the signal becomes vanishingly small ($\rightarrow 0$), maximize the slope of the power function at the zero signal level while keeping a fixed false-alarm probability.

The design of the LOD is based |7,8| on the leading terms of a power series expansion for the generalized average likelihood ratio (GALR) about zero signal level.

The LOD structures have been analyzed extensively |9,10| in the case of lowpass and bandpass signals. Moreover, recently, the authors have carried out |11|, with reference to the coherent reception, the synthesis and the performance analysis of a LOD that, making use of the space-diversity technique, effectively counteracts short-term fading effects.

In weak signal situations a large sample size is generally required to achieve a satisfactory system performance. Such an assumption, however, leads to an increase in the effect of higher-order terms which are dropped in the synthesis of the LOD's. In other words, for a fixed small signal level, the LOD's become increasingly suboptimum as the sample size increases |12|.

In order to assure optimum performance in situations of practical interest it is more suitable to consider the asymptotically optimum detectors (AOD's), whose operating characteristics approach the optimality as the sample size increases and the signal amplitude vanishes.

The synthesis of the AOD's is based |13| on the asymptotic expansion of the GALR for sample size $N \rightarrow \infty$ and for signal amplitude decreasing as $1/\sqrt{N}$ [1].

The present paper deals with the synthesis (Section 2) of the asymptotically optimum space-diversity detector for the noncoherent detection of a bandpass signal in non-Gaussian noise.

For the proposed structure the performance analysis is carried out and numerical results and comments are presented (Section 3). Moreover, a comparison between the performances of the AOD and a suboptimum structure, easier to implement [2], operating according to a scheme recently proposed by the authors in |11|, is made.

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[1] The comparison between the conditions of applicability |12| of the asymptotically optimum and locally optimum approaches shows that, in spite of their similar construction, the AOD and LOD are directed to different classes of signals.

2. ASYMPTOTICALLY OPTIMUM DETECTOR STRUCTURE

The detection problem under consideration can be represented by the following hypothesis test:

$$H_0: \tilde{r}_{pi} = \tilde{n}_{pi}$$
$$H_1: \tilde{r}_{pi} = A_p \frac{\gamma}{\sqrt{N}} \tilde{S}_i e^{j\theta_p} + \tilde{n}_{pi} \quad p=1,2,\ldots,K, \; i=1,2,\ldots,N \quad (1)$$

where $\tilde{r}_{pi}$ and $\tilde{n}_{pi}$ denote the complex amplitudes of the received signal and the noise at the ith instant on the pth diversity branch. $\tilde{S}_i$ is the ith sample of the complex envelope of the bandpass signal. The random variable (r.v.) A_p, which assumes nonnegative real values, takes into account the presence of a slow fading in the pth channel. The r.v. θ_p is assumed uniformly distributed over a 2π interval (noncoherent reception) and, finally, γ^2 is a positive constant proportional to the SNR (after the signal processing) [3].

We assume that the noise samples on the pth branch are identically distributed and statistically independent of each other and of the r.v.'s A_p and θ_p. Moreover, we suppose that the fadings, the noise processes and the phases θ_p on the different branches are mutually independent. Therefore, the logarithm of the GALR can be easily related to the single GALR's in each channel:

$$\ln L(\tilde{\underline{R}}_1, \tilde{\underline{R}}_2, \ldots, \tilde{\underline{R}}_K) = \sum_{p=1}^{K} \ln \left\{ E_{A_p \theta_p} \left[\prod_{i=1}^{N} \right. \right.$$

$$\frac{f_{n_{cp} n_{sp}} \left(R_{cpi} - A_p \frac{\gamma}{\sqrt{N}} \mathrm{Re}(\tilde{S}_i e^{j\theta_p}), R_{spi} - A_p \frac{\gamma}{\sqrt{N}} \mathrm{Im}(\tilde{S}_i^* e^{-j\theta_p}) \right)}{f_{n_{cp} n_{sp}}(R_{cpi}, R_{spi})} \left. \left. \right] \right\} \quad (2)$$

where: $\tilde{\underline{R}}_p = (\tilde{R}_{p1}, \ldots, \tilde{R}_{pN})$ is the vector of the samples at the input of the pth branch; $f_{n_{cp} n_{sp}}(\cdot, \cdot)$ is the joint probability density function (pdf) of the inphase and quadrature components of each noise sample at the input of the pth branch; R_{cpi} and R_{spi} represent the ith observations on the output of the inphase and quadrature pth channels, respectively; $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote the real and imaginary part (respectively); the asterisk identifies the complex conjugate; $E_{A_p \theta_p}[\cdot]$ denotes the statistical expectation with respect to A_p and θ_p.

On the assumption that the joint pdf of the noise components on each branch possesses circular symmetry, i.e.,

$$f_{n_{cp} n_{sp}}(R_{cpi}, R_{spi}) = f_p\left(\sqrt{R_{cpi}^2 + R_{spi}^2}\right) \triangleq f_p(R_{pi}) \quad (3)$$

an asymptotic expansion of the conditional GALR $L_p(\tilde{\underline{R}}_p \mid A_p)$ can be derived by the approach followed in |13, Ch.3|. Therefore, from (2) it follows that

$$\ln L(\tilde{\underline{R}}_1, \tilde{\underline{R}}_2, \ldots, \tilde{\underline{R}}_K) =$$

$$= \sum_{p=1}^{K} \ln E_{A_p}\left[\exp(-A_p^2 \gamma^2 P F_p / 2\sigma_p^2) \, I_0\left(A_p \gamma \sqrt{I_p^2 + Q_p^2}\right) \right] \quad (4)$$

where:

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} |\tilde{S}_i|^2 \quad (5)$$

provides a measure of signal power; σ_p^2 is the common variance of the inphase and quadrature noise components; $I_0(\cdot)$ is the modified Bessel function of the first kind

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[2] Unlike AOD structure, such a detection scheme does not require the evaluation of the mean (over fading) signal-to-noise ratios (SNR's) at the input of the local processors.

[3] The assumed law of decrease for the signal amplitude assures that the SNR is finite and not zero for any value of N.

and zero order; $E_{A_p}[\cdot]$ denotes the statistical expectation with respect to A_p and

$$I_p \triangleq \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Re(\tilde{R}_{pi}\tilde{S}_i^*) \, g_p(R_{pi}) \qquad (6)$$

$$Q_p \triangleq \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Im(\tilde{R}_{pi}\tilde{S}_i^*) \, g_p(R_{pi}) \qquad (7)$$

$$g_p(R_{pi}) \triangleq - \frac{1}{R_{pi}} \frac{d}{dR_{pi}} \ln f_p(R_{pi}) \qquad (8)$$

$$F_p \triangleq \pi \sigma_p^2 \int_0^\infty R^3 g_p^2(R) \, f_p(R) \, dR \qquad (9)$$

Equation (4) shows that the asymptotically optimum space-diversity detector consists of K local processors that evaluate the statistics $I_p^2 + Q_p^2$ and of a central processor. It evaluates the logarithms of the statistical expectations (with respect to the r.v.'s A_p) of the quantities enclosed within square brackets, performs their summation and compares the result with a threshold chosen to satisfy the false-alarm probability requirement.

The proposed AOD structure depends of the fading statistics on the different diversity branches. In the single channel case (K=1), vice versa, the fading, as it results immediately from (4), does not affect the receiver structure.

If one assumes the fading r.v.'s are Rayleigh distributed, from (4) the following decision statistic results:

$$T^{AOD} = \sum_{p=1}^{K} \frac{\rho_p F_p}{2 + \rho_p F_p} (I_p'^2 + Q_p'^2) = \sum_{p=1}^{K} w_p T_p^{AOD} \qquad (10)$$

where

$$w_p \triangleq \rho_p F_p / (2 + \rho_p F_p) \qquad (11)$$

$$\rho_p = E(A_p^2) \gamma^2 P / \sigma_p^2 \qquad (12)$$

is the mean (over fading) SNR at the input of the pth local processor and

$$I_p' = I_p / \sqrt{2 P F_p / \sigma_p^2} \triangleq D_p I_p \qquad (13)$$

and

$$Q_p' = Q_p / \sqrt{2 P F_p / \sigma_p^2} \triangleq D_p Q_p \qquad (14)$$

are normalized versions of I_p and Q_p such that the variance of $I_p'^2 + Q_p'^2$ is unity. In (10) T_p^{AOD} is a sufficient statistic for the AOD corresponding to the single channel case.

The space-diversity detector structure (Fig.1) consists of K local processors that evaluate the T_p^{AOD}'s (Fig.2) and of a central processor that performs weighting and compares the result of the linear combination (10) with a threshold T.

Equation (10) shows that the weight w_p is an increasing function of the product $\rho_p F_p$, where F_p can be readily recognized |10| to be the asymptotic relative efficiency |7,8| of the pth local processor (i.e.,that implementing T_p^{AOD}) with respect to the corresponding optimum detector for Gaussian noise.

Finally, we note that, in the particular case of Gaussian noises, from (8) it follows that $g_p(\cdot)=1/\sigma_p^2$ and, therefore,(6),(7) and (10) lead to the well-known receiver structure optimized against Gaussian noise.
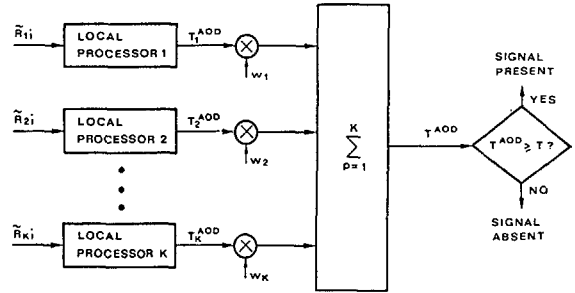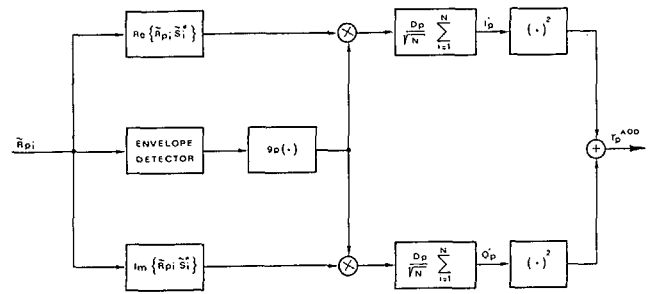

Fig.1 - Asymptotically optimum space-diversity detector.

Fig.2 - Structure of the pth local processor.

3. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

The AOD performance analysis for large sample size N is carried out by means of the characteristic function (CF) approach |14| in that the decision variable T^{AOD},as stated in (10),is a linear combination of statistically independent r.v.'s. Therefore, it follows that |11|

$$\frac{1}{2} - \frac{j}{2\pi} \int_{-\infty}^{\infty} C_{T^{AOD}}(\mu|H_i) \frac{e^{j\mu T}}{\mu} d\mu = \begin{cases} P_{FA} & i=0 \\ \\ P_D & i=1 \end{cases} \qquad (15)$$

where $C_{T^{AOD}}(\cdot|H_i)$ denotes the conditional CF and P_{FA} and P_D are the false-alarm rate and the detection probability, respectively.

Under the hypothesis H_0 the central limit theorem allows one to state that I_p and Q_p are independent zero-mean Gaussian r.v.'s with variance PF_p/σ_p^2 .

Under the hypothesis H_1 and the conditioning upon A_p and θ_p,the r.v.'s I_p and Q_p are asymptotically Gaussian with the same variance PF_p/σ_p^2 and statistical expectations $A_p\cos\theta_p PF_p/\sigma_p^2$ and $A_p\sin\theta_p PF_p/\sigma_p^2$, respectively. Therefore, from (15) it follows that

$$P_{FA} = \frac{1}{2} - \frac{j}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-j\mu T}}{\mu} \prod_{p=1}^{K} (1-j\mu w_p)^{-1} d\mu \qquad (16)$$

$$P_D = \frac{1}{2} - \frac{j}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-j\mu T}}{\mu} \prod_{p=1}^{K} E_{A_p}\left[\frac{1}{1-j\mu w_p} \exp\left(\frac{j\mu w_p}{1-j\mu w_p} \frac{A_p^2\gamma^2 PF_p}{2\sigma_p^2}\right)\right] d\mu \qquad (17)$$

Equations (16) and (17) allow one to evaluate, for a fixed false-alarm probability P_{FA} , the detection rate P_D as a function of the products $\rho_p F_p$, in correspondence of any fading law on each diversity branch.

In the following we present and discuss numerical results on the assumption that the amplitude fluctuations are Rayleigh distributed.

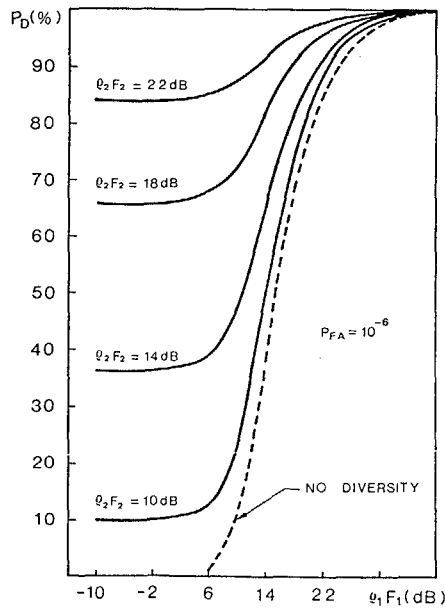In Fig.3, with reference to dual diversity, P_D is

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Fig.3 - Detection probability P_D versus $\rho_1 F_1$ for some values of $\rho_2 F_2$ and for a fixed value of P_{FA}.

plotted as a function of $\rho_1 F_1$ for a fixed value of P_{FA} and for several values of $\rho_2 F_2$. For comparison purposes a dotted curve shows the results concerning the case of absence of diversity. The figure allows one to identify the range of mean SNR's that lead to satisfactory improvements gained by the diversity technique. The presence of a range of values of $\rho_1 F_1$ in which P_D is largely independent of $\rho_1 F_1$ can be readily justified considering that, as long as $\rho_1 F_1 \ll \rho_2 F_2$, the performance is practically that obtainable when only the channel $\rho_2 F_2$ is operating and, therefore, in such a situation, the detection rate can be derived refer-
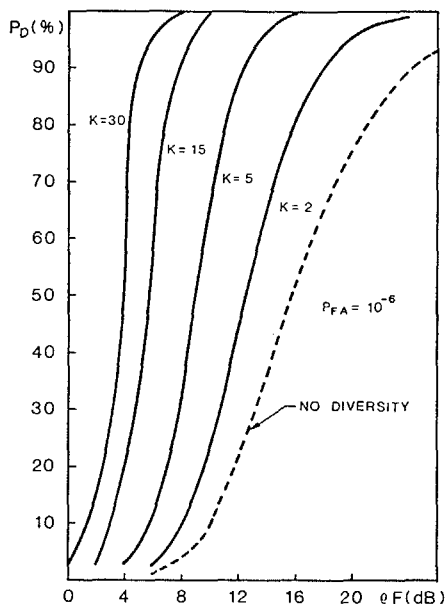

Fig.4 - Detection probability P_D versus ρF in correspondence of some values of the diversity order K and a fixed false-alarm rate P_{FA}.
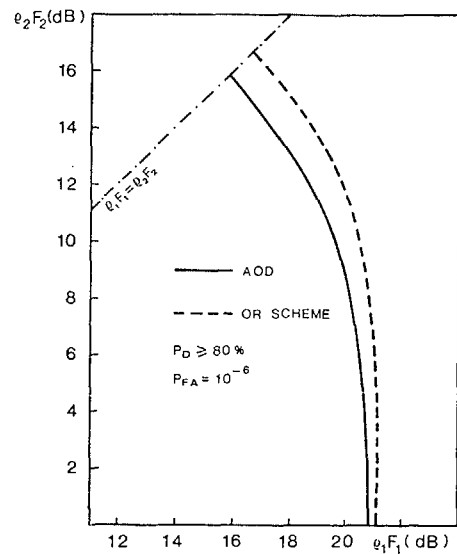

Fig.5 - Contours delimiting regions of operating conditions (with reference to the case of two local processors) that satisfy the requirements: $P_D \geq 80\%$, $P_{FA} = 10^{-6}$.

ring to the curve for the case of absence of diversity.

The performance as a function of the diversity order can be assessed by considering K local processors having the product $\rho_p F_p$ independent of the processor considered (i.e., the same for any value of p), say ρF.

In Fig.4 P_D is plotted as a function of ρF for $P_{FA} = 10^{-6}$ and with the number K as a varying parameter. The curves show the transition from critical to optimum operating conditions is increasingly sharper as the diversity order increases.

Figures 5 and 6 allow one to make a comparison between the proposed AOD (Fig.1) and a suboptimum structure operating according to the scheme proposed by the authors in |11|. The structure consists of peripheral detectors that perform local independent decisions based on the previously introduced statistics T_p^{AOD}'s (see (10) and Fig.2) and of a central processor that, working according to the OR scheme [4], performs the global decision based on the local ones.

With refernce to dual diversity, Fig.5 presents for both detection strategies the contours in the plane $(\rho_1 F_1, \rho_2 F_2)$ delimiting regions of operating conditions specified in terms of overall detection probability for a fixed global false-alarm rate (the straight line $\rho_1 F_1 = \rho_2 F_2$ is an axis of symmetry for both reported curves). The results show that the performance degradation of the OR scheme with respect to the AOD, in terms of power, is bounded to at most 1.5 dB and becomes more and more negligible as the SNR over one of the channels approaches $-\infty$ (in such a limiting case the corresponding local processor is not operating).

Finally, Fig.6 presents the loss (in decibels) in terms of ρF (the same value of $\rho_p F_p$ is assumed for all

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[4] The choice of an OR or AND scheme leads one to a very simple implementation of the central processor. In general it is not possible to state |11| which scheme performs better. However, the OR strategy is considered here because it was found that OR performs much better in all examined cases.

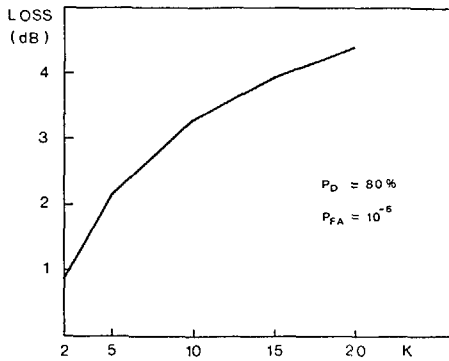OPTIMUM NONCOHERENT SPACE-DIVERSITY DETECTION OF WEAK SIGNALS IN NON-GAUSSIAN NOISE

Fig.6 - Loss (in terms of ρF) of the suboptimum OR scheme with respect to the AOD structure versus number K of peripheral processors.

local processors) of the OR scheme with respect to the AOD. The performance degradation does not exceed a few decibels for any realistic value of the diversity order K.

4. CONCLUSIONS

The theory of the asymptotically optimum detection in non-Gaussian noise environments is considered to synthesize a space-diversity structure for the noncoherent detection of a bandpass signal subject to amplitude fluctuations.

An asymptotic sufficient statistic is derived for arbitrary fading laws on the different diversity branches. It shows that in the AOD structure two processing steps can be recognized: a local processing and a central processing. In the former, which does not depend of the fading law, each observed signal (in terms of inphase and quadrature components) is first processed by a narrowband nonlinearity which depends only of the joint pdf of the noise components and, then, by an envelope correlation detector. In the latter, vice versa, the knowledge of the fading statistics on the diversity branches is required.

In the interesting case of Rayleigh fadings the central processing weights the output of any local processor by emphasizing the contribution of those more reliable, performs the summation of such weighted outputs and, finally, compares the result with a threshold.

The analysis of large-sample performance of the proposed asymptotically optimum space-diversity detector is carried out. Moreover, numerical results are presented and discussed and a comparison between the performances of the AOD and of a suboptimum detector (easier to implement) recently proposed by the authors is made. It turns out that the performance degradation does not exceed a few decibels for realistic values of the diversity order.

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