PREMIER COLLOQUE IMAGE

Traitement, Synthèse, Technologie et Applications

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RESTAURATION D'IMAGE TRANSFORMEE DE WALSH BIDIMENSIONNELLE
IMAGE RESTAURATION BY THE USE OF 2-D DISCRETE WALSH TRANSFORMS

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RESUME

Un 2-D algorithme numerique de filtration, realise a l´aide de la 2-D transformation discrete de Walsh, est presente dans le travail. L´algorithme resulte de l´application sequentielle d´une 1-D filtration aux lignes et colonnes de la matrice des donnees d´entree. L´emploi de cet algorithme de filtration dans le processus de restauration des images est montre. Dans ce but une methode de synthese du filtre de restauration dans le domaine de Walsh est proposee. On suppose, que l´image est contamine avec un bruit additiv stationnaire, non-correle avec l´image original. Pour ces suppositions les matrices de filtration, qui determinent les proprietes du filtre numerique de restauration optimal (dans le domaine de Walsh) sont calculees. Le critere d´optimalisation est exprime par le valeur minimal de l´erreur carre moyen du processus de restauration. Un exemple de restauration de l´image, a l´aide de la 2-D transformation de Walsh, est donne.

SUMMARY

The 2-D digital filtering algorithm, which is realized by the use of 2-D discrete Walsh transform is described. The algorithm is a sequential application of 1-D filtering to the rows and columns of the input data matrix. The use of this filtering algorithm for an image restoration is presented. The method of synthesis of the Walsh domain restoration filter is proposed. It is assumed, that the image is blurred by additive stationary noise. The noise is uncorrelated with the original image. For these assumptions, the filter matrices characterizing the properties of the optimal digital Walsh domain restoration filter are calculated. The optimization criterion is the minimal value of the mean-square-error of the restoration process. An example of the image restoration by the use of the 2-D discrete Walsh transform is shown.

1. INTRODUCTION

One of the first steps in image processing particularly in image restoration, is the noise filtering to improve the signal to noise ratio. This can be accomplished by smoothing the discrete noise corrupted signal with an appropriate linear restoration filter. Usually we are interested in such filtering algorithms, which require as less time as possible. The restoration filters are mainly based on the discrete Fourier series-transform technique [3]. The image restoration can be also achieved by filtering the noise by the use of the two-dimensional (2-D) discrete Walsh transform. The 2-D discrete Walsh transform can be computed much faster than the 2-D discrete Fourier transform. The restoration algorithm described in the paper [4] requires for its realization only $4N^2\log_2 N$ real additions/subtractions and $2N^2$ real multiplications (N^2 denotes the number of samples). In this paper a restoration model in the Walsh domain for a smeared image with additive stationary noise is presented. Moreover the results of the digital image restoration are shown. An image field contained 32 x 32 points.

2. THE RESTORATION ALGORITHM

The block diagram of the image restoration algorithm in Walsh domain is shown in Fig.1.

The corresponding formula can be written as

$$\underline{U} = \underline{W}^{-1}\, \underline{G}_l^W\, \underline{W}\, \underline{V}\, \underline{W}\, \underline{G}_k^W\, \underline{W}^{-1} \qquad (1)$$

where $\underline{G}_k^W$ and $\underline{G}_l^W$ are the filter matrices for the rows and columns of the input noisy image data matrix, $\underline{V}$ is the $N \times N$ noisy image data matrix, $\underline{U}$ is the $N \times N$ restored image matrix, $\underline{W}$ denotes the $N \times N$ Walsh transform matrix with elements "+1" and "-1" , which are the values of the discrete Walsh functions in sequency ordering, $\underline{W}^{-1}$ is the inverse matrix.

In this paper the filter matrices $\underline{G}_k^W$ and $\underline{G}_l^W$, which express the properties of the Walsh domain restoration filter, are assumed to have a diagonal and constant form.

Therefore some image restoration error is to be expected. If the 2D sequency (Walsh) spectrum of the blurred image is known, the above mentioned error can be minimized [5].

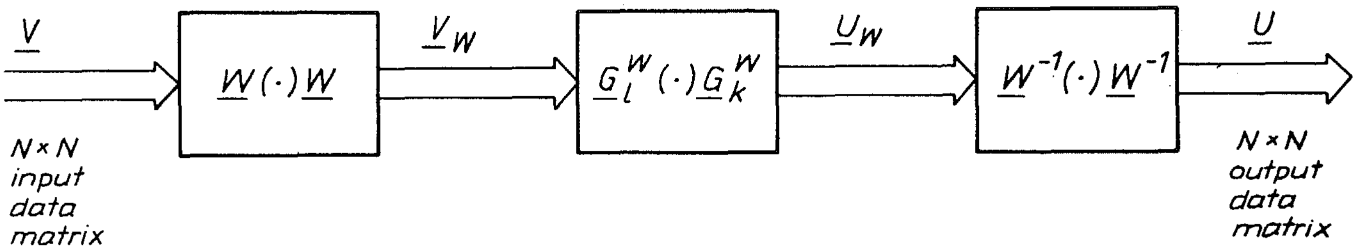

Fig.1. The block diagram of the Walsh domain restoration filter

3. THE RESTORATION FILTER

In the case of additive noise, the most accepted restoration technique is general Wiener filtering, which minimizes the mean square difference between the original and restored images. Our restoration model uses the 2D Walsh domain filter which is defined by the following equations.

$$\begin{aligned} &\sum_{i=0}^{N-1} \left\{ G_l^W(j,j)\, E\left[V_W^2(i,j)\right]\left[G_k^W(i,i)\right]^2 + \right. \\ &\qquad \left. - G_k^W(i,i)\, E\left[V_W(i,j)\, U_W(i,j)\right]\right\} = 0 \\ &\sum_{j=0}^{N-1} \left\{ \left[G_l^W(j,j)\right]^2 E\left[V_W^2(i,j)\right] G_k^W(i,i) + \right. \\ &\qquad \left. - G_l^W(j,j)\, E\left[V_W(i,j)\, U_W(i,j)\right]\right\} = 0 \end{aligned} \qquad (2)$$

where $G^W(j,j)$ and $G^W(i,i)$ are the unknown values of the $N \times N$ diagonal filter matrices G_1^W and G_k^W, respectively.

$E\left[V_w^2(i,j)\right]$ is the Walsh power density spectrum of the smeared image.

E denotes the expectation operator.

$E\left[V_w(i,j)\ U_w(i,j)\right]$ is the Walsh cross-power density spectrum.

The above system of equations can be solved by the use of the iteration method. Equations (2) result from the following condition

$$E\left\{\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}\left[\hat{u}(k,l)-u(k,l)\right]^2\right\}=\min \qquad (3)$$

where $u(k,l)$ and $\hat{u}(k,l)$ are samples of the original and restored images, respectively. According to the Parseval's theorem, condition (3) can be expressed in the following form

$$E\left\{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[\hat{U}_w(i,j)-U_w(i,j)\right]^2\right\}=\min \qquad (4)$$

where $U_w(i,j)$ and $\hat{U}_w(i,j)$ are the Walsh power spectra of the original and restored images.

Taking into account, that

$$\hat{U}_w(i,j) = G_1^W(j,j)\ V_w(i,j)\ G_k^W(i,i) \qquad (5)$$

we have

$$E\left\{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[G_1^W(j,j)\ V_w(i,j)\ G_k^W(i,i)-U_w(i,j)\right]^2\right\}=\min \qquad (6)$$

where $V_w(i,j)$ is the Walsh power spectrum of the smeared image with additive noise. Differentiating (6) with respect to $G_1^W(j,j)$ and $G_k^W(i,i)$ we have

$$\frac{\partial E\left\{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[G_1^W(j,j)V_w(i,j)G_k^W(i,i)-U_w(i,j)\right]^2\right\}}{\partial G_1^W(j,j)}=$$

$$=\sum_{i=0}^{N-1}2\left\{G_1^W(j,j)\ E\left[V_w^2(i,j)\right]\left[G_k^W(i,i)\right]^2-E\left[V_w(i,j)\ U_w(i,j)\right]G_k^W(i,i)\right\} \qquad (7)$$

$$\frac{\partial E\left\{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[G_1^W(j,j)V_w(i,j)G_k^W(i,i)-U_w(i,j)\right]^2\right\}}{\partial G_k^W(i,i)}=$$

$$=\sum_{j=0}^{N-1}2\left\{\left[G_1^W(j,j)\right]^2 E\left[V_w^2(i,j)\right]G_k^W(i,i)-G_1^W(j,j)\ E\left[V_w(i,j)\ U_w(i,j)\right]\right\} \qquad (8)$$

For a minimum, these derivatives must be zero for each value of i and j; hence the best choice of $G_1^W(j,j)$ and $G_k^W(i,i)$ is given by Eqs.(2).

4. RESULTS

Some computer-simulation results are shown in Figs. 2a, 2b, 2c.

An image of a numeral „5" (see [3]) was sampled to 32 x 32 points. Each point in the numeral was given a value 7, while each point outside the numeral was given a value 0. In the images shown in Figs. 2a, 2b, 2c, each point was quantized to 16 levels (0-15), the levels 10-15 being represented by the letters A-F. The original image, shown in Fig.2a was blurred by the additive noise (Fig.2b). The noise was independent of the original signal. Its standard deviation was equal 0,25 and the mean value 4.

The Walsh filtering method was applied to this blurred image. The result after 10 iterations is shown in Fig.2c. It must be noted that even at this relatively low signal-to-noise ratio, the result of applying the Walsh filtering method is quite good.

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a)

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25332432355143334222215534424325
44553214234113334432132541234234
34311333413133133415242333334552
552535545498CBA89BC98CCBA3342442
31422142249999BABBBA9CC8833523312
23451332549AB3341332323244321242
3254533F239CB2243433532411432543
2212322339BA33133515434234543341
323334332ACC41242423443124341315
43353443399A32244153223523523343
5524544529B523443345233452212243
12232322B8BBA99B2552334525332243
442345329A9BA9ACC254113344213423
54431152ABCAA9BCAB93242133352253
3333344BB414334CAA95342134235414
1443221823522343A8AA124232333352
34445222335555442CACA34543351122
24354355132444442AAA845411353124
3234515252553542139A953154435223
454334344134544532C9A93234421422
24544221243222543499B95241233311
434545321432243323ACA25442522533
35332344342432455BABC33331423443
524132A21243243529CAA22442142232
11135C9344135425CC9A432214444342
23422BC98324413ACA84123431422322
442345999B9529BBCBB2442433251433
5213225AACA9A9BBA332224352433544
3223455 52C9CBAB93435322421535335
12223232324C9B114422543252432235
23432222444321413522552544342523
43212345225354213554235434344224
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b)

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21101125000000002100001100000000
31101135000000002100001110000000
11000000001011111100111111110100
21110112111111112111111221110100
31101136112222113211011211110100
31101136112222112210011111110000
7210348F222221016310001200000000
4110225A111211004210001100000000
00000000112210100000000000001111
00000000112210110000000000001111
00000000122211210000000000001111
00000000222222220000000001001111
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00000011001110001000000000110000
00000011001110000000000000010000
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c)

Fig. 2 a) Original image, b) Image with additive noise, c) Restored image from Fig. b) using the Walsh filtering method.

5. CONCLUSION

Many restoring methods have been proposed in the literature [3].
We described the simplified restoration filter which is fast and easy to implement. This restoration filter is based on the 2-D Discrete Walsh transform and obeys a minimum mean-square-error quality criterion.

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