# Scattering of short and long sound pulses:

# connexion with

# the singularity expansion method

Diffusion d'impulsions acoustiques de courte et longue durée :

relation avec la méthode de développement en singularités



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*Electromagnetic waves:* Establishment of the relation between resonance scattering theory and the Singularity - Expansion Method (SEM). Theory of radio-wave propagation in the earth-ionosphere waveguide. *Nuclear theory:* Basic development of coherent bremsstrahlung theory (editor of recent book). Development of a

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### SUMMARY

Baum's "Singularity Expansion Method" (SEM), formulated for radar scattering, is based on the observation that transient scattered echoes appear to be composed of a sum of decaying sinusoids. Fourier-transforming these expressions into frequency space reveals a manifold of simple poles in the complex frequency plane, identical with those of the Resonance Scattering Theory (RST), and commonly grouped in layers. We have carried out a time dependent analysis of scattering from rigid and elastic spheres and cylinders, with the following results. The use of short pulses (of spatial extent small compared to the scatterer's dimension) corresponds to the echo being a residue sum over a large number of SEM poles, and the residue sum over one given pole layer produces a sequence of echo pulses corresponding to a creeping wave repeatedly circumnavigating the sphere with the appropriate group velocity. The use of finite-length sinusoidal wave trains (long compared to the scatterer's extension) produces a reflected wave train, coherently superimposed by a sequence of overlapping creeping wave trains, which cause initial transients as well as a final transient tail following the echo. These transients only appear if the carrier frequency coincides with an eigenfrequency of the target, and the tail amplitude plotted as a function of frequency then reproduces the spectrum of resonances including their widths, leading to a direct target spectroscopy as accomplished experimentally by Ripoche et al. This tail corresponds to the ringing of a given eigenvibration, which is selectively excited when overlapped by the narrow spectrum of the long incident pulse.

#### **KEY WORDS**

Singularity Expansion Method (SEM), transients, decaying sinusoids, complex-frequency poles, pole layers, Resonance Scattering Theory (RST), creeping waves, resonance spectrum, target spectroscopy, ringing.

263

Traitement du Signal

volume 2 - n° 1 sp - 1985

#### SCATTERING OF SHORT AND LONG SOUND PULSES

### RÉSUMÉ

La « méthode de développement en singularités » [Singularity Expansion Method (SEM)] de Baum, formulée pour le cas de la diffusion radar, est basée sur l'observation que les échos de la diffusion transitoire paraissent être composés d'une somme de sinusoïdes amorties. La transformation de ces sinusoïdes dans l'espace des fréquences parla transformation de Fourier, révèle une multitude de pôles simples dans le plan complexe des fréquences, identiques à ceux de la « Théorie de la diffusion résonnante » [Resonance Scattering Theory (RST)], et généralement groupés en couches. Nous avons exécuté une analyse de la diffusion transitoire par des sphères et des cylindres impénétrables ou élastiques, avec les résultats suivants. L'utilisation d'impulsions courtes (d'une extension spatiale petite vis-à-vis des dimensions de l'objet diffusant) correspond au fait que l'écho est une somme sur les résidus d'un grand nombre de pôles SEM, et la somme sur les résidus d'une couche de pôles donnée produit une suite d'impulsions qui correspond à une onde circonférentielle effectuant autour de la sphère plusieurs tours avec la vitesse de groupe appropriée. L'utilisation de trains d'ondes sinusoïdales d'une durée finie (d'une extension spatiale grande vis-à-vis de l'étendue de l'objet diffusant) produit un train d'ondes réfléchies, sur lequel est superposée d'une façon cohérente une suite de trains d'ondes circonférentielles qui se chevauchent et causent des transitoires initiaux, ainsi qu'une traînée transitoire qui suit l'écho. Ces transitoires apparaissent seulement si la fréquence porteuse du train d'ondes incident coïncide avec une fréquence propre de la cible; le tracé de la courbe donnant l'amplitude de la traînée en fonction de la fréquence est alors identique à celui des résonances, avec leur largeur, ce qui autorise une spectroscopie directe de la cible, ainsi que l'ont montré Ripoche et al. expérimentalement. La traînée correspond à la réémission libre d'une vibration propre distincte, excitée d'une façon sélective si sa fréquence se trouve dans la région étroite du spectre de l'impulsion (longue) incidente.

#### MOTS CLÉS

Méthode de développement en singularités (SEM), transitoire, sinusoïde amortie, pôle de fréquence complexe, couche de pôles, théorie de la diffusion résonnante (RST), onde de surface, creeping waves, spectre de résonance, spectroscopie de la cible, résonance.

#### CONTENTS

- 1. Introduction
- 2. Scattering of short pulses
- 3. Scattering of long pulses
- Conclusion
- References

#### 1. Introduction

The singularity Expansion Method (SEM), first formulated for the case of radar scattering [1], is based on the observation that the echoes of radar pulses scattered from a finite target appear as the superposition of damped sinusoids,

(1.1) 
$$f_{sc}(t) = \sum_{\alpha=1}^{N} \mathbf{R}_{\alpha} e^{s_{\alpha} t},$$

with complex amplitudes  $R_{\alpha}$  and exponents  $\hat{s}_{\alpha}$ . Taking the Laplace transform, this pulse shape then indicates the presence of complex-frequency poles in the scattering amplitude,

(1.2) 
$$f_{sc}(s) = \sum_{\alpha=1}^{N} \frac{R_{\alpha}}{s - \hat{s}_{\alpha}},$$

where  $s = \omega/i$ , and  $\omega_{\alpha} = \text{Im } s_{\alpha}$  are the natural frequencies of the target whose existence gives rise to resonances in the scattering amplitude, Equation (1.2). This latter equation corresponds to a single-frequency excitation of the resonances. If the incident amplitude is pulsed in time, it can be represented by the Fourier integral:

(1.3) 
$$f_{\rm inc}(z,t) = \int G(k) e^{ik(z-\tau t)} dk/2\pi$$

where  $k = \omega/c$ . Its spectrum G(k) weighs the factor  $R_{\alpha}$  in Equations (1.1), (1.2).

A short pulse is characterized by a wide spectrum G(k); for  $G(k) \equiv 1$ , one has  $f_{inc}(z, t) = \delta(z - ct)$ . Numerous short-pulse experiments for acoustic scattering from submerged elastic bodies [2-6], or for elasticwave scattering from cavities [7, 8] have shown that the echo then consists of a reflected pulse, with a shape close to that of the incident pulse [9], followed by a sequence of pulses slightly widened as compared to the incident pulse shape [10, 11], which correspond to the multiple encirclement of the scatterer by surface (creeping) waves. Theoretically, the poles  $\hat{k}_{\alpha} \equiv i \hat{s}_{\alpha}/c$  at the scattering amplitude fall into layers in the complex frequency plane, generally parallel to the real axis and displaced from it at successeively increasing imaginary spacings. The wide pulse spectrum covers many poles in the first layer and in succeedings ones, and their coherent residue sums add up to the experimentally observed time sequence of pulses [12], one sequence for each layer sum which hence corresponds to a given mode of surface wave. The pulses were found to propagate with the group velocity of the surface waves.

Traitement du Signal

264

## **DIFFUSION PAR DES SURFACES RUGUEUSES**

An incident pulse of long duration has a narrow spectrum G(k), whose weight in Equation (1.2) can radically limit the number of poles contributing to Equation (1.1). In fact, if the pulse duration is chosen long enough, the width of G(k) may be made less than the spacing between the real parts of the poles,  $k_{\alpha} \equiv \operatorname{Re} \hat{k}_{\alpha}$ . If then the peak of G(k) coincides with the position  $k_{\alpha}$  of the  $\alpha$ th pole, Equation (1.1) becomes:

(1.4) 
$$f_{sc}(t) \cong \mathbf{R}_{\alpha} \mathbf{G}(k_{\alpha}) e^{s_{\alpha} t},$$

having been reduced to the contribution of one single pole  $\alpha$ , which appears in the form of one single damped sinusoid with its decay factor  $\exp(\operatorname{Re} \hat{s}_{\alpha}) t \equiv \exp(-|\operatorname{Im} k_{\alpha}|) ct$ , Im  $k_{\alpha} < 0$ , representing the "ringing" of the resonance  $\alpha$ ; this ringing is continuous and of duration  $(|\operatorname{Im} \hat{k}_{\alpha}| c)^{-1}$ . Such a use of long pulses leads to the possibility of exciting target resonances corresponding to *individual* poles of the scattering amplitude, and hence determining the natural frequencies, the imaginary parts Im  $\hat{k}_{\alpha}$  of the pole positions (from the decay constant of the ringing), and the residues.

The information thus obtained can conceivably be utilized for target identification purposes [13]. The pioneering experiments of Maze, Ripoche et al. [14-18] have clearly demonstrated the ringing of target resonances as excited by long incident sound pulses, thus obtaining the target's eigenfrequency spectrum in a direct fashion, and opening up the way to what may be called "acoustic spectroscopy" [13, 19].

### 2. Scattering of short pulses

The acoustic field in the presence of a spere is [20]:

(2.1) 
$$p = \frac{1}{2} \sum_{n=0}^{\infty} (2n+1) \times i^n \{ h_n^{(2)}(kr) + S_n h_n^{(1)}(kr) \} P_n(\cos \theta),$$

and its poles are given by those of the S-function:

(2.2) 
$$S_{n} = S_{n}^{(r)} \frac{F_{n}^{-1} - z_{n}^{(2)-1}}{F_{n}^{-1} - z_{n}^{(1)-1}},$$
$$S_{n}^{(r)} = -\frac{h_{n}^{(2)'}(x)}{h_{n}^{(1)'}(x)}$$

(where x = ka, a = sphere radius) in the complex frequency plane. These are shown in Figure 2.1 [21] for



the case of a tungsten carbide sphere. Figure 2.1 *a*, top, presents the poles  $\hat{x}_{nl}$  corresponding to the resonances of external surface waves (close to the zeroes of  $h_n^{(1)'}(x)$ ); the pole layers corresponding to the *l*-th wave (*l*=1, 2, ...) are connected by solid lines (while the poles of a given mode n are connected by dashed curves). Part (*b*) of the figure, bottom, shows the poles due to internal surface waves (given by the zeros of  $F_n^{-1} - z_n^{(1)-1}$ , the layer labeled *l*=1 corresponding to the Rayleigh wave).

For the case of a rigid sphere  $(F_n \rightarrow 0)$ , one obtains by an expansion of the denominator of  $S_n^{(r)}$  around its poles  $\hat{x}_{nl}$ , for the *l*-th creeping-wave amplitude in the far field, after application of Equation (1.3):

(2.3) 
$$p_{cr}^{(l)}(\theta, \tau) = \frac{1}{r} \sum_{n=0}^{\infty} \frac{n + (1/2)}{\hat{x}_{nl}} \frac{h_n^{(2)'}(\hat{x}_{nl})}{h_n^{(1)''}(\hat{x}_{nl})} \times P_n(\cos \theta) G\left(\frac{\hat{x}_{nl}}{a}\right) e^{-ix_{nl}\tau}$$

where  $\tau = (ct - r)/a$  is a dimensionless time variable. This is the mentioned weighted residue sum over the SEM poles. For an incident  $\delta$ -pulse (G=1), Figure 2.2 shows the backscattered pulse sequence



arising from a numerical sum over 50 decaying sinusoids in Equation (2.3), corresponding to the l=1 external creeping wave. The sinusoids are seen to cancel, except at the arrival times of the creeping wave which circumnavigated the sphere over 1/2, 3/2... circumferences, the arrival times being determined by the group velocity of the surface wave.

#### 3. Scattering of long pulses

This technique was pioneered experimentally by Maze and Ripoche [14-18]. Figure 3.1 *a* shows their initial pulse, a rectangular wave train long enough to wrap around their cylindrical target 50-100 times. Figure 3.1 *b* presents the reflected echo when the carrier frequency of the train coincides with an eigenfrequency  $x_{nl} = \operatorname{Re} \hat{x}_{nl}$ ; the figure shows both an initial transient and the pulse's settling down to a steadystate plateau, while after the passage of the reflected train, a tail due to the ringing of the resonance appears. Its decay time determines  $\text{Im } \hat{x}_{nl}$ , and the rate of disappearance of the tail as the carrier frequency is moved away from the resonance frequency determines the width of the resonance. In this way, an eigenfrequency spectrum of the cylinder is obtained as shown in Figure 1.1b of Reference [13], by performing a sweep of the carrier frequency.





Fig. 3.1. — Scattered wave train (a) off-resonance (identical to the initial pulse), (b) with its carrier frequency coinciding with a resonance frequency of the target (aluminum cylinder in water), showing initial transient and tail to the ringing of the resonance.

The physics of this ringing has been studied recently both for elastic spheres and cylinders [22]. Figure 3.2 shows an incident wave train (center), its spectrum overlaid on, and coinciding with an interference minimum of the backscattering amplitude for a tungsten carbide sphere (top), and the backscattered signal at resonance  $(x_{nl}=14.07)$ . Figure 3.3 shows similar quantities with the carrier frequency somewhat off resonance (x=13.7). The overlay with the minimum causes a deep constriction in the plateau (steady-state portion) of the reflected signal; an initial transient descends to the plateau and, after the passage of the reflected pulse, a final transient (tail,  $\equiv$ ringing of the

Traitement du Signal

266

volume 2 - n° 1 sp - 1985

resonance) decays in a fashion similar to the initial transient, both showing a pattern of steps first pointed out by de Lustrac and Carbó Fite [23], and Veksler [24].

The transients originate from a coherent superposition of ingredients schematically shown in Figure 3.4. Disregarding the dotted rectangles representing internal reflections in backscattering, one has a specularly reflected wavetrain (large rectangle) and successive circumferential wavetrains (small rectangles). At resonance, all these latter wavetrains interfere constructively with each other and form a tail which consists of steps, hereby approximating the exponential ringing, Equation (1.4), of the resonance. They also form a



Fig. 3.2. – Amplitude of incident wave train (center), its spectrum overlaid on the backscattering amplitude of a tungsten carbide sphere (top), and backscattered signal (bottom), at a resonance  $(x_{nt} = 14.07)$ .

Traitement du Signal **267** volume 2 - n° 1 sp - 1985

SCATTERING OF SHORT AND LONG SOUND PULSES



Fig. 3.3. – As in Figure 3.2, but off resonance (x=13.7).

coherent initial transient, region (i) of Figure 3.4, which may interfere either constructively, destructively or in an intermediate fashion with the specular echo depending on the material parameters. In the case of Figure 3.2, the interference is destructive as indicated by the minimum of the form function; the interference of the initial transient with the specular echo thus leads to a step-wise descent of the signal towards the quasi-stationary regime [region (ii) of Figure 3.4], ending with a constricted plateau in Figure 3.2. For the off-resonance case of Figure 3.3, the tail has decreased in amplitude (indicating the resonance width), and the interference of the initial transient with the specular pulse has changed.

Traitement du Signal 268 volume 2 - n° 1 sp - 1985

# **DIFFUSION PAR DES SURFACES RUGUEUSES**



Fig. 3.4. – Schematic view of specular (large solid rectangle), penetrating and multiply internally reflected (dotted rectangles), and circumferential wave trains (small solid rectangles), before their coherent addition; (i) initial transient region, (ii) quasi-steady state region, (iii) final transient region (or ringing).

#### Conclusion

A complete understanding of the transient scattering of long or short sound pulses from elastic targets is obtained by considering the interplay of the incident pulse spectrum with the SEM pole pattern of target eigenfrequencies in the complex frequency plane, and with the backscattering amplitude as plotted vs. frequency. In the time domain, each pole contributes a residue in the form of a decayins sinusoid-the ringing of the corresponding scattering resonance. Short echo pulses consist of a sum over many poles, with their ringing all adding up to nonvanishing (short) echo signals of circumferential pulses, with arrival times corresponding to the group velocities of the circumferential waves. Long echo pulses contain a coherent sum of overlapping circumferential wavetrains, constructively interfering at resonance but destructively off resonance, hence leading to an approximately exponentially (actually, step-wise) decaying ringing of the resonance, or to the absence of ringing off resonance. These trains interfere arbitrarily with the specular-echo wave train, causing initial transients and a steady-state plateau in the in the latter. The success of MIIR [18] as a tool of acoustic spectroscopy [13, 19] rests in the direct measurement of resonance amplitudes (via the ringing effect), unencumbered by any interference with the specular-reflection background as in the steady-state regime. Due to the prominence and sharpness of (intrinsic) elastic-wave resonance (i. e., their high Q values), and the ensuing capability of the associated surface waves to encircle the target a large number of times before being attenuated, resonance spectroscopy in underwater acoustics may well become a useful means for target recognition if applied properly and with sufficient insight and care.

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Traitement du Signal

volume 2 - n° 1 sp - 1985

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