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SIGNAL ANALYSIS FOR A NEW IMPROVED CW RADAR RECEIVER

Dr. A.S. EL SHERIF, Associate Member IEEE

Laboratoire des Signaux et Systèmes, C.N.R.S. - E.S.E.
Plateau du Moulon, 91190 GIF-SUR-YVETTE, FRANCE.

RESUME

Ce travail étudie le choix d'un signal de radar, et les propriétés de son récepteur et le comportement de ce signal au niveau de celui-ci. Cette approche représente une solution mixte pour les deux principaux problèmes des radars CW : les fuites et la résolution en distance. Le système concerné est basé sur une onde modulée performante. Le signal principal pour modulation est la séquence binaire pseudo-aléatoire ayant la longueur maximum pour la modulation de phase de la fréquence porteuse qui est performante pour réaliser une haute résolution en distance. En même temps, une modulation secondaire est introduite, réalisée par une modulation de fréquence de la porteuse avec un signal sinusoïdal, alors que le récepteur travaille sur une des hautes harmoniques de la modulation de fréquence. Les paramètres de ce signal sont choisis à l'intérieur de certaines limites. Une forme modifiée du récepteur est introduite ce qui augmente considérablement le rapport (signal/bruit plus fuites) par rapport aux autres récepteurs. On explicite l'expression du spectre en fréquence et on mesure celui en amplitude à la sortie du mélangeur. Le comportement du signal dans le récepteur est analysé dans les cas de bonne et de mauvaise détermination de la distance de la cible. Cette analyse montre les propriétés en ambiguité et en résolution du signal, ainsi que les propriétés de rejet des fuites du récepteur et l'effet de la longueur de bande du récepteur sur ces propriétés.

SUMMARY

This work deals with radar signal design, corresponding receiver properties and signal behaviour in receiver. This approach represents a combined solution for the two main problems of CW radar, namely the leakage and resolution in range measurement. The system in concern is based on a suitably designed modulating waveform. The main waveform is the pseudo random binary sequence (P.r.b.s.) with maximum length for PHM the CW carrier which is excellent for realizing high resolution in range. Beside, secondary modulation is introduced realized by sinusoidal FM of carrier while operating receiver on higher harmonic of FM. The parameters of this signal are chosen within presented limits. A modified form of receiver is introduced which increases considerably the signal-to-noise plus leakage compared with other receivers. Expression for frequency spectrum is derived. Amplitude spectrum at mixer output was measured. Signal behaviour in receiver parts is analysed for two cases of correct and incorrect determination of target range. The analysis reveals the resolution and ambiguity properties of the signal, as well as the leakage rejection property of the receiver and the effect of receiver bandwidth on these properties.

1. INTRODUCTION

The form of the signal and the type of modulation are determined according to some theoretical and practical considerations [1]-[4]. The main waveform adopted for use in this application is the binary pseudo-random sequence of maximum length generated by linear feedback shift register. Such signal is used for phase modulating the CW carrier, since it possesses a very desirable correlation function having a sharp central peak and low sidelobes. This waveform has proved to be excellent for realizing high resolution in range measurement [5]. The range resolution is proportional to the time duration τ_o of one segment of the code. The behaviour of the ambiguity function of the sequence on the doppler frequency axis secures unambiguous velocity determination [6]. Beside this phase modulation as a main modulation, another one called secondary modulation is introduced. It is realized by sinusoidal frequency modulation of the CW carrier while operating the receiver on higher harmonic of the modulation frequency spectrum. Since the amplitudes of harmonics of the frequency modulation spectrum are weighted by the bessel functions with argument depending on range, and gets zero for zero range, then filtering only a higher harmonic of this spectrum, in the receiver for example the third harmonic, secures efficiently the suppression of the leakage from transmitter and reflections from close-in targets [7]. The objective of the analysis of this signal in a model of the modified receiver is to determine the resulting resolution and ambiguity properties while suppressing The leakage in the receiver. Particularly is important in further steps to compare this receiver with other CW receivers.

2. WAVEFORM DESIGN LIMITS

If we suppose that the frequency of the carrier varies according to the sinusoidal law
$f_n = f_o + \Delta f_o \sin\Omega t$ and the carrier phase is varied between two states 0 or π according to the change of

the binary pseudorandom sequence between the states ± 1 i.e. $V(t) = \cos[\varphi(t)]$. The transmitted signal S(t) is therefore in the form

$$S(t) = U_t \cos\left[\omega_o t - \frac{\Delta f_o}{f}\cos\Omega t + \varphi(t)\right] \qquad (1)$$

or equivalently $S(t) = U_t . V(t) . \cos[\omega_o t - x\cos\Omega t]$ (2)

The parameters of this signal are chosen within the following limits :

a) Phase modulation : the parameters affecting the maximum length P.r.b.s. are :
   (1) The length of the sequence L where $L = 2^S - 1$
   (2) The duration of the segment pulse τ_o of the P.r.b.s. where in this case the period of the sequence is given as $T = L\,\tau_o$, the parameter τ_o is given as the inverse of the clock frequency F_c. For this sequence to fit a prescribed maximum range R_{max} it must be in the limit $F_c \le L \frac{C}{2R_{max}}$; c : light velocity
b) The frequency modulation : the parameters affecting the resulting waveform are :
   (1) The frequency f is chosen to satisfy the inequality $2F_{d_{max}} < f \le \frac{C}{4R_{max}}$. This condition is imposed to secure the extraction of doppler frequency without distorsion and to avoid any blind ranges or any deterioration in range [8].
   (2) The index of modulation X is chosen to minimize the signal loss at the maximum range. The optimum index of modulation is different depending on which harmonic of the frequency modulation spectrum will be selected in the receiver and is tabulated in reference [8].

3. MODIFIED CW RADAR RECEIVER

A modified form of CW radar receiver processing the above signal is introduced in Fig.1 which increases considerably the signal-to-noise plus leakage. It is used a cross-correlator and a doppler filter bank where the cross-correlator can do for several delay bits given by a shift register. Only the nth harmonic of the FM spectrum with its doppler side band components are allowed to pass through the selective circuit. While the nth harmonic detector will use the reference nth harmonic of FM signal for detecting the doppler signal. This signal will be further integrated in the doppler filter bank. Observing the maximum signal appearing at any of the time delay bits and doppler frequency bits, it would be possible to determine the unknown parameters (range, velocity) corresponding to those bits with maximum outputs. The signal power though is decreased due to the use of only one component of the FM spectrum. On the other hand the leakage is much more decreased. In addition, the noise is decreased compared with simple homodyne CW system due to two factors :

(1) The use of nth harmonic of the FM spectrum for detecting the useful signal shifts the useful signal away from the range of high $\frac{1}{f}$ noise of the RF mixer.
(2) The use of phase modulation distributes the useful signal power (which is distributed among the FM spectrum components) around harmonics of the phase modulation which, in turn, shifts the frequency band among which useful signal energy is distributed from the range of high $\frac{1}{f}$ noise of the RF mixer. These two factors decrease the noise and consequently the net signal-to-noise plus leakage ratio will be much increased than other related systems.

4. FREQUENCY SPECTRUM OF SIGNAL WITH DOUBLE MODULATION

The transmitted signal has the form given by relation (2) where V(t) is P.r.b.s. signal. Since V(t) is a periodic signal it can be expanded in a Fourier series where

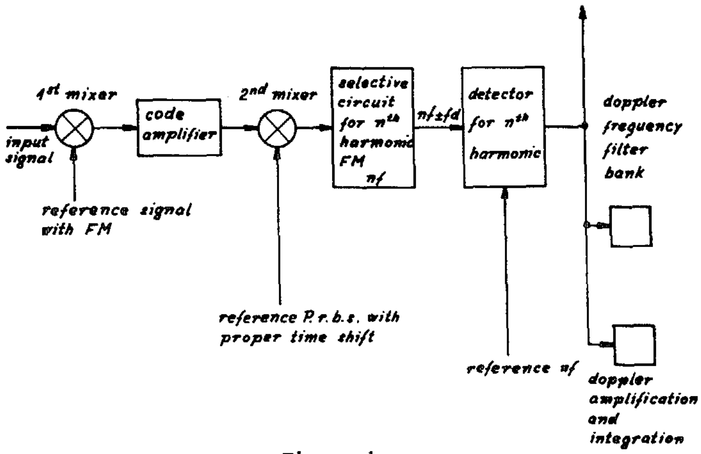


Figure 1

Correlation receiver for signal with double modulation

$$V(t) = \sum_{m=0}^{\infty} C_m \cos(m\omega_p t + \varphi_m) \qquad (3)$$

where C_m, φ_m, ω_p are amplitude and phase coefficients, $\omega_p = 2\pi/T$, is the PHM frequency, the transmitted signal could be in the form

$$S(t) = U_t \left[\sum_{m=0}^{\infty} C_m \cos(m\omega_p t + \varphi_m)\right] . \cos\left[\omega_o t - x\cos\Omega t\right] \qquad (4)$$

Thus the received signal after being delayed in time by t_1 has the form

$$S_r(t) = U_r . V(t-t_1)\cos\left[\omega_o(t-t_1) - x\cos\Omega(t-t_1) + \varphi_c\right] \qquad (5)$$

t_1 for moving target is given as $t_1 = 2\frac{R(t)}{C} = 2\frac{v}{C}t + 2\frac{R}{C}$.
Using relation (3), the expression for $V(t-t_1)$ is

$$V(t-t_1) = \sum_{m=0}^{\infty} C_m \cos\left[(m\omega_p - \omega_{dm})t + \varphi'_m\right] \qquad (6)$$

where $\omega_{dm} = m\omega_p\, 2\frac{v}{C}$, $\varphi'_m = \varphi_m - \varphi_{om} = \varphi_m - m\omega_p . \frac{2R}{C}$

thus the expression (5) of the received signal can be

$$S_r(t) = U_r\left[\sum_{m=0}^{\infty} \cos\left[(m\omega_p - \omega_{dm})t + \varphi'_m\right]\right] . \cos\left[\omega_o(t-t_1) - x\cos\Omega(t-t_1) + \varphi_c\right] \qquad (7)$$

Mixing the received signal with LO signal as CW with FM by sinewave. Then (LO) signal has the form
$U_{LO} = U_o \cos\left[\omega_o t - x\cos\Omega t\right]$.

If we suppose that the mixer acts as a multiplier with transfer coefficient k and the higher harmonics of ω_o are filtered out, thus the difference frequency component at the output of the mixer is

$$S_{d_2} = U\left[\sum_{m=0}^{\infty} C_m \cos\left[(m\omega_p - \omega_{dm})t + \varphi'_m\right]\right] . \cos\left[\omega_o t_1 - \varphi_c + B\sin\Omega(t - \frac{t_1}{2})\right] \qquad (8)$$

$$\text{where } U = \frac{1}{2} k\, U_o\, U_r,\quad B = 2\times\sin\Omega\frac{t_1}{2} = 2\times\sin\Omega\frac{R}{C} \qquad (9)$$

using the relation for t_1

$$\therefore\quad \omega_o t_1 - \varphi_c = \omega_d t + \psi \qquad (10)$$

where $\omega_d = \omega_o . \frac{2v}{C}$; $\psi = \omega_o . \frac{2R}{C} - \varphi_c$

Thus equation (8) can be in the form

$$S_{d_2} = U\left\{\sum_{m=0}^{\infty} C_m \cos\left[(m\omega_p-\omega_{dm})t+\phi'_m\right]\right\}.\left\{\cos\left[\omega_d t+\psi+ +B\sin\Omega(t-\frac{t_1}{2})\right]\right\} \quad (11)$$

The signal expressed by equation (11) is composed of 3 factors, a constant factor U, the second bracket is the P.r.b.s. used for phase modulation but shifted in time by t1 the third bracket corresponds to a signal of doppler frequency which is frequency modulated by frequence f equal to the frequency of the modulation frequency of the transmitter whereas the depth of modulation is B which is function of target range as given by relation (9). The index of modulation B gets zero for values of range given by

$$R = W.\frac{C}{2f} \quad ; \; W = 0,\ 1,\ 2... \quad (12)$$

The expression in the third bracket of equation (11) can be expanded in an infinite series of harmonics of f, so (11) becomes

$$S_{d_2} = U.\left\{\sum_{m=0}^{\infty} C_m \cos\left[(m\omega_p-\omega_{dm})t+\phi'_m\right]\right\}.\left\{J_o(B)\cos(\omega_d t+\psi) + \sum_{n=1}^{\infty}(-1)^n J_n(B).\left\{\cos\left[(n\Omega+\omega_d)t-n\Omega\frac{t_1}{2}+\psi+n\pi\right] + \cos\left[(n\Omega-\omega_d)t-n\Omega\frac{t_1}{2}-\psi\right]\right\}\right\} \quad (13)$$

performing the multiplication of the two infinite series in (13) thus the form of the signal at the output of the mixer is

$$S_{d_2} = UJ_o(B)\sum_{m=0}^{\infty}\frac{1}{2}C_m\cos\left[(m\omega_p\pm\omega_d-\omega_{dm})t+\phi'_m\pm\psi\right] + U\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2}C_m(-1)^n J_n(B) \left\{\cos\left[(m\omega_p\pm n\Omega\pm\omega_d-\omega_{dm})t+\phi'_m\mp n\Omega\frac{t_1}{2}\pm\psi\pm n\pi\right]+ \cos\left[(m\omega_p\pm n\Omega\mp\omega_d-\omega_{dm})t+\phi'_m\mp n\Omega\frac{t_1}{2}\mp\psi\right]\right\} \quad (14)$$

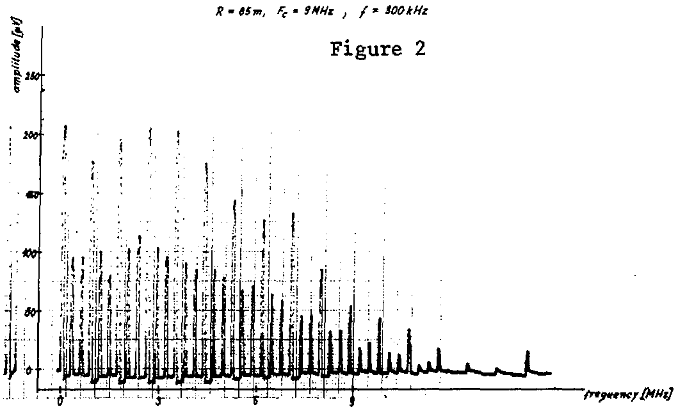
It is seen that the spectrum is composed of doppler frequency components at equidistance from each harmonic of the frequency modulating frequency f whereas all this spectrum is repeated at equidistant from each harmonic of the phase modulating frequency f_p. The amplitudes of these frequency components are, proportional to the corresponding amplitude coefficient C_m of the Fourier series expansion of the P.r.b.s., and proportional to the corresponding bessel function $J_n(B)$. The argument of the bessel function is dependent on range as shown from relation (9). The value of B gets zero for ranges given by (12). The first zero of B is given for W = 0 in relation (12) which secures for us the suppression of leakage since $J_n(0) = 0$, whereas the second zero of B is given for W = 1 corresponding to $R = \frac{C}{2f}$.

This zero amplitude corresponds to the first blind range, therefore R_{max} must be less than R given by this value. It was shown in the inequality satisfied by f that R_{max} has to satisfy relation $R_{max} \leq \frac{C}{2f}$ if it is required to avoid any deterioration of response at R_{max}. The amplitude spectrum at the output of the mixer as given by equation (14) was measured by spectrum analyzer and is given in Figures 2, 4. The received signal is simulated

Measured amplitude spectrum at the output of mixer

R = 85m, F_c = 9MHz, f = 300kHz

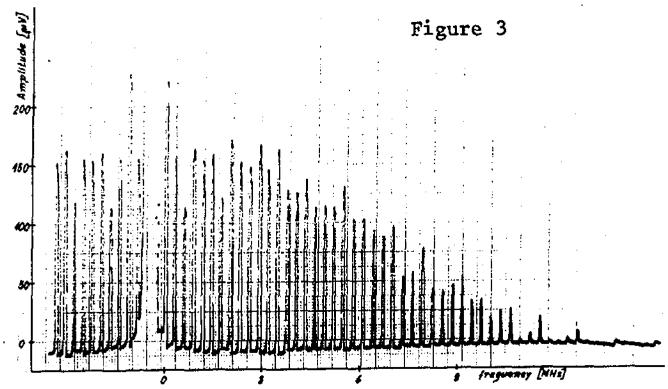
Figure 2



Measured amplitude spectrum at the output of mixer

R = 85m + 6 cm, F_c = 9MHz, f = 300kHz

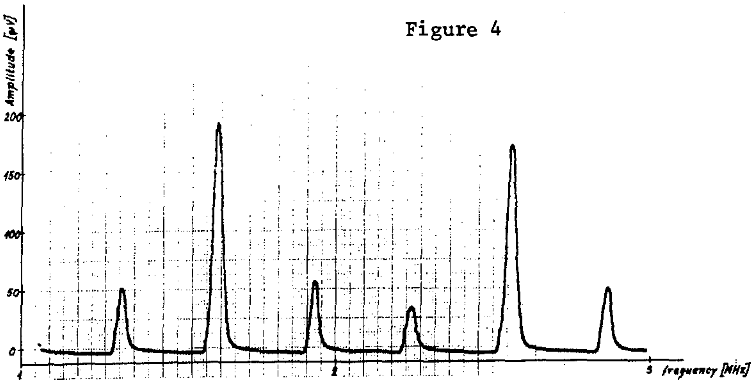
Figure 3



Measured amplitude spectrum at the output of mixer

R = 85m, F_c = 9MHz, f = 300kHz

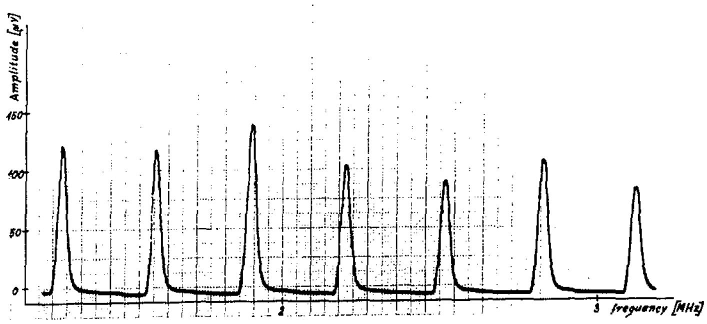
Figure 4



Measured amplitude spectrum at the output of mixer

R = 85 m + 6 cm, F_c = 9MHz, f = 300kHz

Figure 5



by passing the transmitted signal through a coaxial cable with length 106 m, in which the signal is subjected to a phase shift corresponding to target range nearly equal to 85 m. Although the received signal in this case contains no doppler shift since the simulated target exhibits no motion but the motion of the target could be simulated by a small change of the target range. This could be made by a phase shifter calibrated to small changes in range in the order of centimeters, Figures 3, 5. The effect of change in range can be easily recognized when comparing Figures 2, 3 and Figures 4, 5 from the relative change in amplitude of the corresponding components in the two Figures.

5. SIGNAL BEHAVIOUR IN RECEIVER PARTS

In this part we shall suppose ideal behaviour of receiver and receiver parts. This implies that the receiver is noise-free and the individual parts of the receiver have a sufficient large bandwidths to pass the signal without distortion. In addition, constant transfer factors and phase shifts within the BW are supposed.

Although the analysis using these assumptions will not yield exact result but it will be useful to simplify the analysis in order to obtain a form which can be compared with the result obtained from analysis of other CW receivers under the same assumptions and simplifications.

The signal at the output of the RF mixer was illustrated in equation (14). Therefore, the output of the code amplifier $A_1(t)$ under these assumptions will be the same as equation (14) with the exception of multiplication by factor k_1 and addition of phase shift θ_1. This signal will be multiplied in the code demodulator with the reference code. At this point it is useful to deal separatly with case of correct and case of incorrect choice of time delay of the reference code.

a. Case of correct determination of target range

In this case we suppose that the reference code applied to the code demodulator has the same form as given by relation (6) which implies the same doppler shift as given by velocity of target and the same phase shift due to target range. Thus the output $A_2(t)$ of the balanced code demodulator after filtering out ω_p and higher harmonics of ω_p has the form

$$A_2(t) = \frac{1}{2}k_1k_2UJ_o(B)\left[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty}C_m^2\right]\cos(\omega_d t+\psi+\theta_2\pm\theta_1)+$$
$$+\frac{1}{2}k_1k_2U\sum_{n=1}^{\infty}(-1)^nJ_n(B).\left[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty}C_m^2\right]$$
$$.\left[\cos\left\{(n\Omega+\omega_d)t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2\pm\theta_1\right\}+\right.$$
$$\left.+\cos\left\{(n\Omega-\omega_d)t-n\Omega\frac{t_1}{2}-\psi-\theta_2\mp\theta_1\right\}\right] \qquad (15)$$

This can be put in the form

$$A_2(t) = k_1k_2UJ_o(B)\left[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty}C_m^2\right]\cos\theta_1\cos(\omega_d t+\psi+\theta_2)+$$
$$+k_1k_2U\sum_{n=1}^{\infty}(-1)^nJ_n(B)\left[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty}C_m^2\right]\cos\theta_1$$
$$\left[\cos\left\{(n\Omega+\omega_d)t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2\right\}+\right.$$
$$\left.+\cos\left\{(n\Omega-\omega_d)t-n\Omega\frac{t_1}{2}-\psi-\theta_2\right\}\right] \qquad (16)$$

The factor $\left[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty}C_m^2\right]$ in the above equation is in fact the average power of the periodic P.r.b.s. denoted as P. This factor results from multiplication of two P.r.b.s.'s with the same shift which corresponds to maximum correlation and yields the maximum of the correlation function of the P.r.b.s. In fact this factor P is the value of the correlation function deduced from equation (3) and given by equation (28) when τ is substituted by zero.

The signal $A_3(t)$ which results from passing $A_2(t)$ in a selective circuit will be applied to the nth harmonic detector to which is applied also the corresponding nth harmonic of FM spectrum given as $u_2 = U_n\sin(n\Omega t+\varphi)$.
Thus the output will depend on whether only one doppler sideband component of equation (16) is used for detection of the doppler frequency or both nearest doppler sideband components to the nth harmonic are used. If only one component is used we have the doppler frequency signal at the output of the nth harmonic detector after filtering only the difference frequency

$$A_4'(t) = -\frac{1}{2}k_1k_2k_3k_4U.U_n.P(-1)^nJ_n(B)\cos\theta_1\sin\left\{\omega_d t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2+\theta_3+\theta_4-\varphi\right\} \qquad (17)$$

$$\text{or } A_4''(t) = \frac{1}{2}k_1k_2k_3k_4U.U_n.P(-1)^nJ_n(B).\cos\theta_1\sin\left\{\omega_d t+n\Omega\frac{t_1}{2}+\psi+\theta_2-\theta_3+\theta_4+\varphi\right\} \qquad (18)$$

In case of using the two sideband components in equation (16) for detection of the doppler frequency in a balanced detector instead of using only one, then the doppler signal at the output of the nth harmonic detector is

$$A_4(t) = k_1k_2k_3k_4U.U_n.P(-1)^nJ_n(B)\cos\theta_1.\sin\left(n\Omega\frac{t_1}{2}+\varphi-n\frac{\pi}{2}-\theta_3\right).\cos\left(\omega_d t+\psi+n\frac{\pi}{2}+\theta_2+\theta_4\right) \qquad (19)$$

When the signal A_4 is integrated in the corresponding filter of the doppler filter bank over one period of P.r.b.s. thus the average output of the receiver will be

$$A_5 = \frac{1}{T}\int_{-T/2}^{+T/2}A_4(t)\,dt \qquad (20)$$

This can be arranged in the form

$$A_5 = k.U.U_n.P(-1)^nJ_n(B).\cos\theta_1\sin\left(n\Omega\frac{t_1}{2}+\varphi-n\frac{\pi}{2}-\theta_3\right).$$
$$.\cos\left(\psi+n\frac{\pi}{2}+\theta_2+\theta_4\right).\left(\frac{\sin\omega_d T/2}{\omega_d T/2}\right) \qquad (21)$$

From this form of output the dependence on doppler frequency, the dependence on range through B, t_1, ψ and the factor u referring to range as given by the radar equation are clear. The effect of transfer factors and phase shifts of individual parts of the receiver is seen. It is interesting to measure amplitude spectrum at the output of the code demodulator using a spectrum analyzer for 2 cases of correct determination of target range (16). The first case is obtained using a cable of 106 m simulating a target at range 85 m in air and is shown in Fig.6. The second case is measured for target at zero range and is shown in Fig.9. It is clear from Fig.6 that the third harmonic of FM has the greatest amplitude compared with the other harmonics at this range while Fig.9 indicates nearly zero amplitude of the 3rd harmonic at zero range. These figures represent the experimental verification of equation (16).

b. Case of incorrect determination of target range

If we suppose that the reference code differs from relation (6) by a phase shift due to small shift from the correct target range while the doppler frequency shift is the same, thus the reference code takes the form

$$V(t-t_1) = \sum_{m=0}^{\infty} C_m \cos\left\{(m\omega_p - \omega_{dm})t + \varphi''_m\right\} \qquad (22)$$

Thus the output of the balanced code demodulator can be obtained with little manipulation and taking into consideration that $\phi' - \phi'' = -m\omega_p\tau$ where τ is delay due to shift in range and filtering out ω_p and harmonics of ω_p.

$$\begin{aligned} A_2(t) = {} & k_1k_2UJ_o(B)\cos\theta_1\Big\{\cos(\omega_d t+\psi+\theta_2)\cdot \\ & \Big[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\cos m\omega_p\tau\Big] + \sin(\omega_d t+\psi+\theta_2)\Big[\frac{1}{2}\sum C_m^2\sin m\omega_p\tau\Big]\Big\} \\ & + k_1k_2U\sum_{n=1}^{\infty}(-1)^nJ_n(B)\cos\theta_1\cdot\Big\{\Big[C_o^2+\frac{1}{2}\sum C_m^2\cos m\omega_p\tau\Big]\cdot \\ & \Big[\cos\Big\{(n\Omega+\omega_d)t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2\Big\} + \cos\Big\{(n\Omega-\omega_d)t- \\ & -n\Omega\frac{t_1}{2}-\psi-\theta_2\Big\}\Big] + \Big[\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\sin m\omega_p\tau\Big] \\ & \cdot\Big[\sin\Big\{(n\Omega+\omega_d)t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2\Big\} + \\ & + \sin\Big\{(n\Omega-\omega_d)t-n\Omega\frac{t_1}{2}-\psi-\theta_2\Big\}\Big]\Big\} \end{aligned} \qquad (23)$$

Then following the same procedure as in case a with the same denotations, thus the output of the nth harmonic detector is

$$\begin{aligned} A'_4(t) = {} & -\frac{1}{2}k_1k_2k_3k_4U.U_n(-1)^nJ_n(B)\cos\theta_1\Big\{\sin\Big\{\omega_d t-n\Omega\frac{t_1}{2}+ \\ & +\psi+n\pi+\theta_2+\theta_3+\theta_4-\varphi\Big\}\cdot\Big[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\cos m\omega_p\tau\Big] - \\ & - \cos\Big\{\omega_d t-n\Omega\frac{t_1}{2}+\psi+n\pi+\theta_2+\theta_3+\theta_4-\varphi\Big\}\cdot\Big[\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\sin m\omega_p\tau\Big]\Big\} \end{aligned} \qquad (24)$$

or

$$\begin{aligned} A''_4(t) = {} & \frac{1}{2}k_1k_2k_3k_4U.U_n(-1)^nJ_n(B)\cos\theta_1\Big\{\sin\Big\{\omega_d t+n\Omega\frac{t_1}{2}+ \\ & +\psi+\theta_2+\theta_4-\theta_3+\varphi\Big\}\cdot\Big[C_o^2+\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\cos m\omega_p\tau\Big] - \\ & - \cos\Big\{\omega_d t+n\Omega\frac{t_1}{2}+\psi+\theta_2+\theta_4-\theta_3+\varphi\Big\}\cdot\Big[\frac{1}{2}\sum_{m=1}^{\infty} C_m^2\sin m\omega_p\tau\Big]\Big\} \end{aligned} \qquad (25)$$

If two signal components are used for detection of the doppler frequency then with little arrangement and using trigonometric relations, the signal with doppler frequency is

$$\begin{aligned} A_4 = {} & k_1k_2k_3k_4U.U_n(-1)^nJ_n(B)\cos\theta_1.\cos(\omega_d t+\psi+n\frac{\pi}{2}+\theta_2+\theta_3)\cdot \\ & \Big[C_o^2\sin(n\Omega\frac{t_1}{2}-n\frac{\pi}{2}-\theta_3+\varphi)+\frac{1}{2}\sum_{m=1}^{\infty} C_m^2. \\ & .\sin(m\omega_p\tau+n\Omega\frac{t_1}{2}-\theta_3+\varphi-n\frac{\pi}{2})\Big] \end{aligned} \qquad (26)$$

The output of the receiver after integration in the corresponding doppler filter is

$$\begin{aligned} A_5 = {} & k.U.U_n(-1)^nJ_n(B)\cos\theta_1\Big[C_o^2.\sin(n\Omega\frac{t_1}{2}-n\frac{\pi}{2}-\theta_3+\varphi) + \\ & +\frac{1}{2}C_m^2\sin(m\omega_p\tau+n\Omega\frac{t_1}{2}-n\frac{\pi}{2}-\theta_3+\varphi)\Big]\left(\frac{\sin\omega_d T/2}{\omega_d T/2}\right) \end{aligned} \qquad (27)$$

It is clear that this equation is similar to equation (21) with the exception that the factor P which represents the value of the correlation function of the P.r.b.s. at maximum correlation is now replaced by the expresion between brackets which represents in fact the correlation function of the P.r.b.s. with some phase shift added to the components corresponding to target range. This shift is dependent on harmonic order of the FM spectrum selected in the receiver. The measurement of the amplitude spectrum at the output of the code demodulator in case of incorrect adjustement of the delay of the reference code for zero meter cable is shown in Fig.7 and for 106 m cable is shown in Fig.8. It is clear from Fig.7 that this spectrum in fact represents the spectrum of P.r.b.s. and this verifies experimentally the result obtained from equation (23). In that equation if we substitute $B = 0$, $\psi = 0$ for zero range and $\omega_d = 0$ for nonmoving simulated target, further if θ_2 is supposed equal zero then it results directly that $A_2(t)$ is proportional to the factor

$$C_o^2 + \frac{1}{2}\sum_{m=1}^{\infty} C_m^2 \cos m\omega_p\tau \qquad (28)$$

which is the correlation function of the P.r.b.s., Fig.8 represents also in this case the spectrum of P.r.b.s. but the amplitudes have been attenuated in the cable.

6. RECEIVER PROPERTIES

In this part we shall not deal with the properties of the individual parts of the receiver but we shall only illustrate the main properties of the whole receiver as given by its output and its doppler signal. The resolution and ambiguity properties have been illustrated based on equations (21) and (27) which were verified experimentally. The amplitude of the doppler frequency signal as given by equation (17), obtained as a result of mixing only one signal component of (16) with corresponding harmonic of f, is calculated as function of relative range with respect to R_1 where R_1 corresponds to R_{max}. This amplitude S_1 is calculated for different values of BWR which is the ratio of BW for the code to the optimum BW for one segment pulse of the code that is

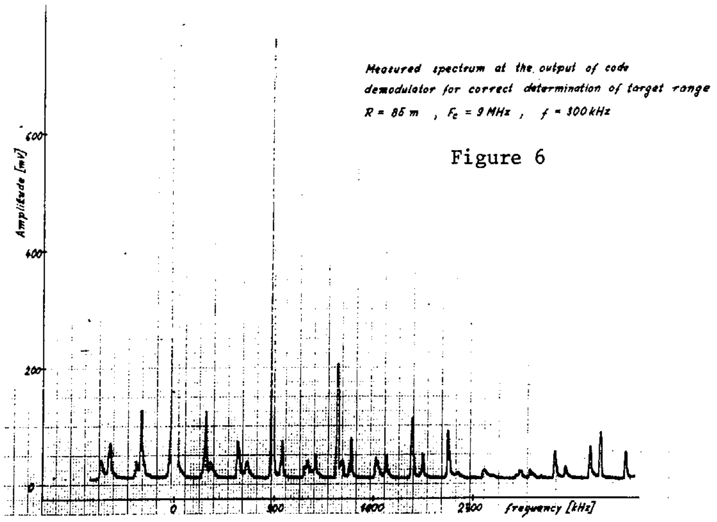
$$BWR = \frac{BW}{BW_{opt}} \quad \text{where } BW = m.\frac{F_c}{L},\ BW_{opt} = \frac{1{,}5}{\tau_o}$$

a. Dependence on range of doppler signal amplitude

The amplitude factor S_1 which is the relative value of $A'_4(t)$ as given by (17) is dependent on range through the bessel function of nth order and depends also on the factor P which in turn depends on the number of components in the Fourier series expansion of P.r.b.s. permitted to pass through the code amplifier. This number depends on the BW of the code amplifier. The dependence of S_1 on R/R_1 for 3 values of BWR is illustrated in Fig.10. From this figure is clear the dependence on range which satisfies the requirement on the receiver for leakage suppression as well as sensitivity range control.

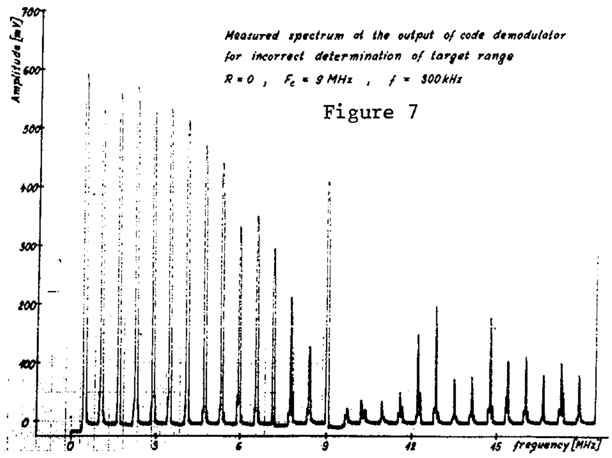
b. Effect of bandwidth on doppler signal

The dependence of S_1 for third harmonic on BWR is plotted in Fig.11. The figure shows a little increase of signal amplitude for values of BWR greater than 1 from which we conclude that the suitable BWR for the code amplifier in our case is 1 which corresponds to BW of code amplifier equal to optimum BW for one segment pulse of the code which implies the same requirement as for CW radar using only P.r.b.s.



Measured spectrum at the output of code demodulator for correct determination of target range
$R = 85\ m$, $F_c = 9\ MHz$, $f = 300\ kHz$

Figure 6

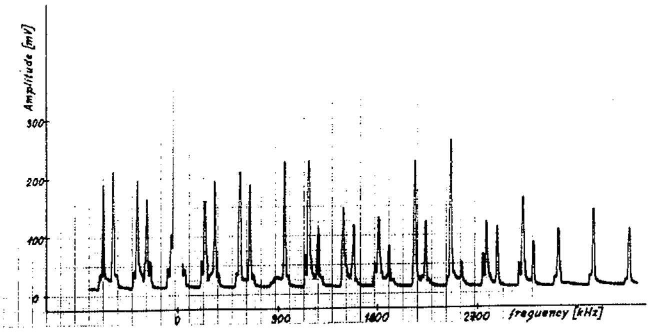


Measured spectrum at the output of code demodulator for incorrect determination of target range
$R = 0$, $F_c = 9\ MHz$, $f = 300\ kHz$

Figure 7

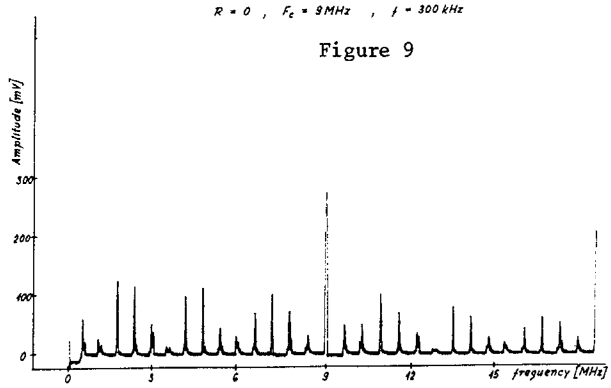
Measured spectrum at the output of code demodulator for incorrect determination of target range
$R = 85\ m$, $F_c = 9\ MHz$, $f = 300\ kHz$

Figure 8



Measured spectrum at the output of code demodulator for correct determination of target range
$R = 0$, $F_c = 9\ MHz$, $f = 300\ kHz$

Figure 9



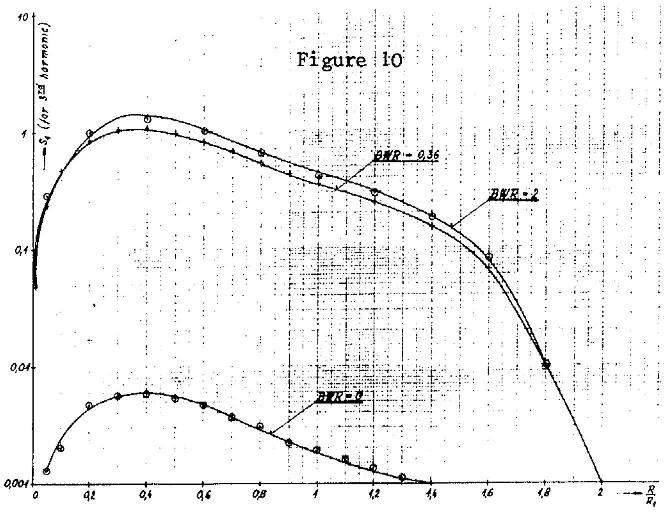


Figure 10

Dependence of doppler signal on BWR

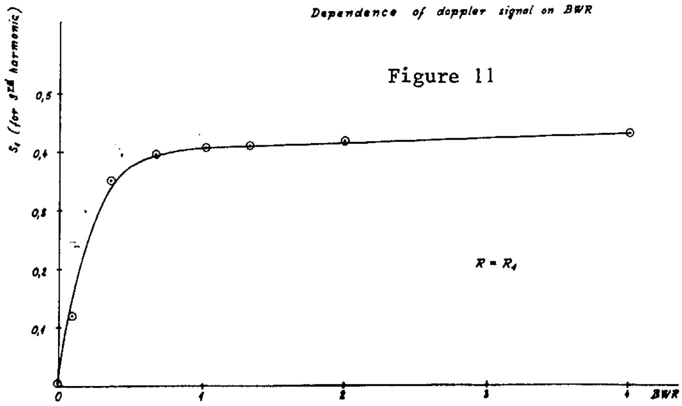


Figure 11

7. CONCLUSION

The analysis carried out in this paper reveals that :

1) Equation (27) represents the general form of the output of our proposed system while equation (21) can be deduced as a special case from equation (27).
2) It is clear from equation (27) that the behaviour of the output for any change of range given by τ around the correct target range is given by the behaviour of the correlation function of P.r.b.s. as illustrated by the expression between brackets.
3) It results from the behaviour illustrated in the previous point that the resolution in range is nearly the same as in system using P.r.b.s. only.
4) A conclusion from point 3 that this signal adds no new requirement on the BW of the system. Thus the BW requirement for system with only P.r.b.s. is also applicable here.
5) From the point of view of ambiguity in range it is clear that equation (27) for $\omega_d = 0$ behaves as the correlation function of P.r.b.s. which has no ambiguity in range between the repetitive peaks.
6) The dependence of the output on the nth order bessel function secures the suppression of leakage and provides the system with sensitivity range control.

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