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DESIGN GUIDELINE FOR DESIRED BEAM RESPONSE IN THE PRESENCE OF RANDOM WEIGHTS, PHASE ERRORS, AND ELEMENT FAILURES

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RESUME

Dans des conditions idéales exemptes d'erreurs, la méthode normale d'obtention de la réponse désirée d'un faisceau d'une antenne à éléments périodiques consiste à pondérer et à retarder convenablement les données fournies par chaque hydrophone. En pratique, une fois l'antenne construite, on se trouve généralement en présence de poids aléatoires, d'erreurs de phase et de défaillances d'éléments. Ces erreurs aléatoires peuvent provenir de fluctuations du front d'onde, d'erreurs dans le placement des hydrophones, de différences entre les caractéristiques des hydrophones, d'erreurs sur la dynamique de l'amplificateur, les quantifications d'amplitude et de phase, et de l'opération de formation du faisceau. A l'analyse, on a découvert que ces erreurs imprévisibles causent une réduction du gain de faisceau, des erreurs dans la direction à donner au faisceau, et une augmentation du niveau moyen du lobe latéral qui est presque indépendant de la direction du braquage du faisceau. Des directives de conception pour obtenir le type de faisceau désiré à partir d'une antenne linéaire et d'une antenne plane en présence de poids aléatoires, d'erreurs de phase et de défaillances d'éléments, sont étudiées en termes de gain de faisceau, d'ouverture de faisceau et de niveaux moyen et de crête du lobe latéral.

SUMMARY

ABSTRACT

In ideal error-free conditions, a standard method for obtaining a desired beam response of a periodic array is to weight and delay the output of each hydrophone. In practice, after an array is built, it is found generally that random weights, phase errors, and element failures have occurred. These problems may originate in wavefront fluctuations, errors in hydrophone placement, different hydrophone characteristics, errors in amplifier dynamics, amplitude and phase quantizations, and beamforming operation. Analysis has shown that these random errors cause a reduction of beam gain, error in beam-pointing direction, and an increase in average sidelobe level that is almost independent of the beam-steering direction. A design guideline for a desired beam pattern of linear and planar arrays in the presence of random weights, phase errors, and element failures is discussed in terms of beam gain, beamwidth, and average and peak sidelobe levels.

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INTRODUCTION

Generally the output of each hydrophone in an array is passed through an analog channel consisting of a signal amplifier, filtering, and analog-to-digital conversion. Then the output of each channel is inserted into digital beamformers where it is appropriately delayed and summed to form beams.

In order to avoid false detections, it is desired that the sidelobe design level of a beam be much below that of the main beam response. The beam pattern of an array can be controlled by shading and controlling the element or hydrophone positions. In a periodic array, the hydrophone positions are fixed; thus, the only remaining control left to the designer is shading. In an aperiodic array, the element weights are usually made equal, and the only design freedom is in the location of the elements. However, we are concerned solely with a periodic array where the design freedom is limited to the shading or weights.

If all channels have identical gain and identical phase shift and if all digital delays are exact for the desired beam, the summed signal will provide a perfect beam pattern, as expected. In practice, each signal channel in the array will have imperfections; therefore, each output is not at its expected amplitude or phase. As a result, an array designed for a certain beam pattern will have one that is different from the expected pattern.

Random shading errors and phase errors will affect beam patterns, beam gain, and beam-pointing direction. The shading errors are introduced through errors in the weighting, variation in the hydrophone sensitivities, amplifier gain errors in the system, and quantization of the element shading coefficients. Phase errors are introduced by errors in the placement of array elements; variations of frequency response of hydrophones; and frequency characteristics of the filter, which is included in the signal channel, time quantization, and digital phase shifts or time delays. Random weight and phase effects also could be introduced owing to wavefront distortions. The most severe type of shading error is the failure of an element, which corresponds to a shading coefficient of zero.

Here our object is to provide a design guideline for a beam response of planar and linear arrays to achieve a specific beamwidth, directivity index, and control of desired average and peak sidelobes in the presence of random weight and phase errors, including element failures. First, in order to provide a design guideline, we shall investigate the effects of random errors on beamforming performance, which deals with various parameters such as beamwidth, beam gain, directivity index, beam-pointing errors, and average and peak sidelobe levels (SLL's). Second, the effects of random errors are discussed in terms of design parameters. Also, the effects of random errors on beamforming parameters are compared with simulated results whenever possible.

Here all derivations and discussions focus on the basis of a planar array; however, the results are also valid for a linear array, assuming the planar array is a multiplication of two linear arrays.

EFFECTS OF RANDOM ERRORS

The beam patterns of a periodic planar array of m x n elements in an xy plane, with spacing dx in the x-direction and dy in the y-direction between elements, can be written as

$$U(f,\theta,\phi,\theta_\ell,\phi_\ell) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}' \exp\Big[j\frac{2\pi f}{c}\cdot m\cdot dx\,(\sin\theta\cos\phi - \sin\theta_\ell\cos\phi_\ell) + j\frac{2\pi f}{c}\cdot n\cdot dy\,(\sin\theta\sin\phi - \sin\theta_\ell\sin\phi_\ell)\Big], \quad (1)$$

where

W_{mn}' is the actual shading coefficient at the mn-th element of the array,

θ and ϕ are the polar and azimuthal angles of the signal,

θ_ℓ and ϕ_ℓ are the look directions of the array beamformer,

f is the frequency of the incoming signal, and

c is the speed of sound in water.

The (power) beam pattern is obtained by multiplying equation (1) by its complex conjugate. The shading coefficient W_{mn}' is the actual shading coefficient at the mn-th element and is related to the designed (error-free) shading coefficient W_{mn} as follows:

$$W_{mn}' = W_{mn}\,\alpha_{mn}\,(1+\Delta_{mn})\,e^{j\delta_{mn}}, \quad (2)$$

where

Δ_{mn} is the fractional error in the weight at the mn-th element and

δ_{mn} is the error in the phase at the mn-th element.

The average value of the phase error and weight error are assumed to be zero. At any element the phase and amplitude errors are taken to be independent of the errors in any other element. Factor α_{mn} accounts for missing elements, such as might be caused by element failure, and has the value of unity with probability P_e and the value zero with the probability $(1 - P_e)$. Thus, the probability of element mn being operative is designated P_e; this probability is assumed to be independent of the location of the element within the array. Also, P_e is equal to the average fractional number of elements that remain operative.

It can be shown that the average power pattern[1-5] is

$$\overline{|U(f,\theta,\phi,\theta_\ell,\phi_\ell)|^2} = P_e^2\, e^{-\overline{\delta^2}}\, |U_o(f,\theta,\phi,\theta_\ell,\phi_\ell)|^2 + \Big[(1+\overline{\Delta^2})\,P_e - P_e^2\, e^{-\overline{\delta^2}}\Big]\sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}^2 \quad (3)$$

where

$\overline{\delta^2}$ and $\overline{\Delta^2}$ are the variances of phase and weight errors, respectively,

W_{mn} is the designed weight, and

$U_o(f,\theta,\phi,\theta_\ell,\phi_\ell)$ is the error-free designed beampattern.

We also have assumed that the phase errors have a Gaussian distribution.

Equation (3) may be simplified further to obtain the normalized average beam patterns:[6]

$$\overline{|U_n(f,\theta,\phi,\theta_\ell,\phi_\ell)|^2} = P_e^2\, e^{-\overline{\delta^2}}\, |U_{on}(f,\theta,\phi,\theta_\ell,\phi_\ell)|^2 + \Big[(1+\overline{\Delta^2})\,Pe - P_e^2\, e^{-\overline{\delta^2}}\Big]\frac{\sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}^2}{\left(\sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}\right)^2}$$

$$= P_e^2\, e^{-\overline{\delta^2}}\, |U_{on}(f,\theta,\phi,\theta_\ell,\phi_\ell)|^2 + \Big[(1+\overline{\Delta^2})\,P_e - P_e^2\, e^{-\overline{\delta^2}}\Big]\Big/L_{effective}, \quad (4)$$

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where

$$L_{effective} = \frac{\left(\sum_{m=1}^{M} W_m\right)^2}{\sum_{m=1}^{M} W_m^2} \cdot \frac{\left(\sum_{n=1}^{N} W_n\right)^2}{\sum_{n=1}^{N} W_n^2} = M_{effective} \times N_{effective} , \; W_{mn} = W_m \cdot W_n \quad \text{and}$$

$[U_{on}(f,\theta,\phi,\theta_\ell,\phi_\ell)]^2$ is the normalized error-free designed beam pattern.

In practice, because the average power pattern may never be realized, sufficient information about the behavior of the SLL's may not be provided. One way to look at the variability of the power pattern, especially in the deep sidelobe region, is to focus attention on the peak SLL, which is one of the central issues associated with practical beam-response design in the presence of random weight and phase fluctuations. Generally, the error-free SLL decreases with increasing angles away from the main lobe. In this deep region, the statistical SLL due to random errors eventually will dominate the error-free SLL. In practice, when designing a beam response of an array in the presence of random errors, the peak SLL should not be over a specified level in order to avoid false target detections.

It can be shown that the ratio of the peak (power) sidelobe (X_o) to average SLL (μ) is[7]

$$\frac{X_o}{\mu} = -\log_e(1 - \beta^{1/n}) \cong \log_e n - \log_e \log_e \frac{1}{\beta} \; ; \; \beta \geq 0.5 \quad , \qquad (5)$$

where

n is equal to the number of points in the sidelobe region and

β is the confidence level.

Now the question arises as to how n is determined from an array. One reasonable way to resolve this problem is to determine the number of sidelobes in the error-free design because it is more likely for the peak sidelobe to occur on the lobes than in the valleys. Notice that the ratio of peak to average SLL's depends only on the number of points n and confidence level β and is independent of the beam steering and element weightings.

Skolnik[5] has shown that the ratio of the error-free directivity index to the directivity index with random errors, when the elements are placed a half-wavelength apart, is given by

$$\frac{DI}{DI_o} \cong \frac{P_e}{1 + \overline{\Delta^2} + \overline{\delta^2}} \quad . \qquad (6)$$

Observe that the reduction of the directivity index due to random errors is only a function of the random errors and is independent of the array size and weights. We have found that effects of random errors on beamwidth and beam-pointing errors are negligible provided $P_e \approx 1$, $\sqrt{\overline{\Delta^2}} < 0.1$, and $\sqrt{\overline{\delta^2}} < 0.15$ radian.

DESIGN GUIDELINE WITH ANALYTICAL AND SIMULATED RESULTS

One way to provide a design guideline for a desired beam pattern, beam gain, and beamwidth of an array can be done by examples, then discussion, and, finally, design guidelines. We shall provide two examples: first, with moderate sidelobe suppression (-25 dB) and, second, with heavy sidelobe suppression (-40 dB).[6]

We start with the first example of a planar array that meets the desired error-free beamwidth and beam-gain. The array has 448 elements (32 elements in the x-direction and 14 elements in the y-direction) that are placed a half-wavelength apart. This array is weighted with Taylor shading in both the x and y directions. Further, we have assumed that the first five sidelobes in the x-direction and the first three sidelobes in the y-direction must not be higher than -25 dB compared to the main lobe.

Figure 1a shows a beam pattern when the beam is steered to broadside; i.e., the polar look (θ_ℓ) and azimuthal look (ϕ_ℓ) angles are both 0 deg. This is a beam response where the signal-arrival angle θ varies from 0 to 90 deg, and ϕ = 0 deg. The effects of random phase errors, weight errors, and element failures are included in the plotted beam pattern.

There are two curves in figure 1a, as indicated by "ideal" and "mean." The ideal curve shows the beam pattern using equation (4), with $P_e = 1$, $\overline{\Delta^2} = 0$, and $\overline{\delta^2} = 0$, i.e., the ideal error-free beam pattern. The mean curve represents an average beam pattern (using equation (4)), with $P_e = 0.9$, $\sigma_a = \sqrt{\overline{\Delta^2}} = 0.1$, and $\sigma_\phi = \sqrt{\overline{\delta^2}} = 0.15$ radian.

Figure 1b is similar to figure 1a, except that 1a is derived from analytical results (equation (4)) and 1b is an average of 20 simulated beam patterns that contain random weight and phase errors, with $\sigma_a = 0.1$ and $\sigma_\phi = 0.15$ radian and also 45 elements of the array (total 448 elements) randomly shorted or set to zero. These 45 elements represent about 10 percent of the total array elements. In addition, figure 1b has two curves that represent the ideal and average beam patterns.

Now let us consider the second example, which shows the effects of random errors on a heavily shaded array. Figures 2a and 2b were obtained from analytic (equation (4)) results and simulation, where the array is shaded for a -40-dB SLL compared to the main lobe.

Equations (4), (5), and (6) provide the design guideline in the presence of random errors. Specifically, equation (4) shows an average (power) normalized beam pattern that includes the effects of random errors such as weight and phase and element failures as a superposition of two terms. The first term contains two factors: one factor is the error-free normalized beam pattern, and the other factor depends on the variance of phase error and the fraction of the elements remaining operative. Therefore, the first term is a normalized error-free beam pattern multiplied by a scale factor that is proportional to the square of the fraction of elements remaining operative and proportional to the variance of phase error. This scale factor is always equal to or less than unity, depending on the errors. Observe that the first term reduces to the error-free normalized beam pattern when the phase error is zero and there are no element failures. The second term of equation (4) depends on two factors: one factor depends on the random errors such as weight and phase and element failures, and the other depends on the effective number of elements. Notice that the second term of equation (4) is independent of the angular coordinates; i.e., the second term is a bias (or pedestal) and independent of beam direction. The effective number of elements depends on the design weights and the actual number of elements used in the array. Observe that the effective number of elements, $L_{effective}$, is M x N when no weights (i.e., uniform weights) are used in the design. The value of $L_{effective}$ depends on the weights used in the design. Therefore, for a constant-error tolerance and for a fixed-array size with M x N elements, the bias is minimum when all weights are equal. However, the bias due to random errors will decrease for a larger array, provided that the error tolerance is constant.

It is evident from equation (4) that all directional properties are in the first term, which contains the normalized error-free beam pattern. Further, it shows that if the bias is negligible the shape of the average normalized power pattern is unchanged from that of the error-free power pattern. Therefore, the beamwidth of the main lobe remains unchanged. The dominant effect is a reduction in gain due to the scale factor applied to the first term of equation (4).

In practice, the system (which includes all signal channels) will not be free of imperfections nor will the wavefront be distortion-free. Therefore, all the random-phase errors can be "lumped" together, as are the weight errors and element failures.

In order to design a specified SLL compared to the main lobe, one must consider the effect of the bias term $[(1 + \overline{\Delta^2})\, P_e - P_e^2\, e^{-\delta^2}]/L_{effective}$ in the average beam pattern. If this bias term is negligible compared to the design SLL, then the design SLL will dominate and the bias term will have a negligible effect on the beam pattern, except perhaps in the deep sidelobe region. However, if the bias level is comparable with the design SLL, then the bias term will dominate, and it will increase the SLL. Therefore, in order to achieve a good design SLL, one should design the weights in such a fashion that the design SLL must be much less than the bias term (design SLL >> $\{[(1 + \overline{\Delta^2})\, P_e - P_e^2\, e^{-\delta^2}\}/L_{effective}]$.

For the periodic array, the average SLL due to random errors is about -34.97 dB, which is about 10 dB less than the designed SLL.

In order to see the effect of random errors on the peak SLL, we have plotted the difference of peak SLL and average SLL in decibels as a function of n for various confidence levels (β = 0.5, 0.7, and 0.9), as shown in figure 3. Equation (5) is utilized to obtain figure 3, which shows that the difference between peak and average SLL's for this particular array is about 8 dB (assuming n ≅ 200, β = 0.7). Four of the 20 different realizations of the simulated beam pattern, which we utilized to obtain the average simulated beam patterns, with σ_a = 0.1, σ_ϕ = 0.15 radian, and -25 dB sidelobe suppression, are shown in figure 4. Figure 5 is similar to 4; the only difference is that figure 5 shows a -40 dB design sidelobe suppression. In all cases, we observed the peak SLL's to be about -26 dB, whereas calculation shows it is to be about -27 dB (-35 dB + 8 dB).

Calculation of the reduction of the directivity index (equation (6)) indicates it to be about -0.6 dB, whereas it is observed to be -0.8 dB.

Figures 1a, 1b, and 4 show that if the array is shaded moderately (-25 dB), then the effects of random errors, with σ_a = 0.10, σ_ϕ = 0.15 radian, and P_e = 0.9, on beam gain, beamwidth, average and peak SLL's are tolerable. However, if the array is shaded heavily (-40-dB SLL), the effects of random errors on array parameters are dramatic, especially on average and peak SLL's, as shown in figures 2a, 2b, and 5. These figures show that the random errors must be reduced to meet the design goal or a tradeoff between design goal and tolerable random errors be made. An increase in array size will decrease the average and peak SLL, but changing design weights would not do very much. (See equation (4).)

CONCLUSION

Here we have attempted to provide a design guide for the passive beamforming performance of planar and linear arrays to (1) achieve a desired beamwidth and directivity index and (2) minimize beam-pointing errors, average and peak sidelobe suppression in the presence of random weight, phase errors, and element failures. To accomplish this, first, we investigated the effects of random errors on the parameters such as beamwidth, directivity index, beam-pointing direction, average and peak SLL's. We found that the effects of random errors on the beam pattern main lobe and in the neighborhood of the main lobe are negligible, provided the random errors are not excessive. Therefore, beamwidth is essentially unchanged and beam-pointing error is negligible. As an example, the beam-pointing error is about 2 percent of the 3-dB beamwidth, with σ_a = 0.1 and σ_ϕ = 0.15 radian.

However, the effects of random errors on the average and peak SLL's and directivity index are critical. We have given an example and found that if the peak SLL is comparable with the error-free designed SLL, then we can not expect that the SLL will be close to the desired level. The loss of directivity index depends only on the random errors and is about -0.6 dB for P_e = 0.9, σ_a = 0.1, and σ_ϕ = 0.15 radian. Also, it is independent of the weights and total number of elements.

In order to achieve the desired beamforming performance of a periodic planar array, the following procedure is helpful:

- Design the element spacings dx, dy, number of elements, and weights to achieve a desired error-free beam pattern with specific beamwidth, directivity index, and SLL.
- Locate all possible sources of random weight, phase errors, and possible number of element failures.
- Combine all weight and phase errors and estimate σ_a, σ_ϕ, and P_e.
- Estimate average SLL due to random errors (second part of equation (4)), which is a function of weights, number of total elements used in the array, standard deviations of weight and phase errors (σ_a, σ_ϕ), and a fraction of elements operating (P_e).
- Estimate peak SLL, which is a function of average SLL, confidence level, and number of points in sidelobe region n (equation (5) or figure 3).
- Estimate directivity index (DI) (equation (6)) and beam-pointing error.
- Compare peak SLL with error-free designed SLL; if they are comparable, i.e., the average SLL is much less than the designed SLL, then the goal is accomplished. Also, compare the estimated directivity with desired directivity.
- Reduce the amount of errors whenever possible if the values of DI and peak sidelobe level are not comparable. However, one may improve performance by increasing the size of the array and using new sets of weights. In other words, tradeoffs among σ_a, σ_ϕ, P_e, M • N, and W_{mn} are necessary to achieve a tradeoff among DI, average and peak SLL's, and beam-pointing error.

REFERENCES

1. R. E. Collins and F. D. Zucker, Antenna Theory, McGraw Hill Book Co., Inc., New York, 1969, pp. 220-232.

2. A. H. Quazi and A. H. Nuttall, "Effects of Random Shading, Phasing Errors, and Element Failures on Beam Patterns of Line and Planar Arrays," 1979 IEEE ICASSP Proceedings, 2-4 April 1979, pp. 290-293.

3. A. H. Quazi, "Effects of Random Weight, Phase Errors, and Element Failures on Beam Pattern of a Planar Array, Using Different Types of Weighting," NUSC Technical Memorandum No. 801023, Naval Underwater Systems Center, New London, CT, 31 January 1980.

4. A. H. Nuttall, Effects of Random Shading, Phasing Errors, and Element Failures on Beam Patterns of Linear and Planar Arrays, NUSC Technical Report 6191, Naval Underwater Systems Center, New London, CT, 30 October 1979.

5. M. I. Skolnik, Introduction to Radar Systems, McGraw Hill Book Co., Inc., New York, 1980, p. 319.

6. A. H. Quazi, "Comparison of Simulated and Analytical Results of the Effects of Random Shading, Phasing Errors, and Element Failures on Beam Gain and Beam Pattern of a Planar Array," NUSC Technical Memorandum No. 801153, Naval Underwater Systems Center, New London, CT, 24 September 1980.

7. D. T. Porter, "Effects of Random Phase Errors on Sidelobe Suppression of Heavily Shaded Arrays," NUSC Technical Memorandum No. 781225, Naval Underwater Systems Center, New London, CT, 22 December 1978.

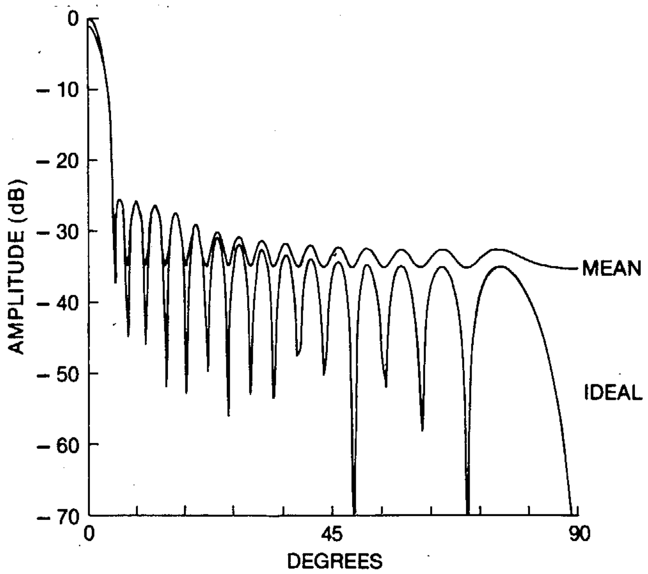

Fig. 1a. Beam pattern of a planar array with 448 elements. Designed sidelobe suppression -25 dB; dx = dy = half wavelength; σ_a = 0.10, σ_ϕ = 0.15 radian, and P_e = 0.9; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.

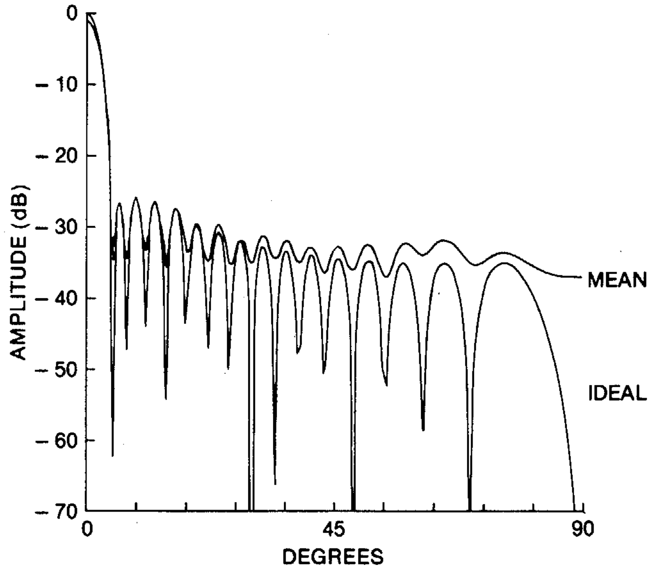

Fig. 1b. Simulated average beam pattern of a planar array with 448 elements. Designed sidelobe suppression -25 dB; dx = dy = half wavelength; σ_a = 0.10, σ_ϕ = 0.15 radian, and 45 elements inoperative; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.

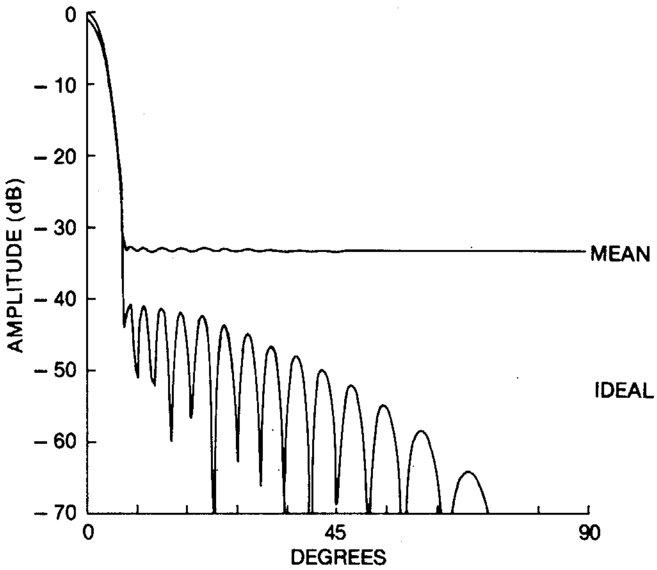

Fig. 2a. Beam pattern of a planar array with 448 elements. Designed sidelobe suppression -40 dB; dx = dy = half wavelength; σ_a = 0.10, σ_ϕ = 0.15 radian, and P_e = 0.9; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.

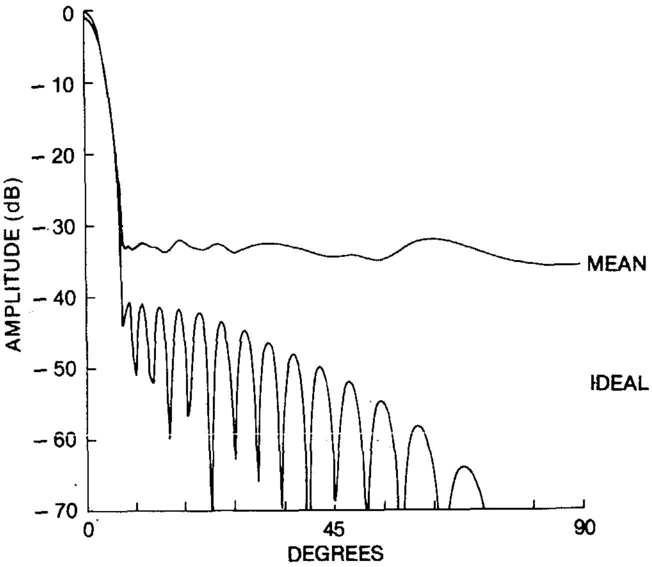

Fig. 2b. Simulated average beam pattern of a planar array with 448 elements. Designed sidelobe suppression -40 dB; dx = dy = half wavelength; $\sigma_a = 0.10$, $\sigma_\phi = 0.15$ radian, and 45 elements inoperative; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.

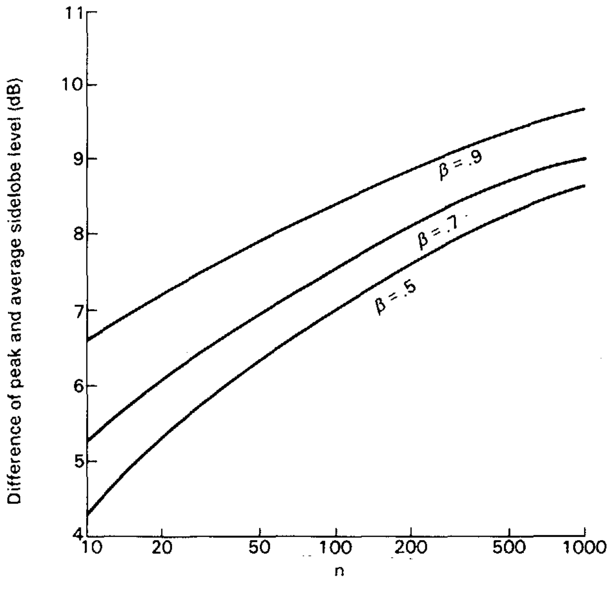

Fig. 3. Difference of peak and average sidelobe level in decibels versus number of points in sidelobe region, n.

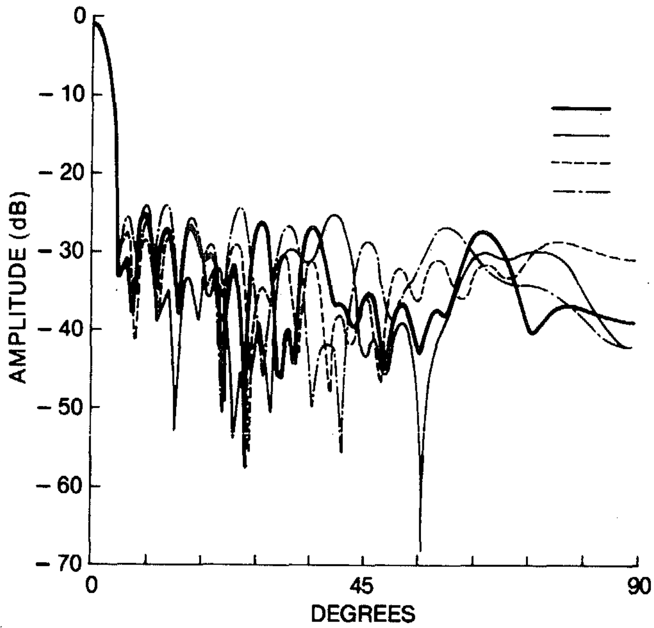

Fig. 4. Four different realizations of a simulated beam pattern of a planar array with 448 elements. Designed sidelobe suppression -25 dB; dx = dy = half wavelength; $\sigma_a = 0.10$, $\sigma_\phi = 0.15$ radian, and 45 elements inoperative; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.

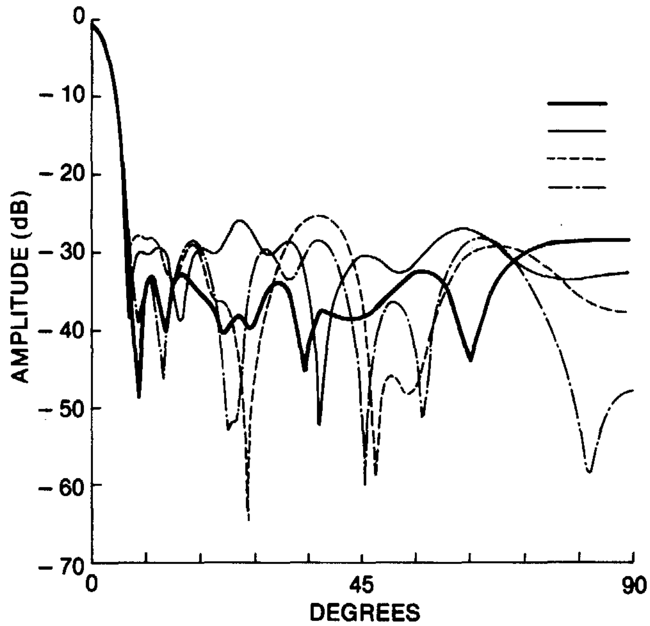

Fig. 5. Four different relizations of a simulated beam pattern of a planar array with 448 elements. Designed sidelobe suppression -40 dB; dx = dy = half wavelength; $\sigma_a = 0.10$, $\sigma_\phi = 0.15$ radian, and 45 elements inoperative; $\theta_\ell = \phi_\ell = \phi = 0°$; θ varies 0° to 90°.