1. Physics of imaging

Unconventional imaging and co-design

Part III: Image reconstruction methods and examples in DH

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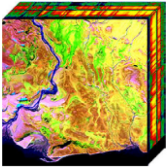
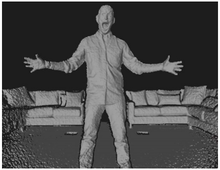
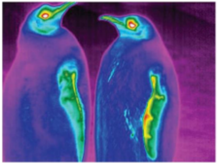
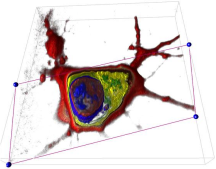
Outline

Extraction of images information

Strength of unconventional imaging = richer information

- Measurements of physical quantities : Depth, humidity, chemical composition, temperature, phase shift (optical thickness, refractive index), ...
- Quantitative imaging.
- Super resolution/Low cost setups...
- Various applications : Biomedical imaging, Chemical engineering, ...

Reconstruction

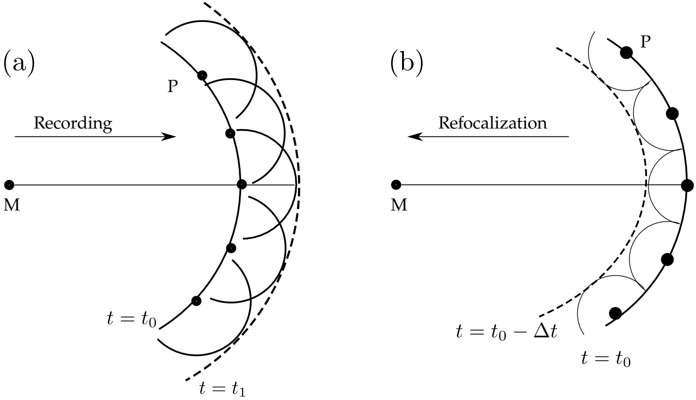
To obtain the measurements of the physical quantities from the data several approaches exist based on :

- physics
- signal processing
- machine learning

These approaches are illustrated in the following in digital holographic microscopy.

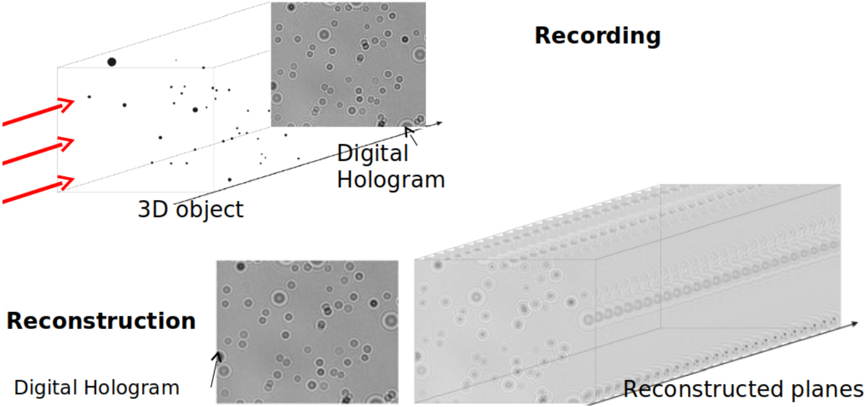
Reconstruction methods based on Physics

Back-Propagation

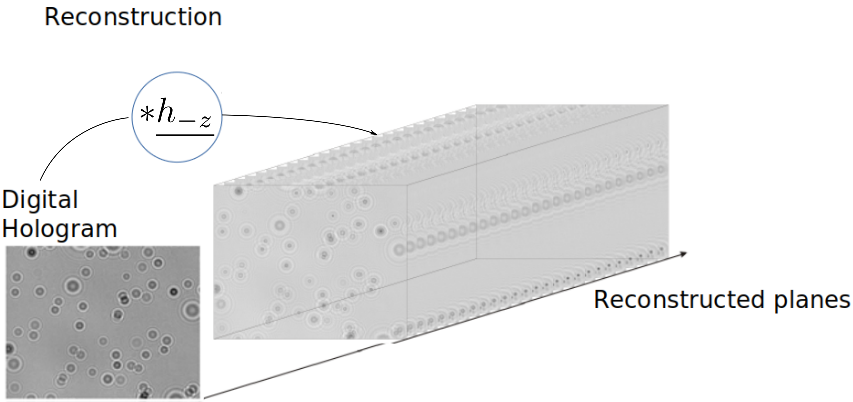


Principle of the reversibility of light.
Propagation models.

Back-Propagation



Back-Propagation



Free space propagation model : Rayleigh-Sommerfeld, Fresnel, Fraunhofer, ...

Mathematical formulation

$$\underline{U}^{Rec}(x,y) = I_z(x,y) \underset{x,y}{*} \underline{h}_{-z}$$

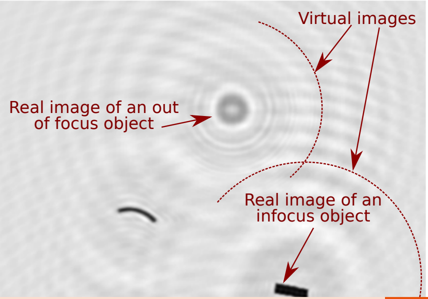
$$= \underbrace{\left(1 + |\underline{\vartheta} \underset{x,y}{*} \underline{h}_z|^2\right) \underset{x,y}{*} \underline{h}_{-z}}_{(0)} \underbrace{- \underline{\vartheta} \underset{x,y}{*} \underline{h}_z \underset{x,y}{*} \underline{h}_{-z}}_{(+1)} \underbrace{- \underline{\vartheta}^* \underset{x,y}{*} \underline{h}_z^* \underset{x,y}{*} \underline{h}_{-z}}_{(-1)}$$

(0) →low frequency pattern

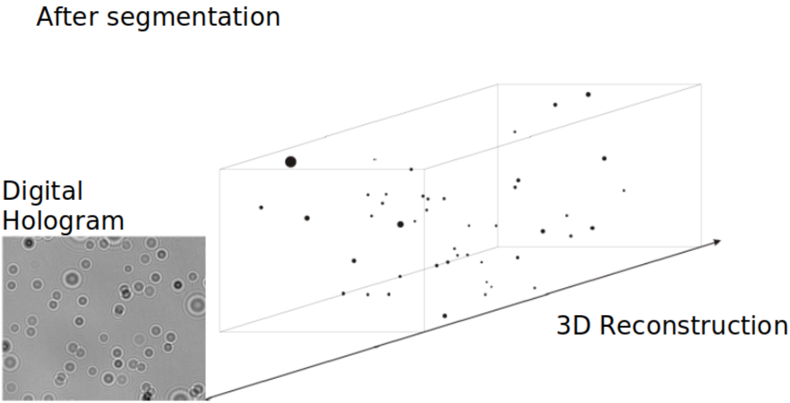
$(+1) : -\underline{\vartheta}$ →real image (in focus)

$(-1) : -\underline{\vartheta}^* \underset{x,y}{*} \underline{h}_{-2z}$ →virtual image (out of focus)

Propagation model with parameter $-z$ does not invert hologram recording because of the non linearity introduced by the intensity.
→Virtual images, borders distortions (numerical artifact), ...



Stack segmentation

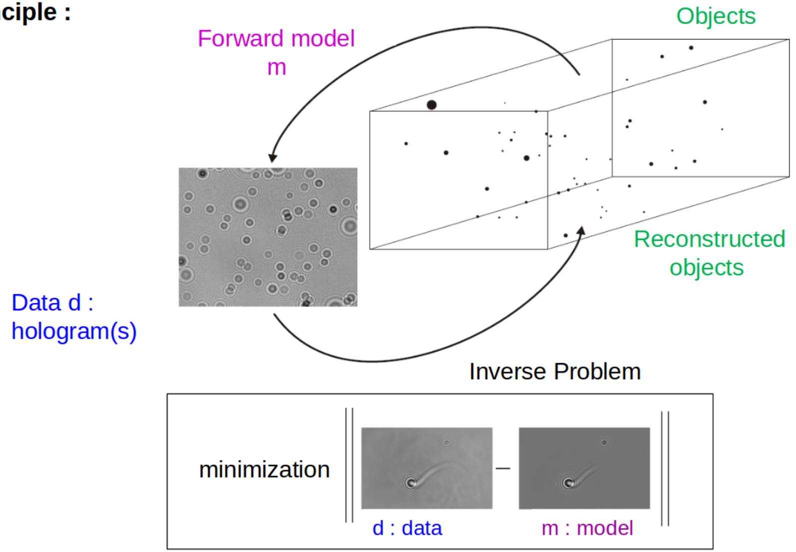


Reconstruction methods based on IP approach

Principle

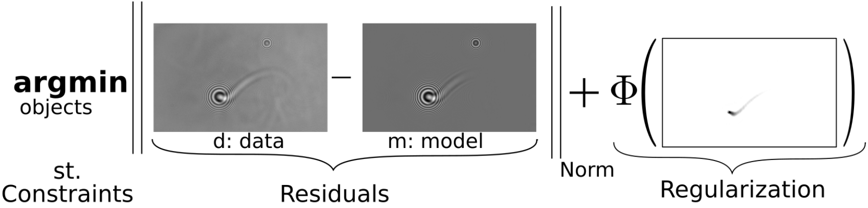
Minimization of data misfits

Principle :

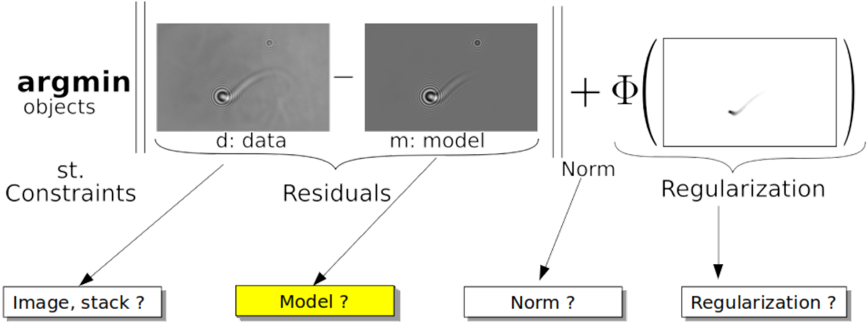


a priori information

The inverse problem is often ill-posed and an a priori information on the object to reconstruct is added → constraints and regularization.



Key questions

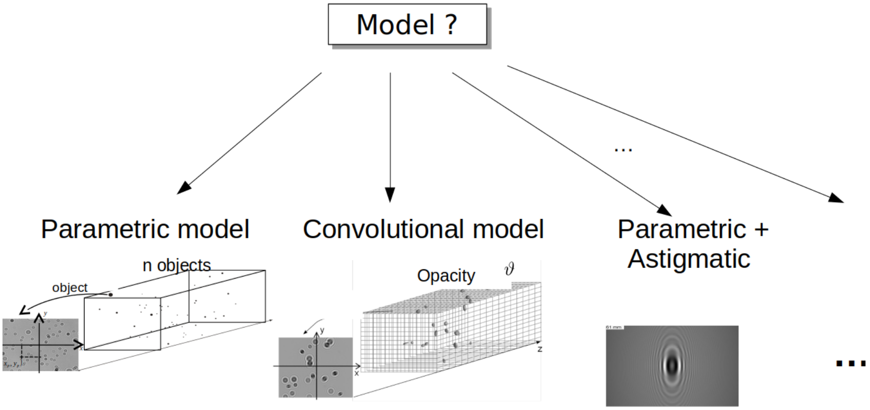


Goals :

- Accuracy
- Robustness vs noise
- Missing data
- Tractability

Forward model

Forward models

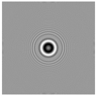


Parametric and convolutional models

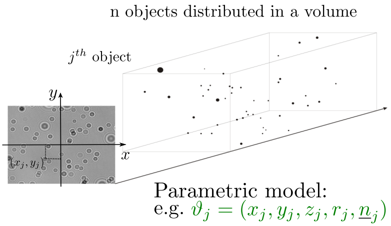
Example of simple, linear additive model : parametric model

(FI) parametric model

$$m(x_p, y_p) = I_0 - I_0 \sum_{j=1}^{n} \alpha_j g_{\vartheta_j}(x_p, y_p) + \epsilon(x_p, y_p)$$



e.g. Mie, Thompson ...



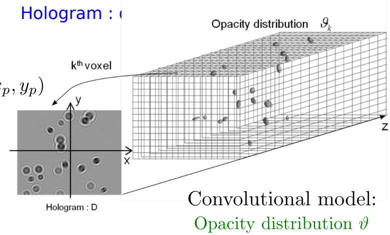
Hologram : d

Parametric and convolutional models

Example of simple, linear additive model : convolutional mode

(FII) convolutional model

$$m(x_p, y_p) = I_0 - I_0 \sum_{k=1}^{N \text{ voxels}} [h_{z_k} * \vartheta_k](x_p, y_p) + \epsilon(x_p, y_p)$$

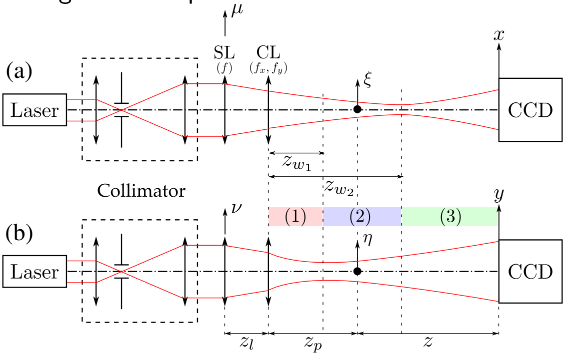


Other non linear and non additive models (accounting for interferences) exist as BPM (Beam Propagation Model) :

* Kamilov, Papadopoulos, Shoreh, Goy, Vonesch, Unser, & Psaltis, "Learning approach to optical tomography". Optica, 2015

Parametric model example

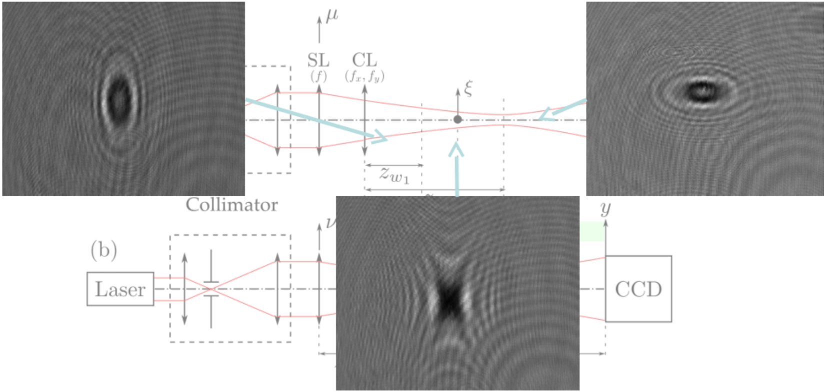
Parametric models are very constrains →used to (self-)calibrate a setup.
Example of an astigmatic setup :



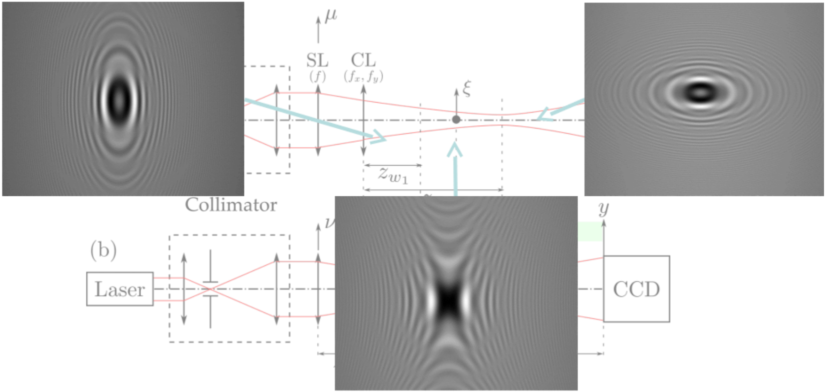
The model depends on the object parameters (calibration bead) and on the setup parameters (astigmatism parameters ...)

N. Verrier et al, "In-line particle holography with an astigmatic beam : setup self-calibration using an "inverse problems" approach", Applied Optics, 2014.

Parametric model example



Parametric model example

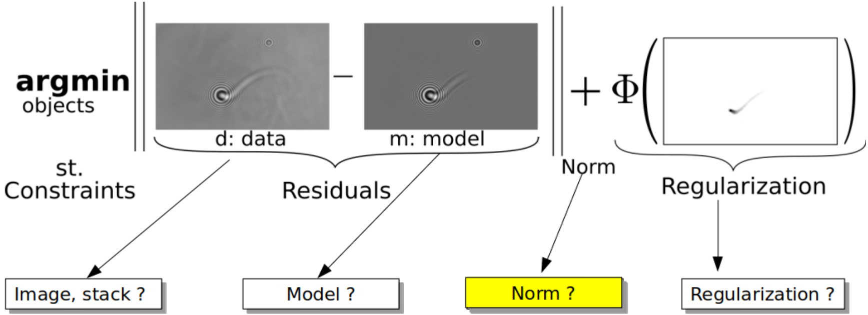


Agreement between the model and the experimental patterns

N. Verrier et al, "In-line particle holography with an astigmatic beam : setup self-calibration using an "inverse problems" approach", Applied Optics, 2014.

Norm

Key question

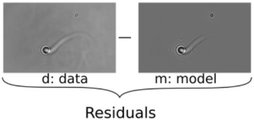


Goals :

- Accuracy
- Robustness vs noise
- Missing data
- Tractability

Norm choice depends on the noise model

Norm ?

$\mathcal{C}(M_\vartheta) = \Big\|$  $\Big\|$

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} |d(a,b) - m(a,b)|^2$

Least Squares

White Gaussian Noise

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)|d(a,b) - m(a,b)|^2$

Weighted Least Squares

Non stationary noise

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)\rho(d(a,b) - m(a,b))$

Robust estimation

Non gaussian noise

$\mathcal{C}(M_\vartheta) = (D - M)^t \mathbf{W} (D - M)$

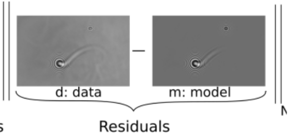
Use of noise covariance matrix

Correlated noise

Weighted least squares

Norm ?

$\mathcal{C}(M_\vartheta) =$



$$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} |d(a,b) - m(a,b)|^2$$

Least Squares

$$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} w(a,b)|d(a,b) - m(a,b)|^2$$

Weighted Least Squares

$$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} w(a,b)\rho(d(a,b) - m(a,b))$$

Robust estimation

$$\mathcal{C}(M_\vartheta) = (D - M)^t \mathbf{W} (D - M)$$

Use of noise covariance matrix

Weight matrix

Weighted Least Squares $$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)\,|d(a,b) - m(a,b)|^2$$

Assumption : additive White Gaussian Noise (WGN) with a varying variance in the field

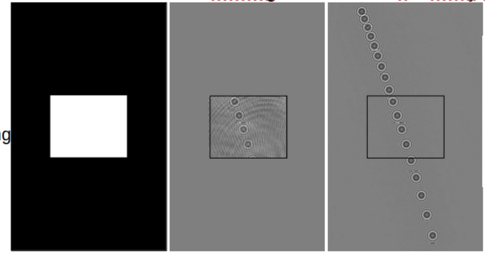
$$w(a,b) = \frac{1}{\sigma^2(a,b)}$$

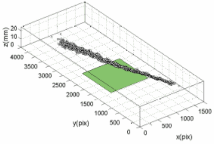
w can be used to account for **dead pixels** $\rightarrow$ $w(a,b) = 0$

missing data $\rightarrow$ w is a binary mask

Binary weights example for missing data

w





3D Reconstruction of 200 holograms of droplets

F. Soulez et al., "Inverse-problem approach for particle digital holography : accurate location based on local optimization", JOSA A,2007.

F. Soulez et al., "Inverse-problem approach for particle digital holography : out-of-field particle detection made possible",JOSA A,2007.

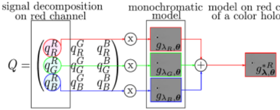
Weight matrix

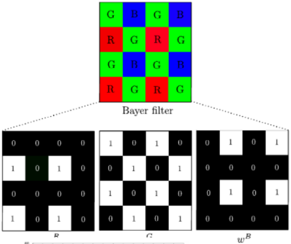
Weighted Least Squares $$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)|d(a,b) - m_\vartheta(a,b)|^2$$

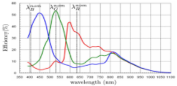
Bayer Color matrix example

$$\mathcal{C}(M_\vartheta) = \sum_{c=1}^{3}\sum_{a=1}^{A}\sum_{b=1}^{B} w^c(a,b)|d^c(a,b) - m^c(a,b)|^2$$

=> avoid interpolation
=> makes possible self calibration :
estimation of the crosstalk coefficients







Spectral responses of Bayer filter

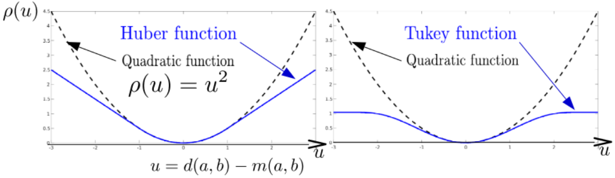
O. Flasseur, C. Fournier, N. Verrier, L. Denis, F. Jolivet, A. Cazier, T. Lépine, "Self-calibration for lensless color microscopy". Applied Optics, 2017.

Robust estimation

Robust estimation $$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)\rho\left(\frac{d(a,b)-m(a,b)}{s}\right)$$

Issue : suppress bias due to outliers

1/ Change the penalization function



- reduce the penalization of largest difference between data and model
- replace the least squares by another objective function
  $\Rightarrow$ Using the M-estimator

P.J. Huber, "Robust statistics. " Springer, 2011.

Penalization function

Robust estimation

$$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A}\sum_{b=1}^{B} w(a,b)\rho\left(\frac{d(a,b) - m_\vartheta(a,b)}{s}\right)$$

Issue : suppress bias due to outliers

2/ Change the outliers weights

- Compute iteratively the weight w accounting for residuals values

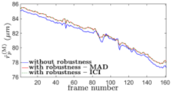
$$\hat{\vartheta}_{\mathbf{k+1}} = \underset{\vartheta}{argmin} \sum_{a=1}^{A}\sum_{b=1}^{B} w_k(a,b)\rho\left(\frac{d(a,b) - m_\vartheta(a,b)}{s}\right)^2$$

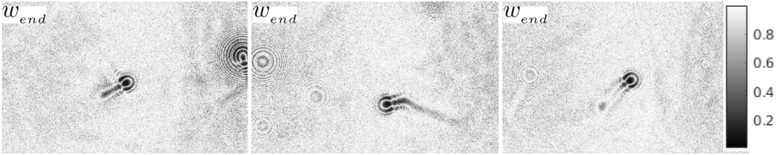
$$w_k(a,b) = w(a,b)\frac{s}{r_k(a,b)}.\frac{\partial\rho(u)}{\partial u}|_{u=r_k(a,b)/s}.$$

Penalization function

video

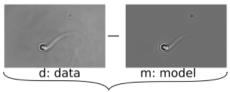
O. Flasseur, et al, "Robust object characterization from lensless microscopy videos", In IEEE European Signal Processing Conference (EUSIPCO), 2017.





Use of noise covariance matrix

Norm ?

$\mathcal{C}(M_\vartheta) = \Big\|$  $\Big\|$

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Use of noise covariance matrix

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} w(a,b)\rho(d(a,b) - m(a,b))$

Robust estimation

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} |d(a,b) - m(a,b)|^2$

Least Squares

$\mathcal{C}(M_\vartheta) = \sum_{a=1}^{A} \sum_{b=1}^{B} w(a,b)|d(a,b) - m(a,b)|^2$

Weighted Least Squares

O. Flasseur, L. Denis, E. Thébaut, et al. "ExPACO : detection of an extended pattern under nonstationary correlated noise by patch covariance modeling", IEEE European Signal Processing Conference (EUSIPCO), 2019.