Scattering from fluid and elastic layers

Diffusion par des couches fluides et élastiques

Herbert ÜBERALL

Department of Physics, Catholic University of America, Washington DC 20064, USA

Professor of Physics, Catholic University of America, Washington DC 20064, USA (since 1964). Field of research: Theoretical Physics (acoustics, ultrasonics, elastic and electromagnetic waves, nuclear physics).
Acoustics: Advancement of the theory of surface waves in acoustic scattering. Pioneering development of acoustic resonance scattering theory. Channeling in under-ocean sound propagation.
Elastic waves: First development of theory of resonance scattering by cavities and inclusions in solids.
Electromagnetic waves: Establishment of the relation between resonance scattering theory and the Singularity Expansion Method (SEM). Theory of radio-wave propagation in the earth-ionosphere waveguide.
Nuclear theory: Basic development of coherent bremsstrahlung theory (editor of recent book). Development of a model of collective nuclear multipole vibrations, with application to electron scattering (author of two books). Photo-pion reactions. Collective model of ion-ion scattering. Neutrino reactions (editor of book).

SUMMARY

In this report, we describe a complete resonance theory for the acoustic transmission and reflection coefficients of an elastic plate imbedded in a fluid medium, including effects of plate viscosity. The purpose of this formulation is to provide a direct means for determining the material parameters of the plate from the measured acoustic resonances of the Rayleigh and Lamb waves in the plate (i. e., their positions in frequency or angle, their widths and their heigths) which are given in our formalism by explicit analytic expressions that depend on the material parameters. Viscosity is seen to manifest itself in a decrease of the resonance heights (especially for the narrow shear-type resonances) and in a broadening and frequency dependence of their widths, which may be used to determine the frequency-dependent loss factor of the plate. This approach then solves the inverse scattering problem for the case of a plate. We also consider the special case of a fluid layer imbedded in another fluid. In addition, a layered ocean floor is modeled by a sediment layer on top of a denser substratum, and overlaid by the water column. It is shown for this case again that resonances in the acoustic reflection coefficient are very prominent and can be used to determine the properties of both sediment layer and substratum.

KEY WORDS

Elastic plates, fluid layers, resonant acoustic transmission, resonant acoustic reflection, plate viscosity, material parameters, inverse scattering, layered ocean floor.

RÉSUMÉ

Dans cet article, nous présentons une théorie complète des résonances dans les coefficients de transmission et de réflexion acoustique par une plaque élastique immergée dans un milieu fluide, en tenant compte des effets dus à la viscosité de la plaque. Le but de ce formalisme est de fournir un moyen direct de détermination des paramètres du matériau constituant la plaque, par la mesure des résonances des ondes de Rayleigh et de Lamb (c'est-à-dire, par leur position en fréquence ou en angle, leurs largeurs et leurs hauteurs). Ces quantités sont données dans notre formalisme par des expressions analytiques explicites qui contiennnent les paramètres du matériau. On trouve que la viscosité se manifeste par une décroissance des hauteurs des résonances (particulièrement pour les résonances étroites du type transversal), et par un élargissement et une dépendance en fréquence de leurs largeurs. Cette méthode permet de résoudre alors le problème de diffusion inverse pour le cas d'une plaque. Nous considérons également le cas spécial d'une couche fluide immergée dans un autre fluide. En outre, nous représentons les couches du fond de l'océan par une couche de sédiment posée sur un fond plus dense, et couverte par l'eau de l'océan. Nous montrons que, pour ce cas également, les résonances dans le coefficient de réflexion acoustique apparaissent de façon très visible, et peuvent être utilisées pour la détermination des propriétés de la couche de sédiment et du fond.

MOTS CLÉS

Plaques élastiques, couches fluides, transmission acoustique résonnante, réflexion acoustique résonnante, viscosité, paramètres du matériau, diffusion inverse, fond d'océan stratifié.

CONTENTS

1. Introduction

A comprehensive theoretical and experimental investigation by Schoch [1] has shown that the process of reflection and transmission of an acoustic wave, incident through a fluid on an elastic plate imbedded in that fluid, is capable of exciting elastic Rayleigh and Lamb-type waves [2] in the plate. Such an excitation takes place when the trace of the phase velocity of the incident wave, as it moves along the plate, coincides [3] with the phase velocity of the plate waves: it manifests itself in the appearance of resonances [4] in the acoustic reflection (R) and transmission coefficient (T) of the plate, either as the frequency f is varied at a fixed angle of incidence θ from the normal, or as θ is varied at a fixed frequency.

Mathematically, such resonances in R or T come about by a vanishing of the common denominator in the fractional analytic expressions for these two quantities. Zeros of this denominator, i. e., poles in R and T, do not occur for real values of both f and θ; they occur, e. g., at complex values of f if θ is kept real (physical), or vice versa. Since experimentally both f and θ are real, measurements with variation of f will carry us successively past the poles which are located somewhat off the real f-axis, thus leading to finite heights and widths of the observed resonance peaks.

The resonance features in R and T can be studied by isolating their pole contributions, that is, by replacing the denominator by its first term in a Taylor-series expansion about each of the poles. This was done by Fiorito, Madigosky and Überall for the cases of a fluid layer in another fluid, the layer being either lossless [4] or lossy [5], as well as for an elastic plate imbedded in a fluid, with the plate being lossless [6] or lossy [7]. It is possible, however, to go beyond this approach, and to represent R and T by an exact series representation as a sum of resonance terms in the sense of Mittag-Leffler [8]; this is also akin to the theory of resonances in nuclear reactions, known as the Breit-Wigner theory [9]. Such an exact representation is discussed here, as given recently by Fiorito, Madigosky and Überall [10] for the case of a fluid layer. The merit of such an exact resonance decomposition rests in the fact that it admits a description of the resonances even for small impedance contrasts between the layer and the ambient fluid.

The mentioned investigations [4-7,10] have provided analytical expressions for the resonance positions (in f or θ), their heights and their widths. These expressions depend on the material parameters of the plate, and the measurement of these quantities thus furnishes a direct means for the determination of the material parameters by remote acoustic sensing. In a way, such a procedure then corresponds to a solution of the "inverse scattering problem", where the properties of the scatterer are determined from the form of the returned echo.

An experimental study of plate resonances has recently been carried out by Izbicki, Maze, and Ripoche [11-13], showing in exact detail the backscattering resonances in the reflection coefficient for aluminium plates immersed in water as plotted vs. f, and featuring their positions in f, their heights and widths.

Fluid layers have also been used to model sediment layers of the ocean bottom, lying on top of a substratum which was also treated as a fluid [14]. Calculated reflection coefficients for a layered ocean bottom show very prominent resonance features [15]. These are explained in the mentioned model, which proceeds with a detailed analysis to show what types of measurements have to be carried out on these resonance features in order to completely determine the geometrical and material properties of layer and substratum. Finally, a way of analyzing the layer resonances via the use of long sound pulses, which are employed in order to cause a ringing of the layer resonances one at a time, has also been pointed out [16].

2. Theory of layer resonances

The exact expression for the transmission and reflection coefficient of an elastic layer as calculated by Schoch [1] is first rewritten in a convenient form:

$$(2.1\,a)\qquad T=i\tau\left(\frac{1}{C_s-i\tau}+\frac{1}{C_a+i\tau}\right),$$

$$(2.1\,b)\qquad R=\frac{C_sC_a-\tau^2}{C_s+C_a}\left(\frac{1}{C_s-i\tau}+\frac{1}{C_a+i\tau}\right),$$

with:

$$(2.2\,a)\qquad \tau=\frac{\rho}{\rho_d}\frac{(n_d^2-\sin^2\theta)^{1/2}}{\cos\theta},$$

and $n_d=c/c_d$, where d is the layer thickness, and $c\,(c_d,\,c_t)$ and $\rho\,(\rho_d)$ are the acoustic (dilatational d, and transverse t) wave speeds and densities in the external (or layer) media, respectively. The quantities C_s and C_a depend [6] on these parameters or combinations these of, such as on:

$$(2.2\,b)\qquad \delta=\frac{1}{2}x(n_d^2-\sin^2\theta)^{1/2},$$

where $x=2\pi fd/c$. Viscosity is described by complex wave velocities c_d^*, c_t^*. For the case of a fluid layer ($c_t=0$), one has $C_s=\cot\delta$, $C_a=\tan\delta$.
A Taylor expansion of the denominators of Equations (2.1) leads to the series of resonance terms (of Breit-Wigner form):

$$(2.3\,a)\qquad T\equiv T_s+T_a=\sum_{m_s}T_{m_s}+\sum_{m_a}T_{m_a},$$

$$(2.3\,b)\qquad R\equiv R_s+R_a=\sum_{m_s}R_{m_s}+\sum_{m_a}R_{m_a},$$

where:

$$(2.4\,a)\qquad T_{m_{s,a}}\cong\mp\frac{(i/2)\,\Gamma_{m_{s,a}}}{x-x_{m_{s,a}}+(i/2)(\Gamma_{m_{s,a}}+\Delta\Gamma_{m_{s,a}})},$$

$$(2.4\,b)\qquad R_{m_{s,a}}\cong\frac{x-x_{m_{s,a}}+(i/2)\,\Delta\Gamma_{m_{s,a}}}{x-x_{m_{s,a}}+(i/2)(\Gamma_{m_{s,a}}+\Delta\Gamma_{m_{s,a}})}.$$

The (normalized) resonance frequencies $x_{m_{s,a}}$ are found as the solutions of the lossless free-plate characteristic equations:

$$(2.5)\qquad C_{s,a}(x_{m_{s,a}})=0,$$

and the resonance widths $\Gamma_{m_{s,a}}$ as well as their loss broadening $\Delta\Gamma_{m_{s,a}}$ are given [6,7] as analytic expressions in terms of the material parameters of the plate. As mentioned, this leads to the possibility of determining these parameters from a measurement of the resonance features.
The derivation of Equations (2.3)-(2.5) was based on the assumption of a large impedance contrast between fluid and plate, i.e., $\tau\ll 1$. A Mittag-Leffler type meromorphic series expansion gives, e.g., for a lossless fluid layer:

$$(2.6)\qquad T=\frac{\tau}{\sigma(1-\tau^2)}\left\{\sum_{m_s=-\infty}^{\infty}\frac{-(1/2)\,i\Gamma}{x-x_{m_s}+(1/2)\,i\Gamma}+\sum_{m_a=-\infty}^{\infty}\frac{(1/2)\,i\Gamma}{x-x_{m_a}+(1/2)\,i\Gamma}\right\},$$

where $\sigma=\tanh^{-1}\tau$,

$$(2.7\,a)\qquad x_{m_s}=(2m_s+1)\,\pi\,(n_d^2-\sin^2\theta)^{-1/2},$$

$$(2.7\,b)\qquad x_{m_a}=2m_a\pi\,(n_d^2-\sin^2\theta)^{-1/2},$$

$$(2.7\,c)\qquad \frac{1}{2}\Gamma=\frac{2\sigma}{(n_d^2-\sin^2\theta)^{1/2}}.$$

Equation (2.6) is an exact representation for T, valid for all $\tau<1$. A corresponding expression can be found for $\tau>1$ [10].

3. Numerical results

Figure 3.1 shows the resonances in $|T|^2$ vs. f at $\theta=19°$, for a one-inch thick absorptive plate in silicone oil, representing plots of the individual resonance terms in the present formalism [7]. The classification in terms of antisymmetric (a) and symmetric (s) plate waves, as well as their dilatational (d) and transverse (t) content follows Brekhovskikh [2].
Figure 3.2 then shows the coherent sum of the terms of the present FMU resonance theory, compared to the result of the exact expression, Equation (2.1 a). The agreement is adequate at the prevailing impedance contrast ($\tau=0.211$ at $\theta=0°$), and it also shows that the representation by individual resonances of the FMU theory provides a means of resolving cases of interfering resonances in the coherent sum, such as at $f\simeq 60$, 105 and 155 kHz.

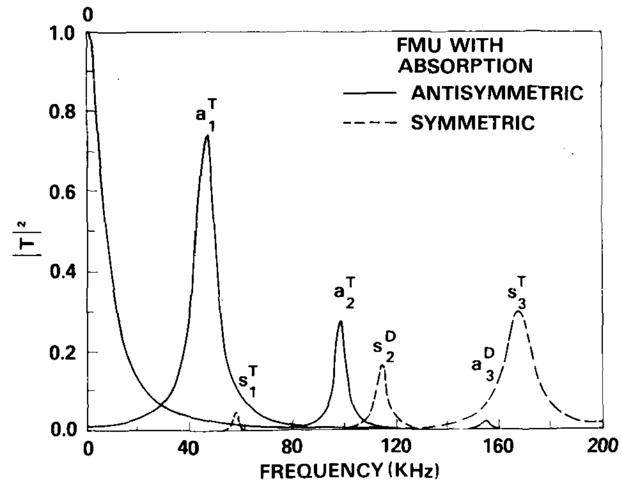

Fig. 3.1. — Individual resonances of $|T|^2$ from the resonance theory, for a 1-inch plexiglas plate *in* silicone oil at $\theta = 19^\circ$.

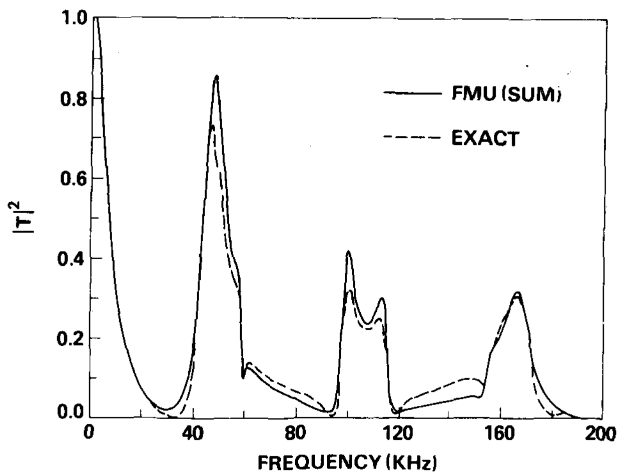

Fig. 3.2. — Coherent sum of resonances in $|T|^2$ for a 1-inch plexiglas plate *in* silicone oil at $\theta = 19^\circ$, compared with exact result.

4. Ocean floor resonances

Calculated reflection coefficients for models of a layered ocean floor often contain prominent resonances [15]. Figure 4.1 shows a two-dimensional plot of $|R|^2$ for two (fluid) sediment layers on top of a fluid substratum, plotted vs. both f and θ [14]. In this reference, a detailed analysis is given how to determine the bottom structure from measurements on the resonances; a large redundancy of measurements was found to apply. In addition, the ringing of layer resonances as manifested by a tail in long, specularly reflected wavetrains (Fig. 4.2) has been suggested [16] as a means of measuring the resonance properties.

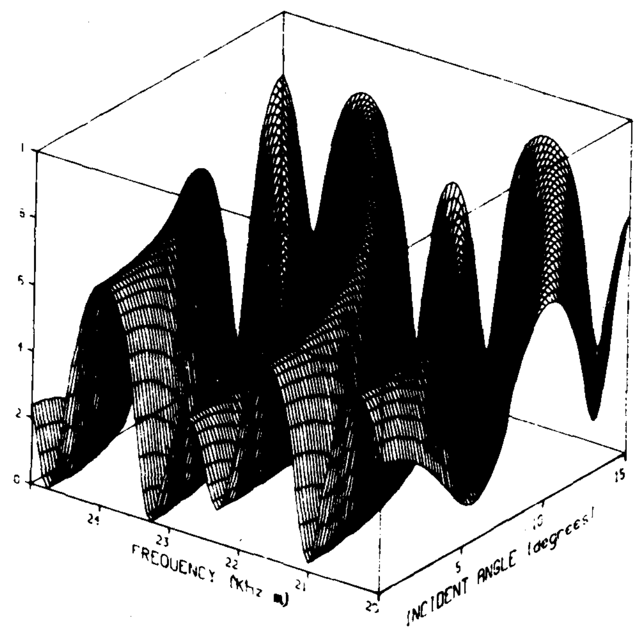

Fig. 4.1. — $|R|^2$ for two ocean-floor sediment layers on top of a substratum, plotted vs. frequency-thickness and incident angle.

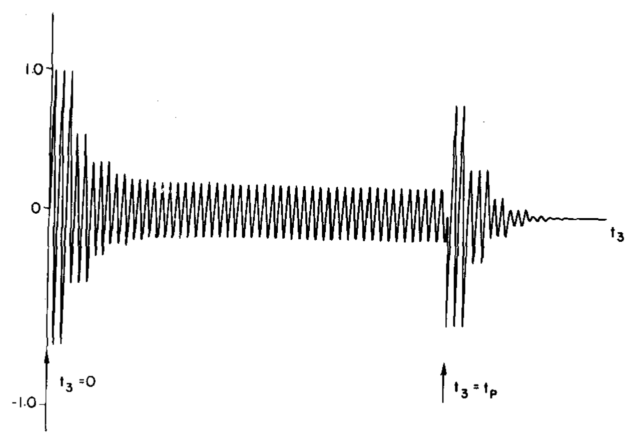

Fig. 4.2. — Reflected signal vs. time, for incident wave train with carrier frequency equalling a resonance frequency of *an* ocean floor sediment layer. Final transient is due to ringing of layer resonance.

5. Conclusion

The analysis of resonances in the acoustic reflection from layers and plates has shown that reflection and transmission coefficients can be represented as a sum of resonance terms, of Breit-Wigner form familiar from nuclear physics, either in the "resonance approximation" or *in* an exact meromorphic series. The resonance positions (in frequency, or in angle of incidence), heights, and widths depend on the material parameters, and their measurement may thus serve to determine these parameters by remote acoustic sensing. An explicit demonstration of such a procedure was provided [14,16] for the case of sediment layers on the ocean floor.

Portions of the results reported here are based on work supported by the ONR and the NSWC.

REFERENCES

[1] A. Schoch, Der Schalldurchgang durch Platten, *Acustica*, 2, 1952, pp. 1-7.

[2] *See*, e.g., L. M. Brekhovskikh, *Waves in Layered Media*, Academic Press, New York, 1960.

[3] L. Cremer, Über die Analogie zwischen Einfallswinkel und Frequenzproblemen, Arch. Elektr. Übertragung, 1, 1947, pp. 28- .

[4] R. Fiorito and H. Überall, Resonance theory of acoustic reflection and transmission through a fluid layer, *J. Acoust. Soc. Amer.*, 65, 1979, pp. 9-14.

[5] R. Fiorito, W. Madigosky and H. Überall, Acoustic resonances and the determination of the material parameters of a viscous fluid layer, *J. Acoust. Soc. Amer.*, 69, 1981, pp. 897-903.

[5] R. Fiorito, W. Madigosky and H. Überall, Acoustic resonances and the determination of the material parameters of a viscous fluid layer, *J. Acoust. Soc. Amer.*, 69, 1981, pp. 897-903.

[6] R. Fiorito, W. Madigosky and H. Überall, Resonance theory of acoustic waves interacting with an elastic plate, *J. Acoust. Soc. Amer.*, Vol. 66, 1979, pp. 1857-1866.

[7] R. Fiorito, W. Madigosky and H. Überall, Theory of ultrasonic resonances in a viscoelastic layer, *J. Acoust. Soc. Amer.*, 77, 1985, pp. 489-498; *see* also R. Fiorito, W. Madigosky and H. Überall, Determination of the material parameters of fluid or viscoelastic layers from their ultrasonic resonances, *Proceed. Ultrasonics International* 81 *Conference*, Brighton, England, 1981.

[8] *See*, e.g., L. Bieberbach, *Lehrbuch der Funktionentheorie*, Chelsea Publ. Co., New York, 1945, I, Chap. 13.1,2.

[9] G. Breit and E. P. Wigner, The scattering of slow neutrons, *Phys. Rev.*, 49, 1936, pp. 519-531.

[10] R. Fiorito, W. Madigosky and H. Überall, An exact resonance decomposition of acoustic transmission and reflection coefficients of a fluid layer, submitted to *J. Acoust. Soc. Amer.*

[11] J. L. Izbicki, G. Maze and J. Ripoche, Résonance d'une plaque élastique, *Revue du CETHEDEC*, 78, 1984, pp. 161-169.

[12] J. L. Izbicki, G. Maze and J. Ripoche, Étude de la réémission libre d'une plaque résonnante, *Acustica*, 55, 1984, pp. 27-29.

[13] J. L. Izbicki, G. Maze, M. Cherif and J. Ripoche, Résonances : ondes guidées dans les cylindres et dans les plaques, this colloquium.

[14] A. Nagl, H. Überall and W. R. Hoover, Resonances in acoustic bottom reflection and their relation to the ocean bottom properties, *IEEE Transactions Geosc. Rem. Sensing*, GE-20, 1982, pp. 332-337.

[15] P. J. Vidmar and T. L. Foreman, A plane-wave reflection loss model including sediment rigidity, *J. Acoust. Soc. Amer.*, 66, 1979, pp. 1830-1835.

[16] A. Nagl, H. Überall and K. B. Yoo, Acoustic exploration of ocean floor properties based on the ringing of sediment layer resonances, Department of Physics, Catholic University of America Report, November 30, 1983; *Journal of Physics H-Inverse Problems*, 1, 1985, pp. 99-110.