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THE ORTHOGONAL EXPANSION METHOD FOR SOLVING ELECTROMAGNETIC SCATTERING PROBLEMS IN FINITE MILLIMETER-WAVE STRUCTURES

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RESUME

Ce document étudie numériquement et en partie avec des mesures l'onde électromagnétique répartissant des guides d'ondes rectangulaires et homogènes vers les guides d'ondes avec une section transversale non homogène. Ces transitions jouent un rôle important dans la recent développement de circuits à onde-millimètre dans le domaine des satellites de communication.

La méthode de calculation basée sur l'expansion de champs dans des modes propres orthogonaux appropriés est introduite avec plus de détail pour l'exemple de la transition directe d'un guide d'ondes homogène vers une microligne écrannée. Ce traitement peut être considéré comme général dans ce sens qu'il faut prendre en considération tous les six composants de champs et les caractéristigues des sections transversales avec des plaques diélectriques.

Pour la transition de guide d'ondes vers la microligne il est montré que la plupart de la puissance est transmise par le mode de premier ordre hybride et non par le mode fondamental de la microligne communément désiré. Cela indique qu'une jonction directe des guides d'ondes avec les microlignes n'est pas appropriée.

Les structures de plaque diélectrique montées dans le plan électrique des guides d'ondes rectangulaires sont très appropriées pour des applications d'ondes-millimètre. Ceci est demontré par les exemples de transition de guide d'ondes vers un guide d'ondes avec une plaque diélectrique et vers une ligne "ailée" ("fin-line") où la plaque diélectrique est partiellement métallisée. La structure de cette ligne est très propre aux circuits integrés des ondes-millimètre. On a dessiné et calculé un filtre avec une ligne "ailée" à neuf sections comme exemple pour 33,7 GHz qui montre une perte d'insertion mesurée d'environ 0,5 d. B. seulement.

SUMMARY

This paper investigates numerically and partially by measurements the electromagnetic wave scattering at transitions from homogeneous rectangular waveguides to waveguides with an inhomogeneous cross-section. Such transitions play a significant role in the recent development of suitable millimeter-wave communication circuits.

The calculation method, which is based on the field expansion into suitable orthogonal eigenmodes, is introduced in more detail for the example of the direct transition from a homogeneous waveguide to a shielded microstrip line. This treatment can be regarded as general in the sense that all six possible field components have to be considered and the features of dielectric slab-filled cross sections are taken into account.

For the investigated waveguide transition to the micro-strip-line it is shown that the principal part of the transmitted power is transported by the first order hybrid mode and not by the commonly desired fundamental microstrip mode. This indicates that a direct waveguide instrumentation of microstrip lines is inappropriate.

Dielectric-slab structures, however, mounted in the E-plane of rectangular waveguides are very suitable for millimeter wave applications. This is demonstrated by the examples of the waveguide transition to a dielec-tric slab filled waveguide and to a fin-line, where the dielectric slab is partially metallized. The fin-line structure is very suitable for designing milli-meter-wave integrated circuits. A nine step fin-line filter is designed and calculated as an example for 33.7 GHz which exhibits a measured insertion loss of only about 0.5 dB.

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1. INTRODUCTION

Recent studies, e.g.[1,2], have indicated that there will be a significant increase in satellite communications in the near future. To overcome the problem of spectral crowding, efforts are being made to expand the communications services upward to the millimeter-wave region of the spectrum. Since the millimeter-wave systems are based on waveguide instrumentation, waveguide scattering problems find increasing interest.

Further, because of their known advantages in the lower frequency range, particularly compactness and lower price, there is a growing demand in the actual satellite communication research activities for suitable millimeter-wave integrated circuit (MIC) structures. The ideal transmission line type for MIC's for millimeter-wave application is one which avoids excessive miniaturization, is suitable for waveguide instrumentation, and yet offers the potential for low cost production through batch-processing techniques. The fin-line and the dielectric image line (Fig. 1) are such line types. However, there is still a paucity of suitable theoretical research on such structures. So far only experimental design data and first order design theories are available, e.g.[3,4], especially concerning three-dimensional problems.

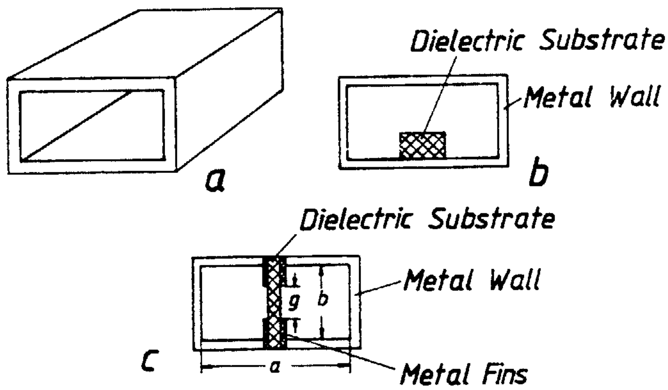

Fig. 1 Millimeter-Wave Structures
a Rectangular Waveguide
b Shielded Dielectric Image Line
c Fin-Line

The purpose of this paper is therefore to show that the orthogonal expansion method, principially introduced by Whinnery and Jamieson[5] is an appropriate numerical method for the rigorous solution of millimeter-wave structure discontinuity scattering problems. An infinite set of linear equations for the amplitudes of the reflected and transmitted modes for any incident mode can be obtained by exactly satisfying the boundary conditions. This theory takes into account the fact that higher mode propagation plays a significant role in the effect of those scattering problems.

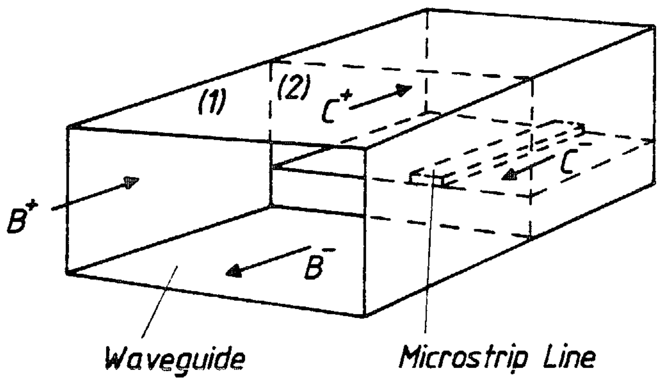

Fig. 2 Transition from a Rectangular Waveguide to a Microstrip Line

The first example investigated is the direct transition from a rectangular waveguide to a microstrip line (Fig. 2). It is shown, that a significant higher order mode propagation is caused at the discontinuity.

The transition from a waveguide to a dielectric loaded waveguide (Fig. 3), the second example, represents the first step towards the investigation of the transition to a dielectric image line (Fig. 1). The H_{10}-wave of the empty waveguide is transmitted almost undisturbed into the dielectric loaded part thus showing the possibility of a direct waveguide instrumentation. This is in clear contrast to the microstrip transition (Fig. 2), where the H_{10}-mode is considerably disturbed, so that suitably tapered transitions are required.

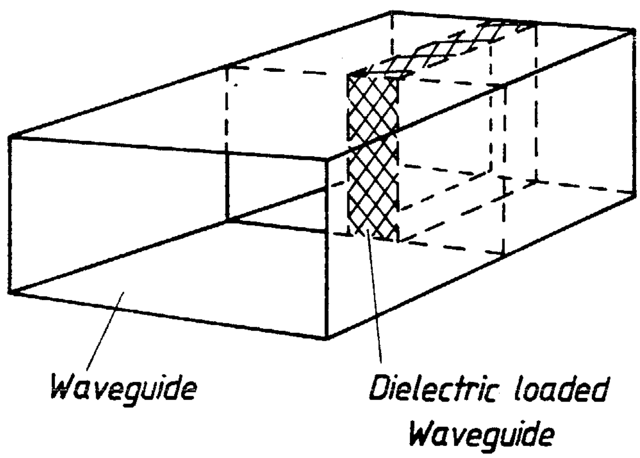

Fig. 3 Transition from a Rectangular Waveguide to a E-plane Dielectric Loaded Waveguide

In a fin-line structure, metal inserts ("fins") are printed on a dielectric substrate which bridges the broad walls of a rectangular waveguide (Fig. 1c, Fig. 4). The fin-line combined with RT/duroid substrate material is very suitable for millimeter-wave integrated circuits. Therefore, as the third example, a fin-line filter for about 34 GHz is designed. A new design theory is introduced, based on the orthogonal expansion method, taking into account the higher mode propagation.

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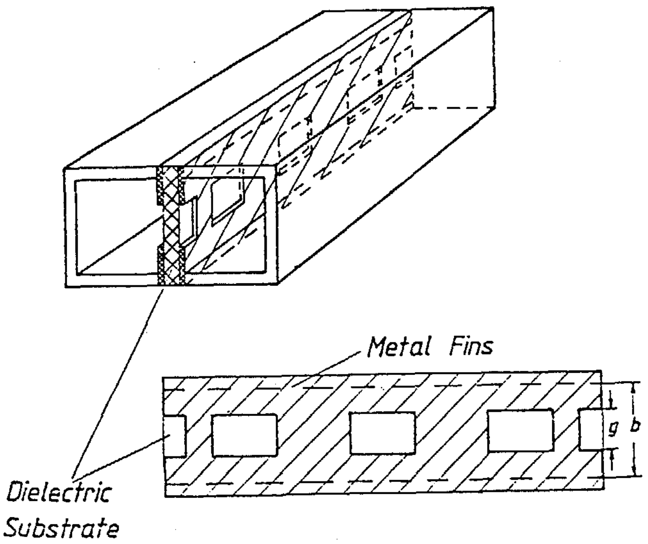

Fig. 4 Fin-Line Filter

2. METHOD

The method of the series expansion into orthogonal eigenmodes is described at the rather general and relatively complicated example of the direct transition from a waveguide to the microstrip[7] (Fig. 2). This example in principle includes all cases investigated in this paper, because all six field components are excited, and it takes into account the aspects of dielectric slab filled cross sections.

The hybrid modes on the shielded microstrip line are derived from the axial z-components of the vector potentials A_h and A_e:

$$\vec{E} = \mathrm{rot}(A_{hz}\vec{e}_z) + \frac{1}{j\omega\epsilon}\,\mathrm{rot}\,\mathrm{rot}(A_{ez}\vec{e}_z)$$

$$\vec{H} = -\frac{1}{j\omega\mu}\,\mathrm{rot}\,\mathrm{rot}(A_{hz}\vec{e}) + \mathrm{rot}(A_{ez}\vec{e}_z). \qquad (1)$$

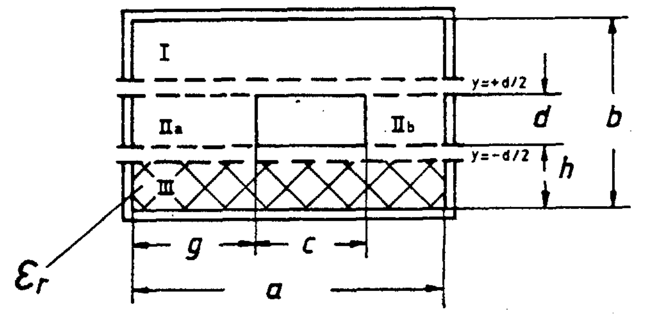

Fig. 5 Cross Section of the Shielded Microstrip Line

The subdivision of the cross section according to Fig. 5 leads to four parallel lines. Their vector potentials can be written as a product of the eigenfunctions $V_{h,e}(x,y)$ with the common propagation expression $e^{-jk_z \cdot z}$ and the square roots of the field impedances

$$Z_h = 1/Y_h = \frac{\omega\mu}{k_z} \qquad Z_e = 1/Y_e = \frac{k_z}{\omega\epsilon} \qquad (2)$$

$$A_{hz} = \sqrt{Z_e^\nu}\, V_h^\nu(x,y)e^{-jk_z z}$$

$$A_{ez} = \sqrt{Y_e^\nu}\, V_e^\nu(x,y)e^{-jk_z z}$$

$$\nu = \mathrm{I}, \mathrm{IIa}, \mathrm{IIb}, \text{ and } \mathrm{III} \qquad (3)$$

with e.g.

$$V_h^I = \sum_{n=0}^{\infty} \frac{\cos\frac{n\pi x}{a}}{\sqrt{1+\delta_{0n}}}\,[A_n^I e^{+jk_{yn}^I y} + B_n^I e^{-jk_{yn}^I y}] \qquad (4)$$

δ_{0n} = Kronecker delta.

The other eigenfunctions can be written in the same manner. This way is analogous to the known Fourier expansion method with the still unknown coefficients A_n, B_n, which are determined by the boundary conditions.

The eigenfunctions $V(x,y)$ can be regarded as representing waves traveling in the $\pm y$ direction, with the still unknown propagation constant k_y. It is possible to define the amplitudes:

$$I_{hn}^\nu(y) = A_n^\nu e^{+jk_{yn}^\nu y} + B_n^\nu e^{-jk_{yn}^\nu y},$$

$$U_{hn}^\nu(y) = \frac{dI_{hn}^\nu}{dy} = jk_{yn}^\nu [A_n^\nu e^{+jk_{yn}^\nu y} - B_n^\nu e^{-jk_{yn}^\nu y}], \qquad (5)$$

and in the same manner the corresponding amplitudes related to the vector potential A_e: U_{en}^ν, I_{en}^ν.

The boundary conditions at the partial waveguides

$$0<x<g \;:\; E_{x,z}^{III} = E_{x,z}^{IIa} \qquad H_{x,z}^{III} = H_{x,z}^{IIa}$$

$$g<x<g+c:\; E_{x,z}^{III} = 0 \qquad (6)$$

$$g+c<x<a \;:\; E_{x,z}^{III} = E_{x,z}^{IIb} \qquad H_{x,z}^{III} = H_{x,z}^{IIb}$$

successively applied to (5) lead finally to the relation between the amplitudes at the lower boundary ($y=y_l^{III}=-d/2-h$) and the amplitudes at the upper boundary ($y=y_u^I=b-d/2-h$, Fig. 6):

$$\begin{bmatrix} U_h^{III} \\ U_e^{III} \\ I_h^{III} \\ I_e^{III} \end{bmatrix}_{y=y_l^{III}} = (M) \begin{bmatrix} U_h^I \\ U_e^I \\ I_h^I \\ I_e^I \end{bmatrix}_{y=y_u^I} . \qquad (7)$$

The still missing boundary condition at the metallic surfaces at a = -d/2-h and y = b-d/2-h leads to the resonant condition:

$$0 = \begin{bmatrix} M_{hh} & M_{he} \\ M_{eh} & M_{ee} \end{bmatrix} \begin{bmatrix} I_h^I \\ I_e^I \end{bmatrix} \qquad (8)$$

This matrix of this characteristic equation (8) is the upper right quarter of the matrix product of (7). The zeros of the determinant which is a transcendent function of $k_{yn}(\omega,k_z)$ provide the interesting dispersion characteristic $k_z(\omega)$ for each mode taken into account.

Fig. 2 shows the step investigated. The transversal E- and H-field strengths of the adjacent transmission (1) and (2) are expressed by the eigenfunctions:

$$\vec{E}_t^{(1)} = \sum_j \sqrt{Z_j}\, \vec{e}_{tj}(B_j^+ e^{-jk_{zj}z} + B_j^- e^{+jk_{zj}z})$$
$$\vec{H}_t^{(2)} = \sum_j \sqrt{Y_j}\, \vec{h}_{tj}(B_j^+ e^{-jk_{zj}z} - B_j^- e^{+jk_{zj}z}) \tag{9}$$

$$\vec{e}_{tj} = \begin{cases} \text{grad } T_h \times \vec{e}_z, & \text{H-mode} \\ -\text{grad } T_e, & \text{E-mode} \end{cases}$$

$T_{e,h}$ = eigenfunctions of the rectangular waveguide

$$\vec{E}_t^{(2)} = \sum_k \vec{e}_{tk}(C_k^+ e^{-jk_{zk}z} + C_k^- e^{+jk_{zk}z})$$
$$\vec{H}_t^{(2)} = \sum_k \vec{h}_{tk}(C_k^+ e^{-jk_{zk}z} - C_k^- e^{+jk_{zk}z}) \tag{10}$$

$$\vec{e}_{tk} = \sqrt{Z_{hk}}\, \text{grad } V_{hk} \times \vec{e}_z - \sqrt{Z_{ek}}\, \text{grad } V_{ek}$$
$$\vec{h}_{tk} = \sqrt{Y_{hk}}\, \text{grad } V_{hk} + \sqrt{Y_{ek}}\, \text{grad } V_{ek} \times \vec{e}_z \, .$$

(The indication ν = I, II, III, Fig. 6 of the subdivision of the cross sections is omitted in this chapter for simplicity.) B_j^+, C_k^+ are the amplitudes of waves traveling in the +z-direction, B_j^-, C_k^- are the amplitudes of waves traveling in the -z-direction, cf. Fig. 2. The indexes j and k designate transversal field vectors $\vec{e}_{tj}$, $\vec{k}_{tj}$, $\vec{e}_{tk}$, $\vec{h}_{tk}$ which cohere with the eigenfunctions and indicate the order of their cutoff frequencies.

The boundary conditions at the step (z=0)

$$E_t^{(1)} = E_t^{(2)} \qquad H_t^{(1)} = H_t^{(2)}$$

lead to the matrix equations

$$B^+ - B^- = (\sqrt{Y})\, K_e(C^+ + C^+)$$
$$B^+ - B^- = (\sqrt{Z})\, K_h(C^+ + C^-) \, . \tag{11}$$

The vectors B^+, B^-, C^+, and C^- contain the amplitudes of the traveling waves, the diagonal matrices (Z) and (Y) contain the square roots of the magnetic- or electric-field impedances Z_h or Z_e, and the magnetic- or electric-field admittances Y_h or Y_e, respectively, (cf. (2)). The electrical K_e and the magnetic coupling matrix K_h are given by the elements

$$(K_e)_{jk} = \int_{Fs} \vec{e}_{tj} \cdot \vec{e}_{tk}\, dF \qquad (K_h)_{jk} = \int_{Fs} \vec{h}_{tj} \cdot \vec{h}_{tk}\, dF. \tag{12}$$

The scattering matrix S of the step is given by eliminating B^- and C^+ in (11):

$$\begin{pmatrix} B^- \\ C^+ \end{pmatrix} = (S) \begin{pmatrix} B^+ \\ C^- \end{pmatrix} \tag{13}$$

3. RESULTS

In order to show the convergence of the orthogonal expansion method, Fig. 6 indicates the normalized propagation factors k_z/k_o (k_o=free space propagation constant) of the shielded microstrip line for four modes as a function of the number of eigenmodes considered. It can be stated that for a number of about 10 to 12 eigenmodes the progation factors converge to a corresponding constant value.

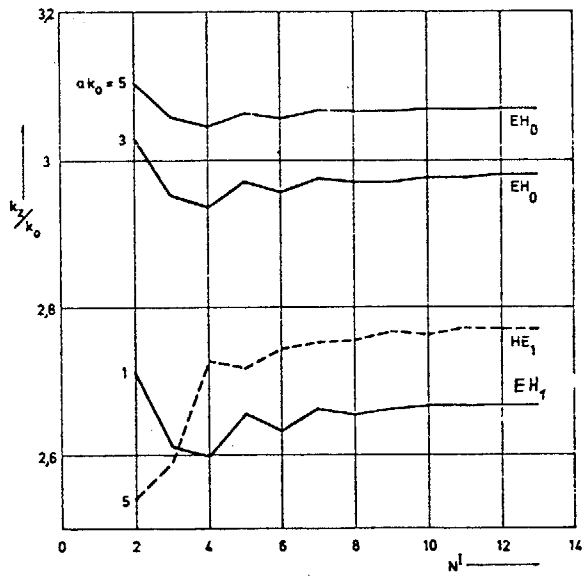

Fig. 6 Normalized Propagation Factor as a Function of the Number of Eigenmodes Considered

Fig. 7 shows the scattering parameters as a function of the normalized frequency a/λ_o (λ_o = wavelength in air) of a direct waveguide to microstrip transition (Fig. 2), if a H_{10}-mode is incident in the rectangular waveguide. Up to a/λ_o = 1 the reflection coefficient in the waveguide is only related to the backward H_{10}-wave. The other modes (dotted lines) do not yet propagate, since their frequency is below the corresponding cut-off frequency.

At the microstrip line (lower figure), however, higher order modes play a significant role. It can be seen that the principel part of the transmitted power is transported by the first higher order HE_1-mode[1] and not by the commonly desired fundamental microstrip mode EH_o. This is because of the incoherence of the waveguide H_{10}-mode with the EH_o-mode. The figure indicates that a direct waveguide instrumentation of microstrip lines is inappropriate. Since suitable transitions (e.g. ridged waveguide tapers) are relatively complicated (the state of the art are insertion losses of about 2.5 dB per transition[8]),other structures, e.g. the two following examples, are much more appropriate for millimeter-wave MIC's.

1) The EH_1-mode is the hybrid-(EH-)mode with the first cut-off frequency and a purely H-mode at cut-off frequency. The other hybrid modes on the microstrip line are indicated in the corresponding manner.

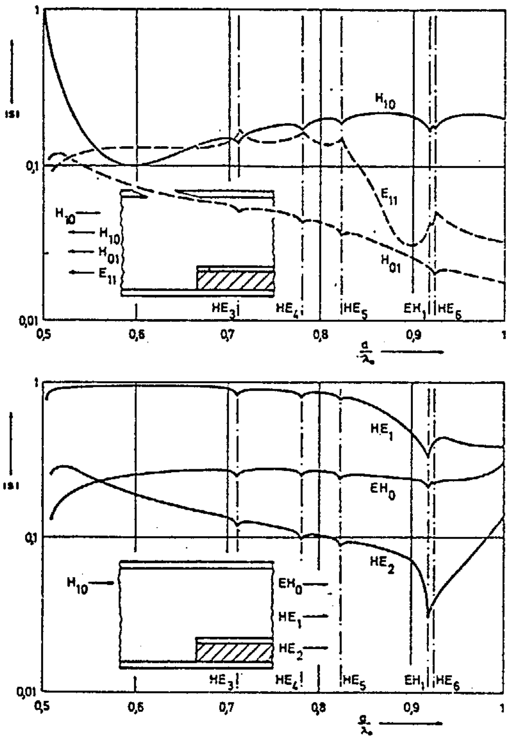

Fig. 7 Scattering Parameters of the Step Waveguide to Microstrip (Fig. 2).
Dimensions (cf. Fig. 5):
b/a = 0.5, c/a = 0.3, d/a = 00.3, g/a = 0.5,
h/a = 0.1, ϵ_r = 9.7 (-.-.- cut-off frequencies)

Line structures with circuits suspended in the E-plane of rectangular waveguides are very well compatible with the H_{10}-mode of the waveguide. Thus for the direct waveguide transition to a waveguide filled with a dielectric slab in the E-plane[9] (Fig. 3) it can be stated (Fig. 8) that below the H_{20}-mode cut-off frequency practically the total energy is transmitted into the desired fundamental H_{10}-mode of the filled waveguide. This is also confirmed by measurements (Fig. 8).

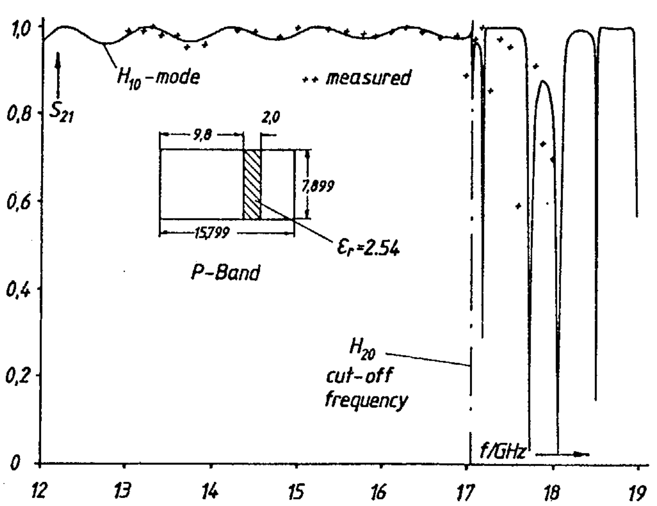

Fig. 8 Transmission Coefficient of the Step Waveguide to a E-Plane Dielectric Loaded Waveguide (Fig. 3)

If the dielectric slab mounted in the waveguide E-plane is partially metallized, a fin-line structure (Fig. 4) is obtained, which is also very adequate for direct waveguide instrumentation. Further it has the advantage that suitable MIC structures, such as filters, can easily be constructed by photoetching techniques. A low insertion loss fin-line filter is designed with the orthogonal expansion method which consists of a fin-line structure with alternatively metallic bridges and gaps in the total height of the rectangular waveguide[9]. Fig. 9 shows the calculated value of the insertion loss in dB as a function of the frequency for a nine-step fin-line filter (four metal inserts,Fig.10),designed for a midband frequency of 33.71 GHz,mounted into an R-band waveguide (a=7.112 mm, b=3.556 mm). The dielectric substrate is RT/duroid 5880 with ϵ_r = 2.22. The theoretical insertion loss in the pass-band is 0.1 dB, the measured value is about 0.5 dB. The slight frequency displacement of the measured values is due to the copper cladding thickness (17.5 μm) which has not been taken into account in the calculations.

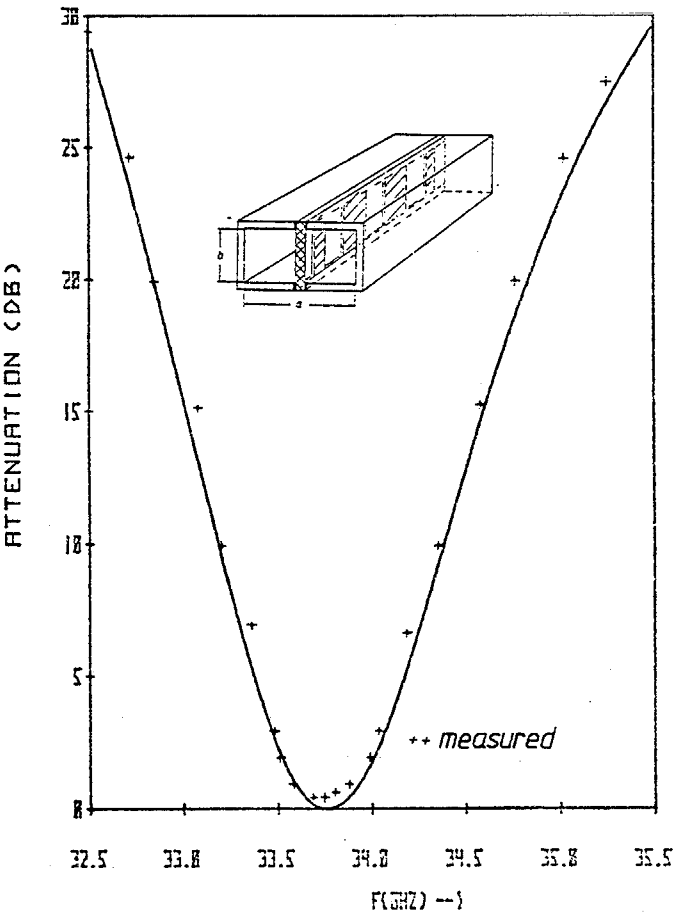

Fig. 9 Calculated and Measured Insertion Loss of Designed High-Q Fin-Line Filter

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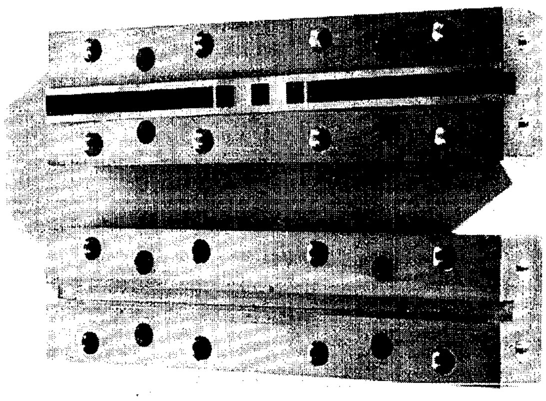

Fig.10 Photograph of the designed Fin-Line Filter for 33.7 GHz

CONCLUSION

Since there is a growing demand for communication systems at higher frequencies, millimeter-wave structures, such as waveguide transitions, dielectric slab filled waveguides, and fin-lines, are of great interest. For these finite structures an orthogonal expansion method is described which takes into account the higher order mode propagation. This method can be interpreted as a generalized Fourier expansion method, where the series terms are eigenmodes of suitably subdivided structures. The orthogonal expansion method allows one to calculate numerically the propagation constant as well as the reflection and transmission coefficients of rather complicated wave structures up to a desired degree of accuracy, depending on the chosen number of eigenmodes. A number of about 10 to 12 eigenmodes has turned out to be sufficient for an error of only about 1 %.

For millimeter waves, waveguide instrumentation is commonly used. There is therefore a great interest in direct waveguide transition scattering problems. As has been calculated a direct transition from waveguide to a microstrip-line, the usual centimeter-wave integrated circuit type, is not appropriate, because the energy of the waveguide is mainly transmitted to an undesirable higher order mode. Dielectric slab structures, mounted in the E-plane of the waveguide, lead, however, to millimeter-wave circuits for which a direct waveguide instrumentation causes no problems. This is demonstrated by the example of the transition waveguide to a dielectric slab E-plane filled waveguide, where the calculated and measured transmission coefficient to the fundamental H_{10}-mode is almost equal to one below the cut-off frequency of the higher order mode. A second very excellent millimeter-wave structure is the fin-line, where the dielectric slab is partially metallized. This circuit structure allows low-cost photoetching techniques to be used and is very suitable for designing millimeter-wave integrated circuits. A fin-line filter is calculated, designed and measured for about 33.7 GHz. The measured insertion loss is only about 0.5 dB.

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