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DECENTRALIZED DETECTION OF WEAK SIGNALS IN NON-GAUSSIAN NOISE†

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RESUME

Nous étudions ici le problème de la détection d'un signal en nous référant à des récepteurs géographiquement distants noyés dans des bruits non gaussiens.

La structure de détection envisagée consiste de quelques détecteurs périphériques et d'un processeur central qui, dans le but de semplifier l'implémentation, opère suivant une règle de décision OR ou AND.

La performance asymptotique (i.e. dans l'hypothèse d'un nombre élevé d'échantillons dans l'intervalle d'observation) des structures (OR et AND) proposées est évaluée dans le cas de la détection cohérente d'un signal faible à bande étroite affecté de fluctuations d'amplitude ("fading") lentes.

Les performances des structures proposées sont comparées entre elles et, de plus, avec celle de le détecteur optimal centralisé.

SUMMARY

With reference to geographically dispersed sensors, operating in non-Gaussian noise environments, the decentralized detection problem is considered.

For the sake of easy implementation the fusion center is assumed to work according to an OR or an AND scheme.

The asymptotic (i.e. for large sample size) performance of both proposed structures is evaluated with reference to the coherent detection of a weak bandpass signal subject to amplitude fluctuations.

A comparison between the performances of the decentralized structures (OR and AND) is made and, furthermore, the performances of such suboptimum detection scheme are compared with that of the optimum centralized detector.

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1. INTRODUCTION

In recent years an increasing attention has been devoted to the distributed sensor systems (e.g. multistatic radar) mainly because of the more and more severe requirements of the surveillance systems.

The classical detection theory, at least in principle, allows one to derive optimum detection structures provided that all sensor signals are available at some central location. However, even if it is possible to overcome the problems of implementation, the total centralization of information is never adopted in practice because of such considerations as cost, reliability, survivability, flood of the fusion center with more information than it can process and (particularly) communication bandwidth.

The decentralized detection structure of Fig.1, which consists of some peripheral detectors and of a fusion center that performs the global decision based on the local ones, can be proposed to overcome the above mentioned difficulties and, in particular, to reduce the bandwidth required for the communications between the local stations and the fusion center. Of course, such a detection strategy is suboptimum with respect to a completely centralized structure implementing the likelihood ratio test (LRT) for the observed data, owing to the loss of information due to the local signal processing.

In general in the optimum decentralized structure each local processor does not operate independently and, moreover, the synthesis is very difficult also for a small number of peripheral stations [1].

The interesting assumption that the input signals of the local detectors, conditioned to each hypothesis, are mutually independent leads [1] to a decentralized detection structure which consists of processors implementing local independent LRT's whose thresholds, however, are dependent of one another.

Another difficulty arises from the implementation of the local LRT that results easy only when the input signal is corrupted by Gaussian noise. Such an assumption, however, is not always justified because the noise generated by a variety of man-made and natural electromagnetic sources exhibits highly non-Gaussian characteristics [2-5], expecially at frequencies below 100 MHz. In these situations the LRT implementation of the peripheral detector can be reasonably achieved [6], under the weak-signal assumption [1], by expanding the likelihood ratio in power series about zero signal level and dropping the higher order terms.

In the present paper, on the reasonable assumption that the input signals of the local detectors, conditioned to each hypothesis, are mutually independent, a complexity reduction of the decentralized structure is reached by considering independent peripheral decisions (i.e., by not accounting for the above mentioned dependence among the thresholds of the local LRT's) by means of locally optimum detectors. In such a case the fusion center test, according to the Neyman-Pearson criterion, is based on a weighted sum of the binary outputs of the peripheral detectors. The fixed global false-alarm rate can be always achieved by choosing the local false-alarm probabilities (and, consequently, the thresholds of the peripheral detectors) such as to reduce the fusion center to an OR or an AND scheme whose easy implementation does not need the evaluation of weights (Section 2). Obviously the overall detection probability depends also of the fixed set of the local thresholds (chosen, as evidenced, to satisfy the fixed global false-alarm rate) that, in its turn, fixes the scheme in accordance with the fusion center operates.

In the paper we evaluate (Section 3) the performances of the proposed suboptimum structures (some independent local detectors and a fusion center operating according to an OR or an AND scheme) with reference to the coherent detection of a weak bandpass signal, affected by slow (in comparison with the observation interval) amplitude fluctuations, in narrowband non-Gaussian noise. A comparison between the performances of the OR and AND schemes is made and, furthermore, the performances of such decentralized structures are compared with that of the optimum centralized detector [9].

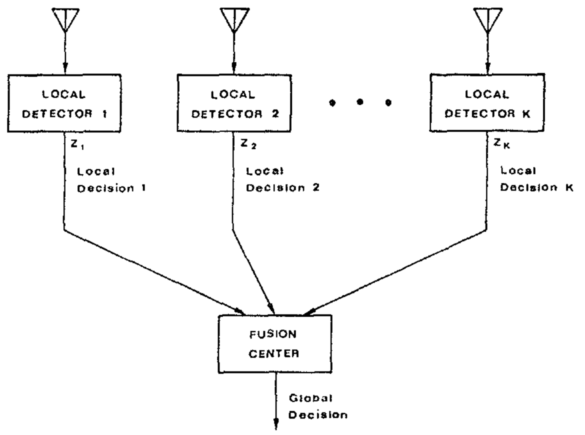

Fig.1 - Decentralized detection structure.

2. DECENTRALIZED DETECTION STRUCTURES

The decentralized detection structure we will consider consists (Fig.1) of K independent peripheral detectors implementing LRT's and a fusion center that, according to the Neyman-Pearson criterion, performs the global decision based on a set of binary observations Z_p ($p=1,2,\dots,K$) where

$$Z_p = \begin{cases} 0 & \text{if the pth detector chooses the hypothesis } H_0 \\ 1 & \text{if the pth detector chooses the hypothesis } H_1 \end{cases}$$

Under both hypotheses such random variables (RV's) Z_p are Bernoulli distributed with

$$\Pr(Z_p=1|H_0) = P_{Fp} \qquad (1)$$

$$\Pr(Z_p=1|H_1) = P_{Dp} \qquad (2)$$

where P_{Fp} is the false-alarm rate of the pth local detector and P_{Dp} the corresponding detection probability.

The reasonable assumption that the input signals of

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[1] Recently the weak-signal detection has received much attention [7,8] because, in order to realize low intercept probability and high antijamming capability, the communication system is frequently designed to minimize and disguise the transmitted signal by spreading it in time and frequency. On the other hand, usually it is only necessary to have a near-optimum detector for weak signals since strong signals will be detected in any case with satisfactory accuracy.

the peripheral processors, conditioned to each hypothesis, are mutually independent leads to the following test:

$$T = \sum_{p=1}^{K} w_p Z_p \underset{H_0}{\overset{H_1}{\gtrless}} t_h \qquad (3)$$

where t_h denotes the fusion center threshold and the nonnegative weights[2] w_p are given by

$$w_p = \ln\left(\frac{P_{Dp}}{1-P_{Dp}} \cdot \frac{1-P_{Fp}}{P_{Fp}}\right) \qquad (4)$$

As evident from eq.(4) each weight is an increasing function of the local detection probability and, therefore, the test statistic (3) emphasizes the most reliable peripheral decisions.

According to the Neyman-Pearson criterion, the threshold t_h of the test of the fusion center is to be chosen to achieve the desired overall false-alarm rate P_{FA}. On the other hand, for any fixed set of P_{Fp}, the P_{FA} can assume only a discrete number of values, as is evident, with reference, for example, to the case K=2, from Fig.2. However the required global false-alarm rate can be always assured by fixing an arbitrary threshold value that, in its turn, specifies the relationship between P_{FA} and the P_{Fp}'s. Such a relationship allows one to choose appropriately the values of P_{Fp}'s and, consequently, the thresholds of the local detectors[3].

The choise of the threshold t_h is therefore no longer conditioned by the P_{FA} constraint and can be always such that the fusion center test (3) becomes simply an OR or an AND decision scheme. More precisely:

$$0 < t_h < \min_p(w_p) \Rightarrow \text{OR}$$

$$\sum_{p=1}^{K} w_p - \min_p(w_p) < t_h < \sum_{p=1}^{K} w_p \Rightarrow \text{AND}$$

In this way a significant reduction of the detection complexity is obtainable in that both decision rules do not require the evaluation of the weights[4].

As regards, finally, the thresholds of the peripheral detectors, in order to gain further simplicity of implementation, it is advisable to set

$$P_{Fp} = P_F \qquad p=1,2,\ldots,K \qquad (5)$$

[2] In the following the case w_p=0 for some value of p will not be considered in that it occurs in the presence of a zero signal-to-noise ratio at the input of the local detector.

[3] Such an approach is closely related to that proposed in [10] with reference to multistatic radar systems operating in Gaussian noise environments.

[4] We note that in general it can occur for some integer L (1<L<K) that

$$\min(w_{i_1}+w_{i_2}+\ldots+w_{i_L}) > \max(w_{j_1}+w_{j_2}+\ldots+w_{j_{L-1}})$$

where $\{i_1, i_2, \ldots, i_L\}$ $[\{j_1, j_2, \ldots, j_{L-1}\}]$ are all possible combinations (without repetitions) among the first K integers, taken L [L-1] at a time (For the OR (L=1) and AND (L=K) schemes the above relation is always valid). In such a case the test (3) includes the decision rule based on the statement:"The fusion center decides H_1 if at least L local detectors decide H_1".

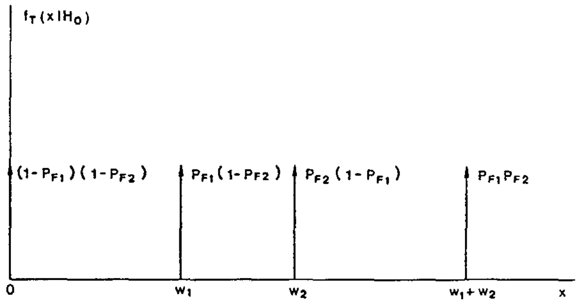

Fig.2 - Conditional probability density function of the decision variable of the fusion center test (with reference to the case of two peripheral detectors).

Obviously different values of P_F need to assure the desired global false-alarm probability according to scheme (OR or AND) adopted for the fusion center.

3. PERFORMANCE ANALYSIS

The overall performance of the proposed structures can be evaluated by the easily derivable relations

$$P_D^{OR} = 1-\prod_{p=1}^{K}(1-P_{Dp}^{OR}) \qquad (6)$$

$$P_{FA}^{OR} = 1-(1-P_F^{OR})^K \qquad (7)$$

$$P_D^{AND} = \prod_{p=1}^{K} P_{Dp}^{AND} \qquad (8)$$

$$P_{FA}^{AND} = (P_F^{AND})^K \qquad (9)$$

with obvious significance of the symbols.

It is useful to note that the global detection probability, for a fixed false-alarm rate, depends of the scheme (OR or AND) assumed for the fusion center and, moreover, that in general it is not possible to state which scheme performs better. In fact, for fixed noise environments and signal-to-noise ratios (SNR's) at the input of the peripheral detectors, it results that, for any given global false-alarm rate, $P_{Dp}^{AND} > P_{Dp}^{OR}$ for any value of p, but the relationships (expressed by eqs.(6) and (8)) between the global detection probability and the local ones, are such that it can occur $P_D^{OR} > P_D^{AND}$.

Equations (6)-(9) show that, in order to evaluate the overall detection rate for a fixed global false-alarm probability, it is only necessary to calculate the local detection probabilities that depend of the statistics of the received signals. The proposed structures of decentralized detection are here analysed with reference to the case of a weak bandpass signal, subject to slow amplitude fluctuations, in narrowband non-Gaussian noise.

With reference to the pth local detector the observables will be assumed to consist of the sequence of complex amplitudes[5] $\tilde{r}_{p1}, \tilde{r}_{p2}, \ldots, \tilde{r}_{pN}$ whose real and immaginary parts are obtained by sampling the outputs of the inphase and quadrature channel filters (respectively). The peripheral detection problem can be represented by the hypothesis test

[5] Complex quantities will be identified by a tilde.

$$H_0:\ \tilde{r}_{pi} = \tilde{n}_{pi}$$
$$H_1:\ \tilde{r}_{pi} = A_p\tilde{S}_i + \tilde{n}_{pi} \qquad i=1,2,\ldots,N \tag{10}$$

Here $\tilde{n}_{pi}$ is a complex noise amplitude obtained by sampling at the ith instant the inphase and quadrature components of the narrowband noise $n_p(t)$ at the detector input. $\tilde{S}_i$ denotes the ith sample of the complex envelope of the bandpass useful signal. The RV A_p, which assumes nonnegative real values, takes into account the presence of amplitude fluctuations in the pth channel. Since such fluctuations are supposed to be slow, A_p is random, but constant over the observation interval, so that it is independent of the index i.

Let us note that the hypothesis test stated above assumes coherent reception. In fact the considered complex envelopes are defined with respect to an assumed known carrier frequency and reference phase.

The practical implementation of the LRT for the peripheral detector is extremely difficult in non-Gaussian noise environment. Therefore, rather than maximizing the local detection probability for a fixed local false-alarm rate, one can maximize the slope of the power function at zero signal level while keeping a fixed false-alarm rate, obtaining so a locally optimum detector (LOD) which asymptotically approaches optimum performance as the signal becomes small.

By assuming that the joint probability density function (pdf) of the inphase and quadrature noise components possesses circular symmetry and by expanding the likelihood ratio in power series about zero signal level and dropping all terms of degree two and higher, the LOD test is given by [7,9]

$$T_p^{LOD} = \sum_{i=1}^{N} \mathrm{Re}\{\tilde{R}_{pi}\tilde{S}_i^*\} g_p(R_{pi}) \underset{H_0}{\overset{H_1}{\gtrless}} t_{hp} \tag{11}$$

where Re{·} and the asterisk denote the real part and the complex conjugation (respectively), R_{pi} is the envelope[6] of the ith observation and

$$g_p(R_{pi}) \triangleq -\frac{1}{R_{pi}}\frac{d}{dR_{pi}} \ln f_p(R_{pi}) \tag{12}$$

In eq.(12) $f_p(\cdot)$ denotes the joint pdf (possessing circular symmetry) of the inphase and quadrature components of each noise sample at the detector input.

The structure of the peripheral detector (implementing the LOD test) is illustrated in Fig.3. In the particular case of Gaussian noise it is easily established that

$$g_p(R_{pi}) = 1/\sigma_p^2 \tag{13}$$

where σ_p^2 is the common variance of the noise components. The structure of the local detector becomes in such a case (see eqs.(11) and (13)) the well-known linear one.

An exact evaluation of the detection probability of the peripheral detectors would generally require extensive numerical computation or computer simulation and, therefore, it is reasonable to resort to a performance estimation on the assumption of large sample size N. In fact, under the hypothesis H_1, the conditioning upon the RV A_p allows one to derive, via the

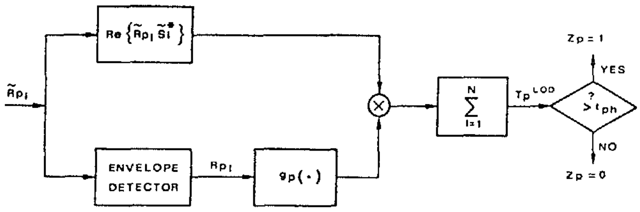

Fig.3 - Structure of the pth peripheral detector.

central limit theorem, the conditional detection probability and, then, to evaluate the detection rate of the local detector by

$$P_{Dp} = \int_0^{\infty} Q(\alpha - y)\, f_{Y_p}(y)\, dy \tag{14}$$

with

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-t^2/2)\, dt \tag{15}$$

In eq.(14) $Q(\alpha-y)$ represents the probability of detection conditioned to a fixed value of A_p, α is such that $Q(\alpha) = P_F$ and $f_{Y_p}(\cdot)$ is the pdf of

$$Y_p \triangleq \frac{\sqrt{\mathcal{E}\, ARE_p}}{\sigma_p} A_p \tag{16}$$

where

$$\mathcal{E} = \sum_{i=1}^{N} |\tilde{S}_i|^2 \tag{17}$$

provides a measure of signal energy and

$$ARE_p = \pi\sigma_p^2 \int_0^{\infty} R^3 g_p^2(R) f_p(R)\, dR \tag{18}$$

denotes the asymptotic relative efficiency [6,7] of the local detector under consideration with respect to the corresponding linear one ($ARE_p=1$ in the Gaussian noise case).

Numerical results and comments

In the following we present and discuss numerical results on the assumption that the RV's A_p, which take into account the amplitude fluctuations on the different channels, are Rayleigh distributed. In such a case from eq.(14), by integration by parts, one obtains that

$$P_{Dp} = P_F + G(\rho_p ARE_p)\exp\{-\frac{\alpha^2}{2}[1-G^2(\rho_p ARE_p)]\}Q[-\alpha G(\rho_p ARE_p)] \tag{19}$$

where

$$G(\rho_p ARE_p) \triangleq (1+2/\rho_p ARE_p)^{-1/2} \tag{20}$$

and, denoting by m_{2p} the second-order moment of the RV A_p,

$$\rho_p \triangleq \mathcal{E}\, m_{2p}/\sigma_p^2 \tag{21}$$

is the mean (over fading) SNR at the input of the pth peripheral detector.

Equation (19) shows that the same local detection probability is to be expected in different noise environments (i.e. for different values of the asymptotic relative efficiency) in correspondence of values of SNR such that $\rho_p ARE_p$=constant.

In order to make a comparison between the proposed

6 We are using R_{pi} to represent an envelope value and $\tilde{R}_{pi}$ to denote a complex amplitude.

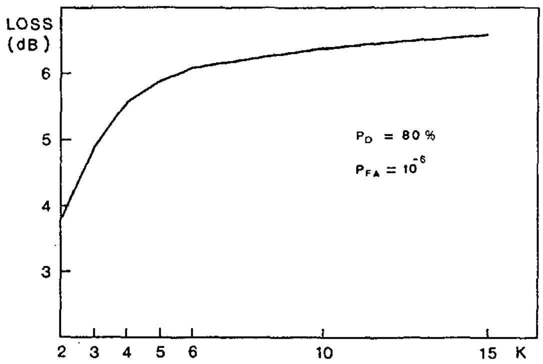

Fig.4 - Loss (in terms of ρARE) of the AND scheme with respect to the OR one versus number K of local detectors.

decentralized structures (in which the fusion center operates according to an OR or an AND scheme),we test, for a fixed pair of P_D and P_{FA} of practical interest, the case of K peripheral detectors having, in both structures, the product $\rho_p ARE_p$ independent of the local detector considered (i.e. the same for any value of p), say ρARE. Figure 4 shows the OR scheme performs significantly better than the AND one. More precisely it presents the loss (in decibels) in terms of ρARE of the AND scheme with respect to the OR one and, therefore, allows one to evaluate the incremental transmission power that needs in order to achieve by an AND decision rule the same performance as with the OR scheme.

Figure 5 presents, with reference to the OR scheme, the overall detection rate P_D as a function of ρARE (again the same value of $\rho_p ARE_p$ is assumed for all local detectors) in correspondence of $P_{FA} = 10^{-6}$ and with the number K of peripheral detectors in the role of a varying parameter. As K increases the curves are characterized by a more and more sharp increase of the performance with ρARE.

The performance degradation of the OR scheme with respect to the optimum (under the weak-signal assumption) centralized structure [9] is readily assessed, with reference to the case K=2, from the Fig.6. For both detection strategies it presents the contours in the plane (ρ_1,ρ_2) delimiting regions of operating conditions specified in terms of overall detection probability for a fixed global false-alarm rate, in correspondence of some values of ARE_1=ARE_2 (The straight line $\rho_1=\rho_2$ is an axis of symmetry for all curves reported, in that, on the assumption ARE_1=ARE_2, the roles of ρ_1 and ρ_2 are interchangeable). The figure shows that the performance degradation, in terms of power, is bounded to at most 1 dB or slightly more and becomes more and more negligible as the SNR over one of the channels approaches $-\infty$ (in such a limiting case the corresponding local detector is not operating).

Finally it is useful to note, with reference to both detection structures, that any curve of Fig.6 can be utilized to obtain the contour delimiting the regions of operating conditions concerning any pair of values of the asymptotic relative efficiency. In fact the coordinates (ρ_1',ρ_2') of any arbitrary point of the contour concerning a pair of values (ARE_1',ARE_2') are obtained by

$$\rho_i' = \rho_i ARE_i / ARE_i' \qquad i=1,2 \qquad (22)$$

where (ρ_1 ,ρ_2) are the coordinates of an arbitrary point belonging to the considered curve of Fig.6.

4. CONCLUSIONS

With reference to geographically dispersed sensors, operating in non-Gaussian noise environments, the decentralized detection problem is considered.

The proposed detection structures consist of peripheral detectors that perform independent local decisions and a fusion center, working according to simple schemes

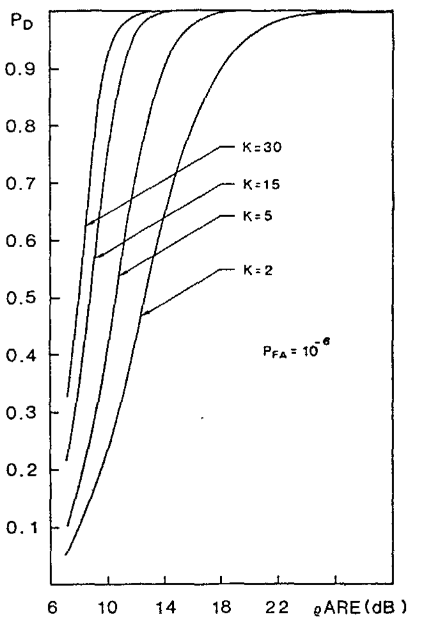

Fig.5 - Overall detection probability P_D for the OR scheme versus ρARE in correspondence of some values of K and a fixed global false-alarm rate P_{FA} .

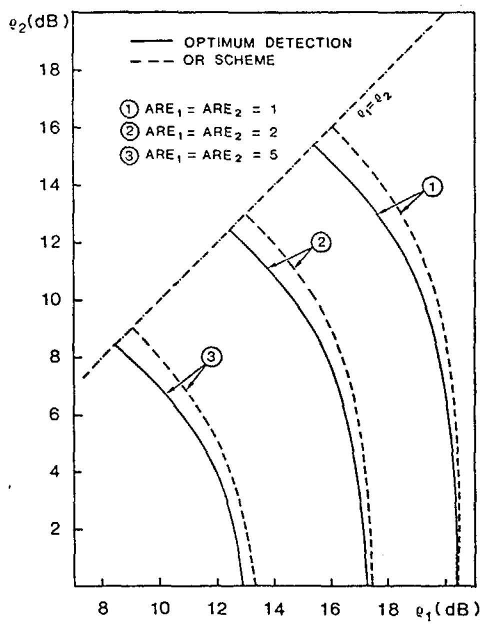

Fig.6 - Contours delimiting regions of operating conditions (with reference to the case of two peripheral sensors) that satisfy the requirements: $P_D \geq 80\%$, $P_{FA} = 10^{-6}$.

(OR or AND), that performs the global decision based on the local ones. The complexity of the peripheral detectors (operating in non-Gaussian noise), implementing likelihood ratio tests, is greatly reduced referring to structures that asymptotically approach optimum performance as the signal becomes small.

The performance evaluation of the considered decentralized structures would generally require extensive numerical computation or computer simulation. Therefore it is suitable to calculate the overall detection probability, for a fixed false-alarm rate, on the reasonable assumption of large sample size (asymptotic performance) that allows one to benefit by the central limit theorem.

With reference to the coherent detection of a weak bandpass signal, corrupted by narrowband non-Gaussian noise and subject to amplitude fluctuations, a comparison between the performances of the decentralized structures (whose fusion center works according to an OR or an AND scheme) is made. The OR scheme is shown to perform much better than the AND one in the operating conditions of practical interest. Moreover the OR structure represents, at least in the case of two peripheral sensors, a reasonable alternative to the optimum centralized detector, in that the performance loss due to decentralization is largely acceptable.

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