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SOURCE MOTION INDUCED ACOUSTIC FLUCTUATIONS IN THE OCEAN

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### **RESUME**

On presente des effets acoustique du movement du source dans un environ sous-marin des plusier vois de propagation. Pour un source monochromatique en motion constant, le coherence temporel du champs recevoit a un point fixé arbitraire est obtenú comme un expression integral, donné que les components du champ totale consistent des ondes statistiquement independent et localement plan. Le formulation est appliqué a un channel iso-velocité, representatif des environs sous-marines du bas profondeur.

Pour le cas de plusiers vois de propagation, des formulations assez simple sont derivé pour des fluctuations du signal dans léau du bas profondeur. Ces expressions depend precisement des parametres de l'environment et du motion du source.

## SUMMARY

A treatment of the effects of acoustic source motion in a multipath ocean environment is presented. For a monochromatic source in constant motion, the temporal coherence of the field received at an arbitrary fixed point is obtained as an integral expression under the assumption that the components of the total field are statistically independent locally plane waves. The formulation is applied to an isospeed channel representative of shallow water.

For the situation where the total field contains many multipaths, simple formulae are derived for signal fluctuation statistics in shallow water. These expressions depend explicitly on the environmental and source motion parameters.



#### INTRODUCTION

Consider an underwater acoustic channel excited by a unit strength, monochromatic source in motion with a constant horizontal velocity. Let P(t) denote the complex pressure field received at an arbitrary fixed receiver location at time t. For sufficiently large separations of source and receiver, P typically will change significantly over short time intervals. In practical situations, with source speeds in excess of a few meters/sec, this phenomenon can be readily understood to be a consequence of changing interference among the received multipath components. Indeed the relative phase between pairs of multipath contributors is very sensitive both to small changes in geometry with time and also to variability in the ocean environment.

Precise predictions of the detailed variations in P are generally impossible to perform owing to random perturbations in the relative phases of its multipath components. However, it is usually the case that precise knowledge is not required; rather what is often needed in practice is a statistical characterization of the sound field. In this regard, the time autocorrelation function of the received field is a useful descriptor of source motion induced fluctuations. The temporal autocorrelation is formed as

$$K(\tau, t) = \langle P(t+\tau)P*(t) \rangle , \qquad (1)$$

where  $\tau$  is a time displacement in the observation of the received field, and the brackets indicate averages over an ensemble of environmental conditions. In general, as indicated, the received process will not be stationary in the wide sense. However in most practical situations  $K(\tau,t)$  will vary slowly enough with t that the observed acoustic field can be considered to be locally stationary.

The procedure that will be employed to obtain analytical formulations of the autocorrelation function have been thoroughly developed by Preston Smith [1] in his investigations of spatial coherence in multipath environments. The derived expressions for temporal autocorrelation will then be used to investigate signal fading statistics following a model studied by Dyer [2].

# I. MULTIPATH DESCRIPTION

It is assumed that the received kth multipath contribution  $P_k(t)$  is transmitted from the omnidirectional moving source as  $P_0(t_k)$ . The relation between the source emission time  $t_k$  and its time of reception t at the receiver is

$$t_k = t - \int_{s_k} ds/c \equiv g_k(t) , \qquad (2)$$

where the line integral is taken over the kth ray path,  $s_k = s_k(t)$ , and where c is the local acoustic speed, assumed dependent on depth only.

The component  $P_k(t)$  can be expressed as

$$P_{k}(t) = A_{k}(t)e^{i\phi_{k}(t)} P_{O}(g_{k}(t))$$
, (3)

where  $A_{k}$  accounts for spreading loss, and  $\phi_{k}$  includes the deterministic and stochastic effects of boundary interactions and of volume scattering. As a result

of the phase randomness introduced in (2), it is supposed that

$$\langle P_k(t) P_k^*(t) \rangle = A_k^2(t) e^{-B_k(t)} \delta_{k\ell}$$
, (4)

where  $B_K(t)$  is a function incorporating volume attenuation and boundary reflection loss. A more detailed discussion of the assumptions inherent in (4) may be found in [1] and the references listed therein.

The total received field is the sum of the multipath contributions; in particular for a monochromatic source of circular frequency w the received pressure is

$$P(t) = \sum_{k} P_{k}(t) = \sum_{k} A_{k}(t) \exp \left[-iw_{0}g_{k}(t) + i\phi_{k}(t)\right], \quad (5)$$

For the autocorrelation function there is obtained  $\operatorname{immediately}$ 

$$K(\tau,t) =$$

$$\approx \sum_{t} A_{k}^{2}(t) e^{-iw_{O}\left[g_{k}(t+\tau)-g_{k}(t)\right]-B_{k}(t)}$$
(6)

If the geometry is sufficiently slowly varying, a further simplification is realized in (6) by performing a Taylor expansion of  $g_k$  about  $t\colon$ 

$$K(\tau,t) \approx \sum_{k} A_{k}^{2}(t) e^{-\beta_{k}(t) - iw_{0} \dot{g}_{k}(t)\tau} . \tag{7}$$

For ocean channels having cylindrical symmetry with respect to the source, the fixed receiver location can be identified by cylindrical coordinates: horizontal range R and depth z. It is useful at this point to characterize multipath components by the ray depression angle  $\theta(t)$  measured with respect to the horizontal at the receiver. Then from the definition (2) it can be shown that

$$\dot{g}_{k}(t) \rightarrow \dot{g}(\theta(t)) = 1 - v\cos\theta/c(z)$$
, (8)

where v=v(t) is the (horizontal) radial component of source velocity.

Following Smith [1], the correlation function can now be written as a single integral rather than as a sum:

$$K(\tau,t) = \int d\theta D(\theta;t) \exp \left[-iw_0 \tau \left(1 - \frac{v}{c} \cos \theta\right)\right] , \qquad (9)$$

where D represents the angular spectrum of intensity [1,3]. This energy flux formulation, to use Weston's terminology [3], has been independently considered by



a number of investigators to study averaged intensity behavior; application of the method to an analysis of spatial coherence was first indicated by Smith.

Consider now the form of the intensity spectrum D. The path of a ray in a range-independent ocean channel is periodic in range with cycle distance  $X(\theta)$  dependent upon the ray arrival angle. Then the intensity spectrum of rays reaching the receiver is given by

$$D(\theta,t) = \frac{2\exp[-B(\theta,t)]}{R(t)X(\theta)\tan[\theta_{o}(\theta)]}, \qquad (10)$$

where  $\theta_0$  is the depression angle at the source depth for a ray with arrival angle  $\theta,$  and B incorporates volume and boundary losses. The derivation of (10), as well as discussion of the approximations and assumptions inherent in the approach, can be found in the cited references.

# II. ISOSPEED CHANNEL

As the simplest application to the study of motion induced fluctuations, the general results obtained above will be considered for the case of a constant sound speed channel of depth H. This standard model in underwater acoustics is often considered as being characteristic of shallow water situations.

The rays in the isospeed channel are straight between boundary reflections; thus  $\left|\theta\right|$  = constant and the cycle range of a ray is

$$X = 2Hcot[\theta] \tag{11}$$

It is supposed that, because of bottom losses, only rays with shallow grazing angles propagate efficiently, the bottom loss per bounce of a ray is taken to be  $\theta\theta$  for small grazing angles, and since there is one bottom bounce every 2H/ $\theta$  the total attenuation of intensity of ray  $\theta$  is

$$B(\theta,t) = (\alpha + \frac{\beta \theta^2}{2H})R(t) , \qquad (12)$$

where  $\boldsymbol{\alpha}$  is the volume absorption coefficient. Thus

$$D = (RH)^{-1} e^{-\alpha R} \exp \left[ -\frac{\beta R}{2H} \theta^2 \right] . \tag{13}$$

There is obtained for the total intensity of the received field  $% \left( 1\right) =\left( 1\right) \left( 1\right)$ 

$$K(0,t) = (2\pi/\beta H)^{\frac{1}{2}} R(t)^{-3/2} e^{-\alpha R(t)}$$
, (14)

a well known result. If a small angle approximation is used in the exponent of (8) and if the integration limits are extended to  $\pm \infty$ , there is obtained for the autocorrelation function

$$\frac{K(\tau,t)}{K(0,t)} = \frac{e^{-iw_D \tau}}{(1-i\tau/\tau_0)^{\frac{\tau}{2}}} ,$$
(15)

where

$$W_D = W_O[1-v(t)/c(z)]$$
,  $\tau_O = \frac{\beta cR(t)}{W_OHv(t)}$ 

It is observed in (15) that  $\tau_0$  defines a correlation time that increases directly with the range from source to receiver. Since decorrelation of the received field is caused by multipath interference, the increase of correlation time with range can be understood to be a consequence of multipath stripping by the boundaries.

#### III. SIGNAL FLUCTUATION RATES

Of considerable practical interest is the rate at which the acoustic radiation received from the moving source experiences deep fades. If the number of contributing multipaths is large, the behavior of the received field is well known. Indeed all the relevant statistical theory has been formulated in [2].

Referring to (15), let the complex received pressure be written

$$P(t) = E(t)e^{-iw_D t}, (16)$$

where the complex envelope E(t) can be expressed as

$$i\phi(t)$$

$$E(t) = A(t)e (17)$$

It is convenient to introduce the notation

$$\sigma^2 = K(o,t)$$

and

$$2\sigma^{2}v = -Re\left\{\frac{d^{2}}{d\tau^{2}}\left[K(\tau,t)e^{iw_{D}\tau}\right]_{\tau=0}\right\}.$$
 (18)

It will be recognized that  $\boldsymbol{\sigma}$  is the rms level of the received signal.

Suppose that it is desired to know the rate at which the amplitude A (see (17)) of the received signal falls below some fraction  $\epsilon$  of the rms level  $\sigma$ . Then, following the development in [2], it can be shown that the mean time, T, between signal fades below the level  $\epsilon\sigma$  is given by

$$T^{-1} = 4\sqrt{\frac{v_{\varepsilon}}{\pi}} \exp(-\varepsilon^2/4) , \qquad (19)$$

or, when expressed in terms of the environmental parameters of the shallow water channel model, by:

$$T = \frac{\beta R(t) c}{2w_0 H v(t)} \sqrt{\frac{\pi}{3\epsilon}} exp(\epsilon^2/4)$$
 (20)

Similar fade statistics can be derived for the phase,  $\boldsymbol{\phi}$  [2].

It should be noted that physical mechanisms connecting ocean and source characteristics with signal fluctuations are not required in deriving (19). The result derives simply from summing a large number of independent multipath components [2].

# IV. DISCUSSION

A theory has been formulated which yields simple statistical descriptions of source motion induced fluctuations in a multipath environment. Several



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critical assumptions were necessary to the development, and it is appropriate to review those assumptions now. First it was assumed that the natural vagaries of the environment result in different received multipath components being independent. This assumption permitted introduction of the angular intensity spectrum [3] as a means of simplifying the analysis. A second assumption, then, is that the energy flux approximations implicit in the intensity spectrum have some useful region of applicability. A third assumption in the formulation was, essentially, that each multipath component propagates locally as a plane wave; this is equivalent to invoking the validity of the eikonal approximation. Finally, in the discussion of signal fading rates it was assumed that a sufficient number of multipaths participate to insure that the received pressure obeys Gaussian statistics. It is noted that this requirement should be rather easily satisfied in studying amplitude fading rates.

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