Ultrasonic scattering in textured polycrystals

Diffusion ultrasonore dans les polycristaux alignés

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University education in theoretical physics in Braunschweig (finals 1973, Ph. D. 1978), since Aug. 1978 working as a scientist in IzfP, Saarbrücken; main field of activity is the calculation of ultrasonic propagation in inhomogeneous materials: determination of the total ultrasonic cross sections and backscattering amplitudes of single scatterers in homogeneous isotropic materials, calculation of the dispersion relations of scattering coefficients and velocities of ultrasonic waves in macroscopically isotropic as well as in textured polycrystals; applications of these theoretical results in nondestructive testing problems as there are characterization of defects and material structure and stress determination in textured components.

SUMMARY

The theory of ultrasonic propagation in macroscopically isotropic polycrystals presented in previous papers is generalized to calculate the scattering coefficients and velocities of plane compressional and shear waves in textured polycrystals. The analytical calculations were carried out for polycrystals of cubic symmetry with rolling texture in second order perturbation theory using the assumption that the changes in the material constants from grain to grain are small. For some examples, the scattering coefficients and sound velocities are calculated numerically and plotted as a function of wave number times grain radius for different propagation directions and in the case of shear waves also for different polarizations.

KEY WORDS

Ultrasonic scattering cross section, scattering amplitude, scattering coefficient, velocity dispersion relation, nondestructive testing, defect characterization, material structure, texture.

RÉSUMÉ

La théorie de la propagation des ultrasons dans des polycristaux macroscopiquement isotropes présentée dans des travaux antérieurs a été généralisée pour calculer les coefficients de diffusion et de vitesse des ondes planes longitudinales et transversales dans des polycristaux alignés. Les calculs analytiques ont été effectués pour des polycristaux de symétrie cubique après laminage, dans le cadre d'une théorie de perturbation de second ordre, et en admettant que les variations des constantes du matériau d'un grain à un autre grain sont faibles. Pour quelques exemples, le coefficient de diffusion ainsi que la vitesse des ondes planes ont été calculées numériquement et tracées comme fonction du produit du nombre d'onde et du rayon des grains pour différentes directions de propagation et aussi, dans le cas des ondes de cisaillement, pour différentes polarisations.

MOTS CLÉS

Section de diffusion ultrasonore, amplitude de diffusion, coefficient de diffusion, relation de dispersion de vitesse, contrôle non destructif, caractérisation des défauts, structure des matériaux, texture.

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References

1. Introduction

In previous papers a theory has been developed which describes the scattering of ultrasonic waves by grains of polycrystals [1,2]. It allows calculation of the scattering coefficients and velocities of plane longitudinal and transverse ultrasonic waves in polycrystals as a function of its wave number times grain radius without limitation to the Rayleigh region. The theory includes mode conversion and multiple scattering. An application for compressional and shear waves in polycrystals of cubic symmetry with independent and uniformly distributed orientations of the grains was already presented [1, 2]. The calculation was done in second-order perturbation theory using the assumption that changes in the material constants from grain to grain are small. To this order of accuracy the solutions do not account for multiple scattering by different grains.

Recently, the theory is generalized to textured polycrystals and applied to plane waves in polycrystals of cubic symmetry with rolling texture [3]. In order to limit the analytical calculations, till now only waves with propagation and polarization parallel to symmetry axes of the texture are considered. This paper gives a short description of the calculation of ultrasonic propagation in textured polycrystals and indications to further generalizations and practical applications.

2. Calculation of the complex propagation constants of plane ultrasonic waves in textured polycrystals

2.1. Description of the approach

To calculate the complex propagation constant of an ultrasonic wave $\mathbf{s}^0$ in an inhomogeneous medium the equation of motion of the complete displacement vector $\mathbf{s}=\mathbf{s}^0+\mathbf{s}'$:

$$\sum_{j,k,l=1}^{3} \frac{\partial}{\partial x_j}\left(C_{ijkl}\frac{\partial S_k}{\partial x_l}\right)+\rho\omega^2 s_i=0,$$

which contains the incident wave $\mathbf{s}^0$ and all scattered waves $\mathbf{s}'$ is used. The x_i are Cartesian coordinates. In an inhomogeneous medium the elastic constants C_{ijkl} and the density ρ are position dependent.

Just in the same way as in the case of untextured materials [1, 2] the equation of motion is solved approximately using the perturbation method, and from its solutions the complex propagation constants of plane waves in the polycrystal considered are determined. The calculations are based upon three assumptions:

1. The polycrystal is weakly anisotropic; that means the relative mean quadratic difference of the wave number in the inhomogeneous polycrystal k_p and that one of the used homogeneous approximation k has to be much smaller than 1, $\varepsilon^2=\langle(k_p-k)^2\rangle/k^2 \ll 1$. In a single-phase polycrystal, ε is mainly determined by the ratio of the anisotropy factor and the single-crystal constants.

2. All grains have the same size and shape, and integrals over a grain volume are approximately equal to the corresponding integrals over a sphere of the same volume.

3. The probability to have a special set of material constants is the same for each grain, and there is no correlation between them from grain to grain.

The angled brackets represent the averaging of the material constants which may occur for each position. Because perturbation theory (equivalent to Born approximation) is used, the validity of the theory is limited for large ka (k = wave number, a = grain radius),

$$ka \ll 1/\varepsilon = k/\{\langle(k_p-k)^2\rangle\}^{1/2}$$

is required.

The zeroth order of the equation of motion is chosen to be homogeneous and isotropic with wave numbers k and κ for compressional and shear waves, respectively. The appropriate choice of k and κ depending on the polycrystal and the incident plane wave considered allows to keep the deviations of the elastic constants from the homogeneous isotropic approximation of the same order of magnitude as the deviations from their averaged values. In the case of the incident compressional wave:

$$S^0(\mathbf{r},t)=\mathbf{s}^0\exp(i\omega t)=\exp\{i(\omega t-k_u x_u)\}\,\mathbf{e}_u,$$

propagating parallel to the symmetry direction of texture $\mathbf{e}_u$ one has $k=k_u$ and $\kappa=\kappa_I$. For the incident shear wave $S^0(\mathbf{r},t)=\exp\{i(\omega t-\kappa_{uv}x_u)\}\,\mathbf{e}_v$ propagating in the same direction and being polarized parallel to another symmetry direction of texture $\mathbf{e}_v$ the values $k=k_I$, $\kappa=\kappa_{uv}$ are taken. k_u and κ_{uv} are related to the averaged elastic constants of the textured polycrystal considered, $\rho_0\,\omega^2/k_u^2 = \langle C_{uuuu}\rangle$, $\rho_0\,\omega^2/\kappa_{uv}^2 = \langle C_{uvuv}\rangle$, while k_l and κ_l belong to the corresponding macroscopically isotropic polycrystal,

$$\rho_0\,\omega^2/k_l^2 = C_{11} - \frac{2}{5}(C_{11} - C_{12} - 2\,C_{44}),$$

$$\rho_0\,\omega^2/\kappa_l^2 = C_{44} + \frac{1}{5}(C_{11} - C_{12} - 2\,C_{44}).$$

ρ_0 is the averaged value of the density.

The approach outlined above can be used to determine the complex propagation constants of plane ultrasonic waves in macroscopically isotropic as well as in textured single-phase polycrystals.

2.2 Numerical results

The analytical results are evaluated for some examples. Figures 1-12 show the normalized phase and group velocities as well as the scattering coefficients of plane waves in 70% cold-rolled steel. The texture data are taken from the literature [4]. The normalization was done with the Voigt approximation of the velocities and wave numbers, respectively, of the corresponding macroscopically isotropic polycrystal. For the special example considered the limit in validity for large frequencies is given by $k_l a \ll 1/\varepsilon_u \approx 23.57$ and $\kappa_l a \ll 1/\varepsilon_{uv} \approx 9.85$ for compressional and shear waves, respectively.

3. Concluding remarks

The theory presented allows to calculate the orientation dependent velocities and scattering coefficients of ultrasonic waves in textured polycrystals. The weak points as there are limitation of validity for large frequencies and assuming grains of spherical shape all of the same size can be suppressed by a combination of this theory with the fundamental ideas of the "Unified Theory of Elastic Waves in Polycrystalline Materials" of Stanke and Kino [5]. The generalization of the analytical results to waves of arbitrary propagation direction and polarization is no problem in principle but requires an immense expansion of the analytical calculations. They will be finished in the early future. Furthermore, it is planned to modify the theory so that the restriction to single-phase polycrystals can be dropped.

Finally, the practical applicability of the theory is pointed out. The interdependence between ultrasonic propagation and material structure can be exploited for its characterization [6]. E. g. the ability to calculate sound velocities and scattering coefficients for materials with known texture can be used inversely to determine texture from measured complex propagation constants of ultrasonic waves. Furthermore, stress determination with ultrasonic velocity measurements are no longer restricted to macroscopically isotropic materials since the texture induced velocity alterations can be calculated [7].

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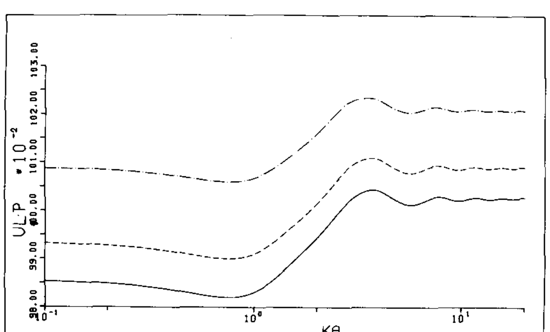

FIG.01 NORMAL. PHASE VELOCITIES OF COMPRESS. WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING(—), TRANSVERSE(--), AND NORMAL(-·-) DIRECTION

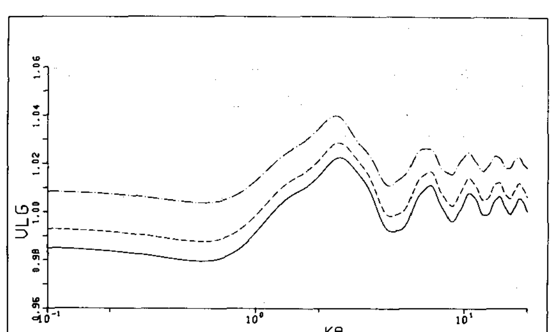

FIG.02 NORMAL. GROUP VELOCITIES OF COMPRESS. WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING(—), TRANSVERSE(--), AND NORMAL(-·-) DIRECTION

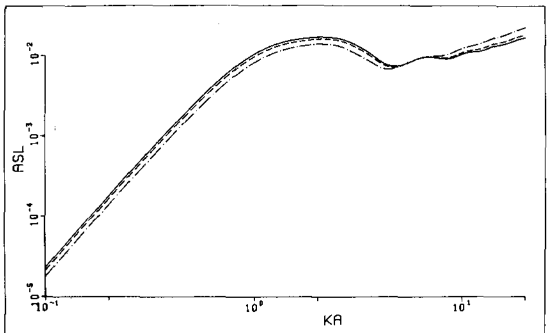

FIG. 03 NORMAL. SCATTERING COEFF. OF COMPRESS. WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING(—), TRANSVERSE(--), AND NORMAL(-·-) DIRECTION

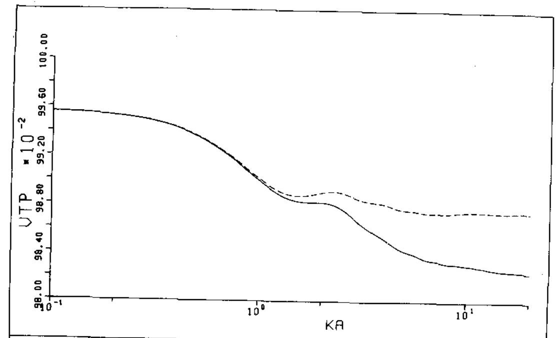

FIG. 04 NORMAL. PHASE VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN TRANSVERSE DIREC.(—) AND V. V.(--)

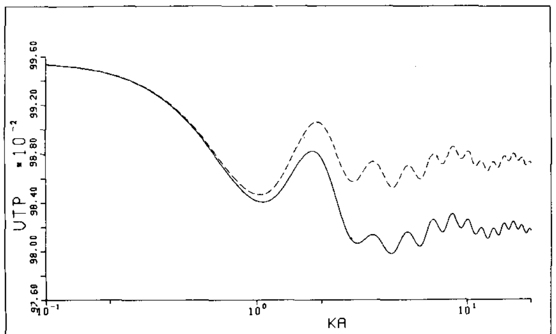

FIG. 05 NORMAL. GROUP VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN TRANSVERSE DIREC.(—) AND V. V.(--)

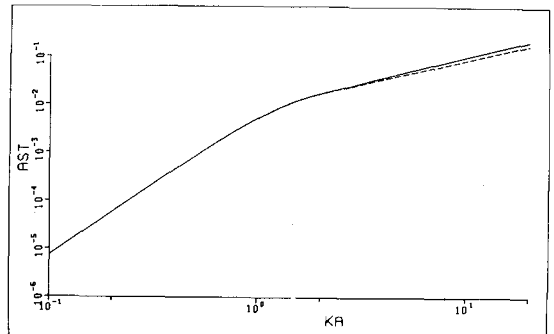

FIG. 06 NORMAL. SCATTERING COEFF. OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN TRANSVERSE DIREC.(—) AND V. V.(--)

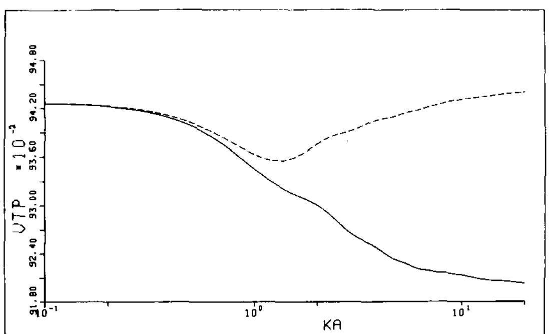

FIG. 07 NORMAL. PHASE VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)

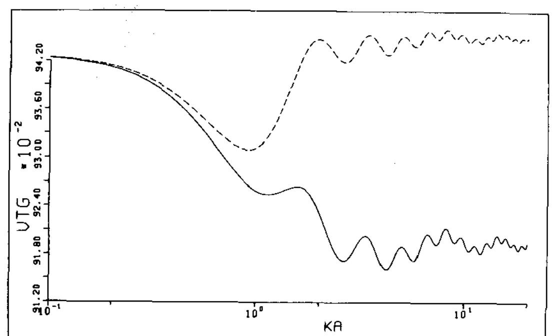

FIG. 08 NORMAL. GROUP VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)

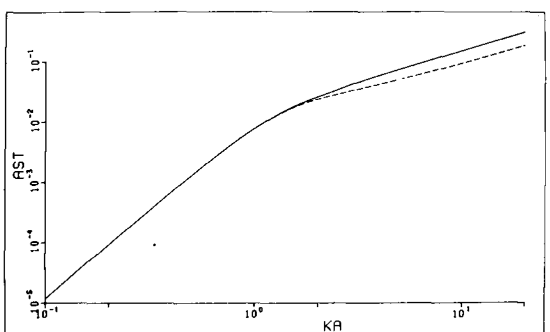

FIG. 09 NORMAL. SCATTERING COEFF. OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN ROLLING WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)

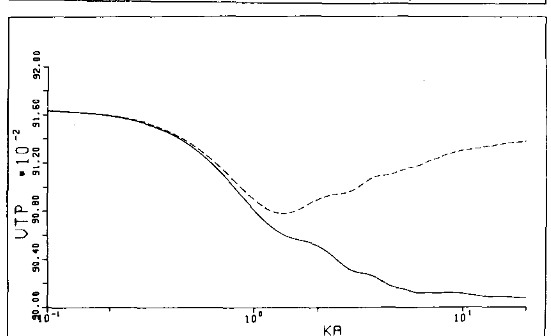

FIG. 10 NORMAL. PHASE VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN TRANSVERSE WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)

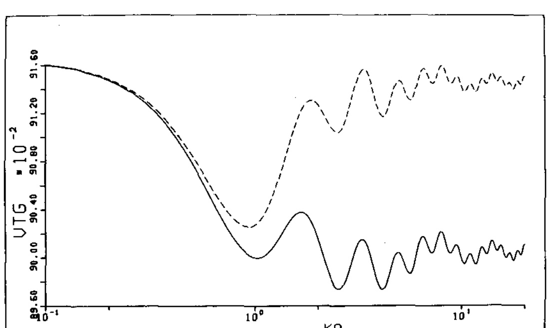

FIG. 11 NORMAL. GROUP VELOCITIES OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN TRANSVERSE WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)

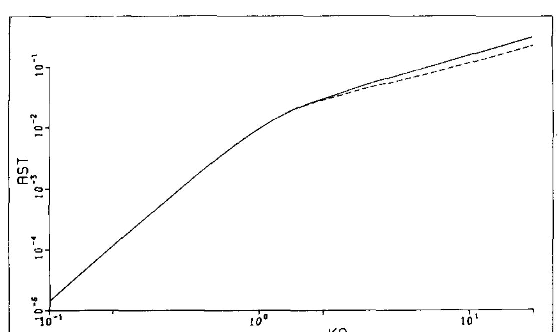

FIG. 12 NORMAL. SCATTERING COEFF. OF SHEAR WAVES IN 70% COLD ROLLED STEEL PROPAGAT. IN TRANSVERSE WITH POLAR. IN NORMAL DIREC.(—) AND V. V.(--)