

Generating Variable Frequency Independent Beam-Pattern by Spatial Frequency Modulation

*Génération de diagrammes de directivité
variables indépendents de la fréquence
par modulation de fréquence spatiale*

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Abstract

A method to generate a desired two-dimensional beam pattern of specified form without loss of energy by phasing a two dimensional array is given. By changing the phase terms it is possible to adapt the beam pattern of the array for different applications.

Key words : Beamforming, phased array, spatial frequency modulation, broadband beam-pattern, array processing.

Résumé

Nous présentons une méthode d'obtenir un diagramme de directivité de forme donnée sans perte d'énergie par modulation de fréquence spatiale appliquée a une antenne en deux dimensions. En changeant le déphasage spatial, il est possible d'adapter la forme du diagramme de directivité aux besoins d'applications diverses.

Mots clés : Formation de voies, traitement d'antenne, diagramme de directivité large bande, modulation de fréquence spatiale.

1. INTRODUCTION

Without phasing the narrowband horizontal and vertical beam pattern of an array are mainly determined by the dimensions of the array. For a line array of length L with uniform weighting, the beamwidth in degrees is given approximately by $50 \frac{\lambda}{L}$, where λ denotes the wavelength [2]. The shape of the beam pattern, especially the sidelobes, can be influenced by an amplitude weighting of the individual sensors. This is a standard topic treated in many books and papers on windows, array and antenna theory, see for example [2;5;7;10;11;12]. Amplitude weighting leads to

an undesired energy reduction. Especially, if beamwidths much larger than the natural beamwidth of the uniformly weighted array are required, the amplitude weighting leads to an important reduction of the effective aperture. This results in a large loss of signal-to-noise ratio for receiving arrays or signal power for emitting arrays.

In practice the possibility of generating beam-pattern with variable shapes and large beamwidths without loosing energy is of interest. For example, in many sonar applications, the emitted signal energy must be very large, leading to a considerable physical extension of the emitting array. On the other hand, operational aspects require that a large sector has to be illuminated instantaneously, hence a large beamwidth is required.

Both requirements cannot be satisfied simultaneously by the conventional approach of amplitude weighting or windowing.

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In fact, in the narrowband case a solution can be obtained by phasing the array, which means applying a complex weighting with (nearly) constant amplitude, see sections 3 and 4.

For time signals the energy-spectrum can be transformed into a specified form by frequency modulation (see Papoulis [8]).

Equivalently for spatial signals the “spatial energy-spectrum” can be forced into a specified shape by a corresponding “spatial frequency modulation”. This means, phasing the aperture function of an array in a certain way results, for a specified frequency, in a prescribed energy-pattern.

It turns out that combining the calculated phasing for different frequencies leads to a well defined broad-band time-delay. Applying this time delay to the individual sensors leads to a frequency independent prescribed broadband beampattern.

Of course, time-delay beamforming is required to achieve this in a hardware implementation.

In [9] the broadening of the natural narrow beam of an unphased planar or cylindrical array by *linear* spatial frequency modulation is described. In [3;4] the 1-dimensional and narrowband case is treated.

In this paper the *general* method of 2-dimensional spatial frequency modulation is presented.

It should also be noted that the approach presented here is an analytical method which can be applied successfully to very large arrays with several hundreds of sensors. Search methods or numerical optimization methods as in [1;6] are very difficult to apply in this case.

2. THEORETICAL DERIVATION OF BEAMFORMING WITH SPATIAL FREQUENCY MODULATION

In general the pattern function $B(\underline{f}_r)$ of an arbitrary three-dimensional array with aperture function $b(\underline{r})$ is given by the three-dimensional Fouriertransform

$$B(\underline{f}_r) = \int b(\underline{r}) e^{i2\pi \underline{f}_r \cdot \underline{r}} d\underline{r}, \quad \underline{f}_r = \underline{u}_r / \lambda,$$

where \underline{u}_r is the vector indicating the direction of a monochromatic plane wave of wavelength λ . Using azimuth and elevation angles Θ_h and Θ_v , the components of \underline{u}_r are given by

$$\{u_x, u_y, u_z\} = \{ \sin\Theta_h \cos\Theta_v, \cos\Theta_h \cos\Theta_v, \sin\Theta_v \}.$$

In this paper we restrict ourselves to a two-dimensional array $b(x, y)$ in the $x - y$ plane, which leads to a pattern function $B(f_x, f_y)$.

We start from an arbitrary desired magnitude pattern $S(u_x, u_y)$, which may depart strongly from the natural pattern of the unweighted aperture (figure 1 shows the unweighted pattern of a planar array). Dependent on the application, a fan beam, a cosecant-square pattern, or a sector beam with beamwidths much larger than the natural beamwidth is desired. We want to find the (frequency dependent) phased aperture function that gives the desired magnitude pattern $S(u_x, u_y)$.

PROBLEM

A separable two dimensional aperture function

$$a(x, y) = a_x(x) \cdot a_y(y)$$

with

$$a_x(x) \geq 0 \quad \text{and} \quad a_y(y) \geq 0$$

and a separable two-dimensional real function

$$S(u_x, u_y) = S_x(u_x) \cdot S_y(u_y)$$

with

$$S_x(u_x) \geq 0 \quad \text{and} \quad S_y(u_y) \geq 0$$

are given.

Further the energy of a_x and S_x as well as a_y and S_y must be the same (by Parseval's theorem) :

$$\int a_x^2(x) dx = \int S_x^2(u_x) du_x, \quad \int a_y^2(y) dy = \int S_y^2(u_y) du_y. \quad (1)$$

The task is to find a spatially frequency modulated aperture function

$$b(x, y) = a(x, y) e^{i2\pi\Phi_\lambda(x, y)}, \quad (2)$$

with a frequency dependent phase function $\Phi_\lambda(x, y)$, such that the absolute value of its Fourier transform approximates the specified frequency independent function S :

$$|B(f_x, f_y)| \cong \lambda S(u_x, u_y). \quad (3)$$

This means, by phasing the two dimensional aperture function $a(x, y)$ the two dimensional beam pattern shall be forced into a specified shape.

An overview of the procedure is given in the scheme “Concept of Phase Modulation”.

It is now possible in the following way to obtain a

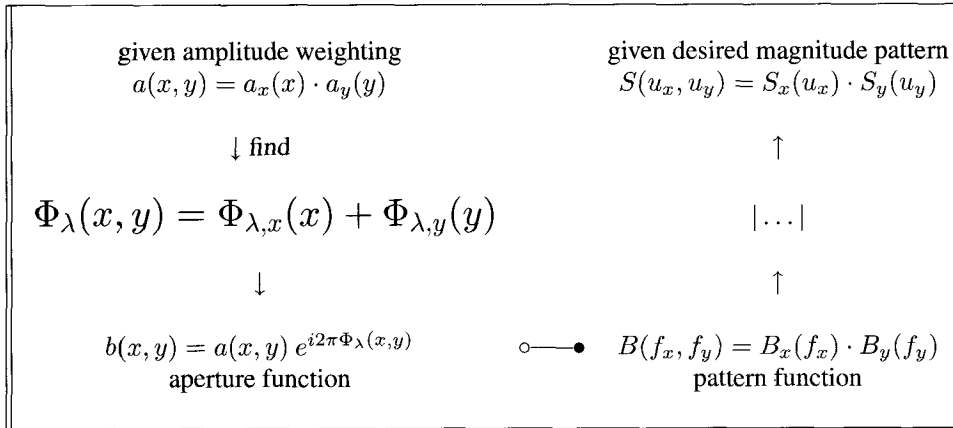
SOLUTION

Define the “instantaneous spatial frequency components”

$\underline{f}_{r,i}(x, y) := (f_{x,i}(x), f_{y,i}(y))$ as the gradient of the phase function $\Phi_\lambda(x, y)$:

$$f_{x,i}(x) = \frac{\partial}{\partial x} \Phi_\lambda(x, y), \quad f_{y,i}(y) = \frac{\partial}{\partial y} \Phi_\lambda(x, y). \quad (4)$$

CONCEPT OF PHASE MODULATION



To determine the desired frequency dependent phase function $\Phi_\lambda(x, y)$, we first restrict ourselves to the normalized case $\lambda = 1$. For $\lambda = 1$, the instantaneous frequency is denoted as $u_{r,i}(x, y) = (u_{x,i}(x), u_{y,i}(y))$ and is given by the gradient of a frequency independent reference phase function $\Phi(x, y)$:

$$u_{x,i}(x) = \frac{\partial}{\partial x} \Phi(x, y) \quad , \quad u_{y,i}(y) = \frac{\partial}{\partial y} \Phi(x, y) \quad (5)$$

Following Papoulis proof for one-dimensional time signals [8], the instantaneous spatial frequency components can be found by solving the integral equations :

$$\int_{-\infty}^{u_{x,i}(x)} S_x^2(u'_x) du'_x = \int_{-\infty}^x a_x^2(x') dx' \quad , \quad \int_{-\infty}^{u_{y,i}(y)} S_y^2(u'_y) du'_y = \int_{-\infty}^y a_y^2(y') dy' \quad (6)$$

In contrast to the certainly difficult task of finding a closed solution of these equations, the numerical solution by straightforward numerical evaluation of the integrals is performed quickly by a simple computer program, and for each (x, y) , the corresponding instantaneous spatial frequency $(u_{x,i}(x), u_{y,i}(y))$ is obtained.

Using the relation (5), the corresponding reference phase function is given by

$$\Phi(x, y) = \Phi_x(x) + \Phi_y(y) \quad (7)$$

with components

$$\Phi_x(x) = \int_{-\infty}^x u_{x,i}(x') dx' \quad , \quad \Phi_y(y) = \int_{-\infty}^y u_{y,i}(y') dy' \quad (8)$$

In the separable case this is a direct consequence of the one-dimensional result and again, the solution by numerical integration is straightforward.

For the reference case $\lambda = 1$, the phase function $\Phi(x, y)$ results in a desired magnitude pattern

$$|B(u_x, u_y)| \approx S(u_x, u_y) \quad (9)$$

By substitution methods, it can be shown easily that a frequency dependent phase function

$$\Phi_\lambda(x, y) := \frac{1}{\lambda} \Phi(x, y)$$

yields the desired frequency independent magnitude pattern :

$$|B(f_x, f_y)| \approx \lambda S(u_x, u_y) \quad (10)$$

It should be noted that the wavelength-dependent phase $\Phi_\lambda(x, y) = \frac{1}{\lambda} \Phi(x, y)$ corresponds to a uniform time-delay for all signal frequencies. This means that in a broadband case, time-delay beamforming can be used to realize the desired beampattern.

HOW GOOD IS THE APPROXIMATION (10)?

By using the symmetry property of the Fourier transform, we can extend the result from [8] to obtain the following.

The approximation (3) is satisfactory at $u_x = u_{x,i}(x)$ resp. $u_y = u_{y,i}(y)$ if the gradients of the aperture function $a(x, y)$ and the magnitude pattern $S(u_x, u_y)$ are both bounded in a neighbourhood of (x, y) resp. (u_x, u_y) in the following way :

$$\sqrt{\lambda} \frac{\frac{d a_x(x)}{d x} |_{x=x'}}{a_x(x)} \ll \frac{a_x(x)}{S_x(u_x)} \ll \frac{1}{\sqrt{\lambda}} \frac{S_x(u_x)}{\frac{d S_x(u_x)}{d u_x} |_{u_x=u'_x}}$$

for

$$|x - x'| < \sqrt{\lambda} \frac{S_x(u_x)}{a_x(x)} \quad \text{and} \quad |u_x - u'_x| < \sqrt{\lambda} \left(\frac{S_x(u_x)}{a_x(x)} \right)^{-1} \quad (11)$$

$$\sqrt{\lambda} \frac{\frac{d a_y(y)}{d y} |_{y=y'}}{a_y(y)} \ll \frac{a_y(y)}{S_y(u_y)} \ll \frac{1}{\sqrt{\lambda}} \frac{S_y(u_y)}{\frac{d S_y(u_y)}{d u_y} |_{u_y=u'_y}}$$

for

$$|y - y'| < \sqrt{\lambda} \frac{S_y(u_y)}{a_y(y)} \quad \text{and} \quad |u_y - u'_y| < \sqrt{\lambda} \left(\frac{S_y(u_y)}{a_y(y)} \right)^{-1} \quad (12)$$

Some comments are necessary at this point. It is very difficult to obtain precise bounds in the conditions (11) and (12). But in fact it is possible to extract guidelines for successful pattern design from these conditions.

To get good approximation results, firstly the aperture function $a(x, y)$ as well as the desired magnitude pattern $S(u_x, u_y)$ have to be sufficiently smooth, high slopes have to be avoided, otherwise (11) and (12) cannot be fulfilled and a bad approximation will result. This is illustrated in the simulation section.

Also, the bounds are less tight if the wavelength λ decreases, leading to a better approximation; see figures (8) to (10).

Secondly (11) and (12) mean that $\frac{a_x(x)}{S_x(u_x)}$ and $\frac{a_y(y)}{S_y(u_y)}$ have to be bounded from below **and** from above simultaneously. This means that the aperture function and the magnitude pattern should have a similar shape! This is reflected in the well-known fact that the time envelope of a linear frequency modulated signal is reproduced in its magnitude spectrum.

3. SIMULATION RESULTS

As examples some horizontal and vertical pattern of a planar array in the x-z plane are shown. This orientation is typical for most Sonar and Radar applications. Without amplitude weighting as well as phasing, the aperture function is given by $a(x, z) = \text{rect}(\frac{x}{L_x}) \cdot \text{rect}(\frac{z}{L_z})$ with dimensions length $L_x = 40 \text{ cm}$ and height $L_z = 40 \text{ cm}$. If there are no further remarks, the simulations are carried out for $\lambda = 2 \text{ cm}$.

Figure 1 shows the 'natural' three-dimensional pattern of the unphased planar array. To reduce the side lobes, a typical amplitude weighting for $a_x(x)$ and $a_z(z)$ (Hamming) is used.

Choosing the desired magnitude pattern $S(u_x, u_z) = \text{rect}(\frac{u_x}{w_x}) \cdot \delta(u_z)$ with a rectangular part of width w_x and an impulse function $\delta(u_z)$, the resulting phase function $\Phi_\lambda(x, z) = \Phi_{\lambda,x}(x)$, and the phasing only affects the horizontal pattern.

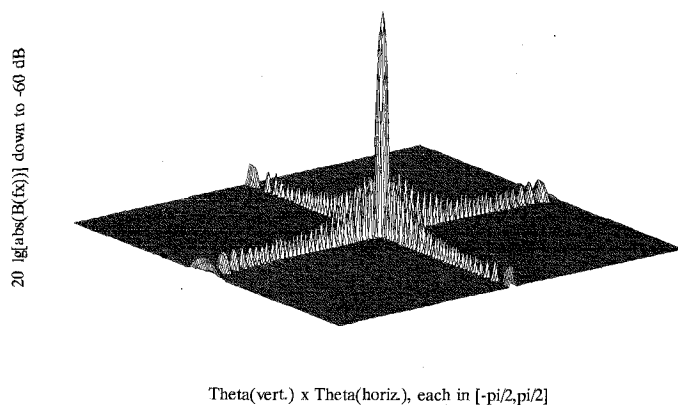


Figure 1.

To get a good approximation, the aperture function $a_x(x)$ is smoothed by flattening slightly the steep flanks at the ends. This takes into consideration conditions (11), (12) and removes the points of infinite gradients.

Again the vertical side lobe level is reduced by using a Hamming weighting $a_z(z)$.

Figure 2 shows the aperture functions $a_x(x)$ and $a_z(z)$ and the phase function $\Phi_{\lambda,x}(x)$ and $\Phi_{\lambda,z}(z)$ in degree.

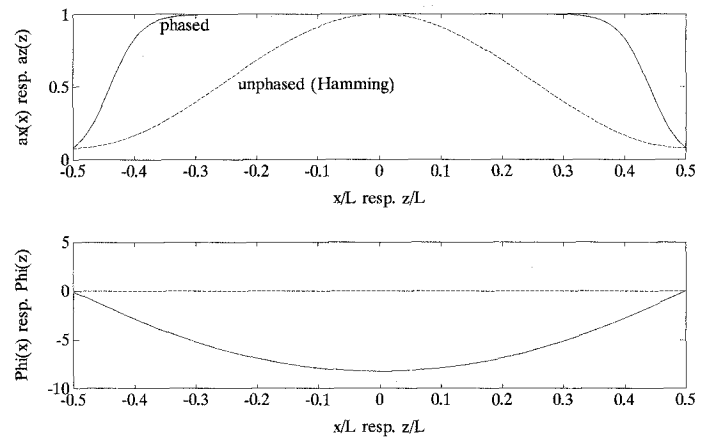


Figure 2.

Figure 3 demonstrates the corresponding three-dimensional pattern, a fan beam.

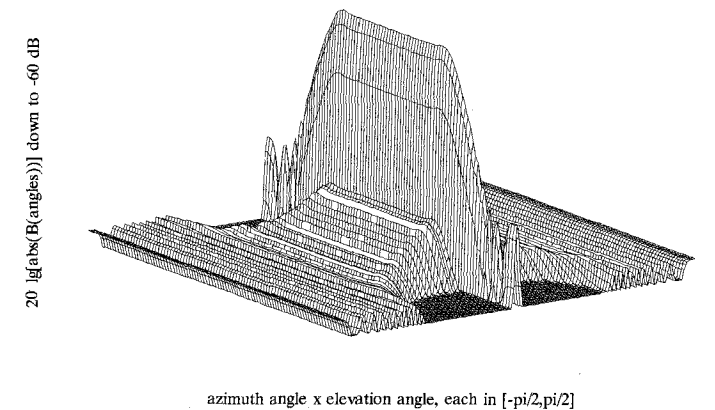


Figure 3.

Figures 4 and 5 show, for a one-dimensional cut, the case that conditions (11), (12) are not fulfilled. In this case the desired magnitude pattern is not differentiable at several points. Therefore, the approximation in the neighbourhood of these points is obviously not good.

Figures 6 and 7 show the effect of slightly smoothing the aperture (resulting in a small loss of energy) and the desired pattern. The result is much better.

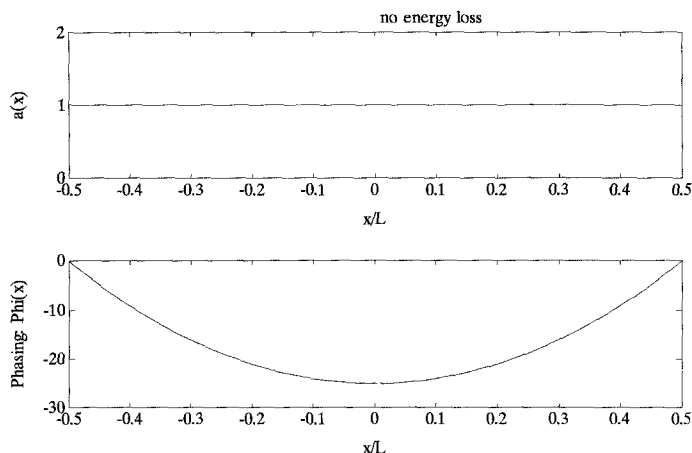


Figure 4.

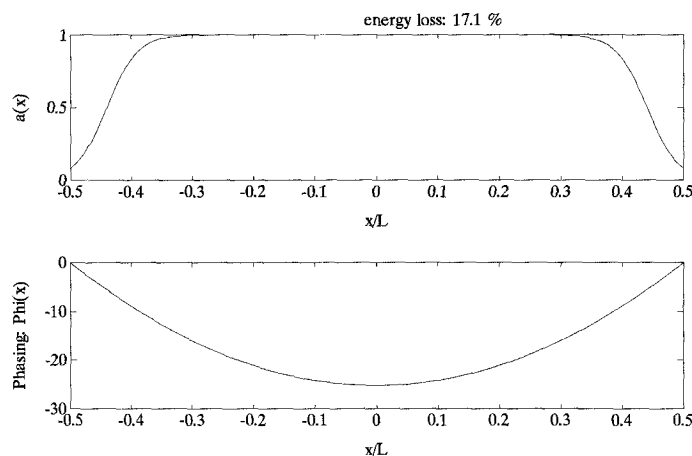


Figure 6.

Figures 8, 9 and 10 show similar pattern as in Figure 7 for different wavelengths from $\lambda = 1 \text{ cm}$ to $\lambda = 3 \text{ cm}$ together with the 'natural' (unphased), hamming weighted pattern. In the approximated pattern functions for different wavelengths, small differences are observed and the approximation is better for the smaller wavelength, as can be expected from condition (11) and the comments thereafter.

Figure 11 and 12 show the aperture functions, desired pattern and the resulting 2-dimensional pattern of a ramp beam.

Figure 13 and 14 show the aperture functions, desired pattern and the resulting 2-dimensional pattern of a sector beam.

As the title of the paper indicates, of course many other shapes can be produced by the method presented. We only gave some examples.

4. CONCLUSION

In practice with this method it is possible to generate variable beams with the same array. In this paper several examples are

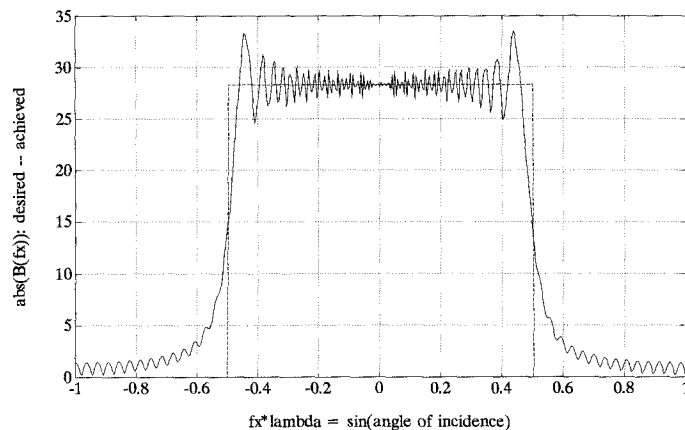


Figure 5.

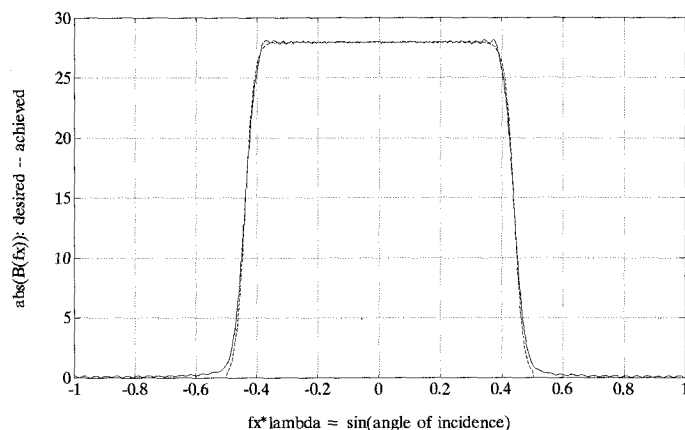


Figure 7.

shown. In fact, depending on the application of the array, the phasing function can be adapted according to the specific problem. As a typical example, the problem of transmitting wide beams of high level with extended arrays can be solved.

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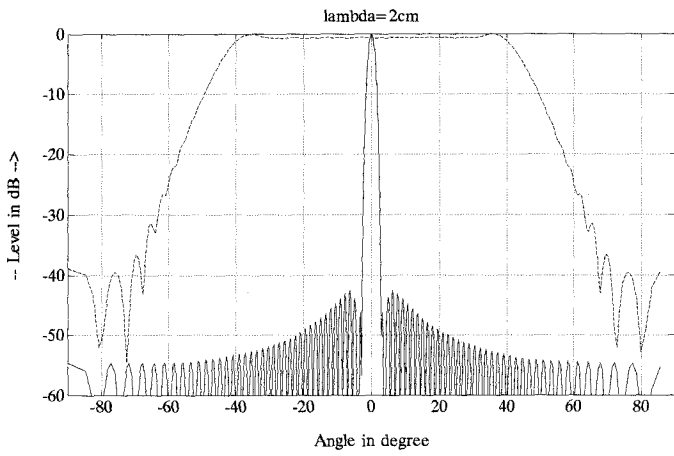


Figure 8.

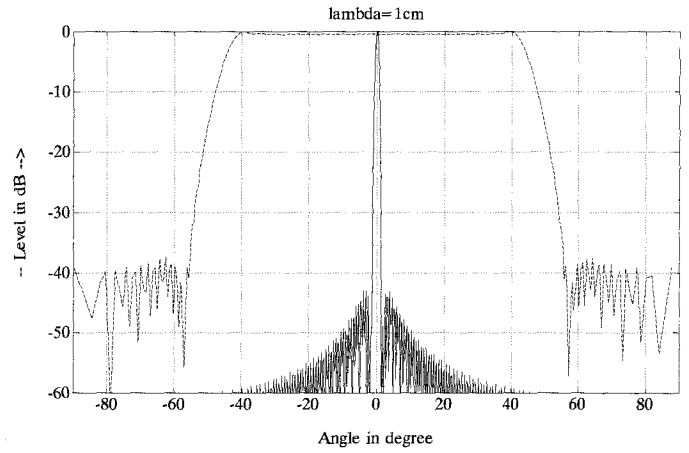


Figure 9.

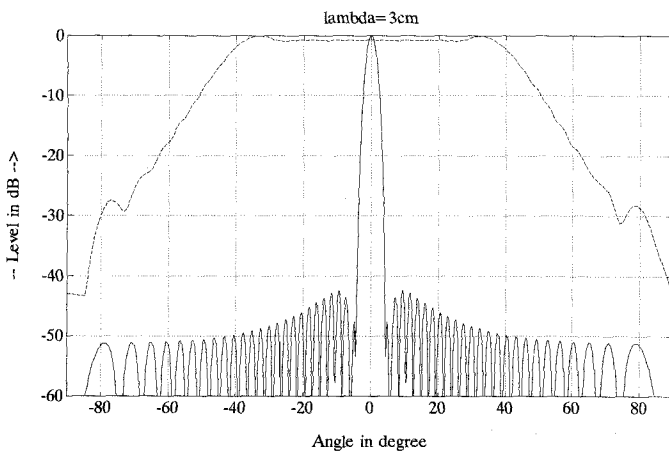


Figure 10.

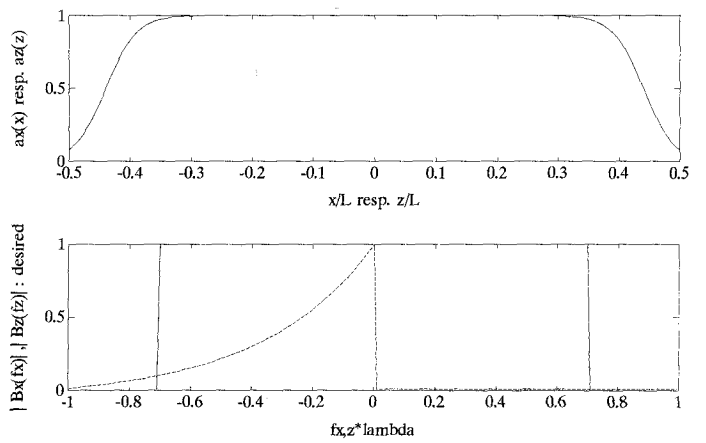


Figure 11.

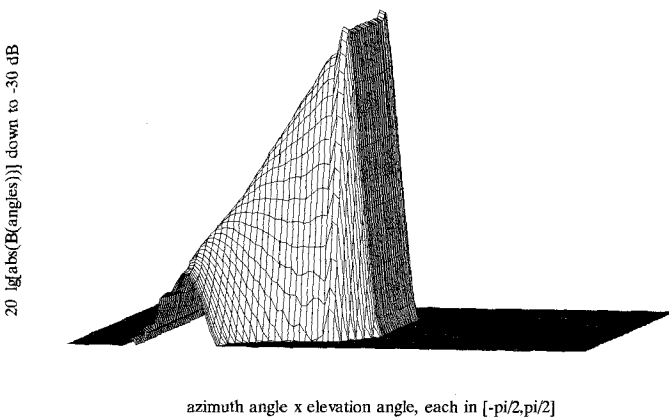


Figure 12.

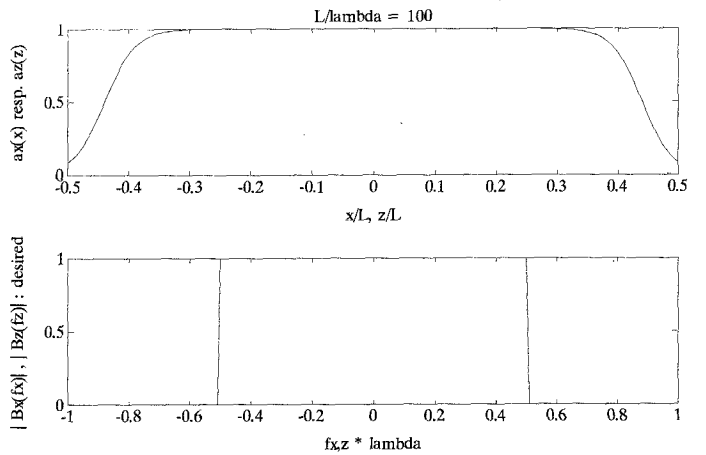
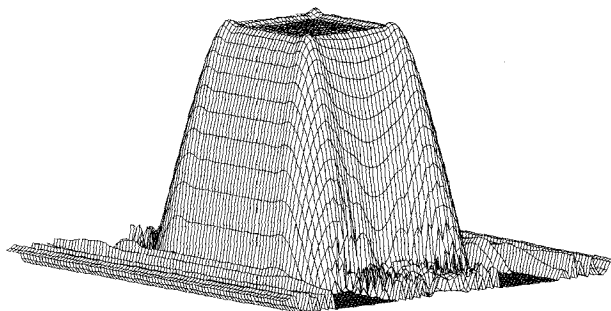


Figure 13.

20 - lg(|a_{bs}(B(angles))|) down to -45 dB



azimuth angle x elevation angle, each in $[-\pi/2, \pi/2]$

Figure 14.

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