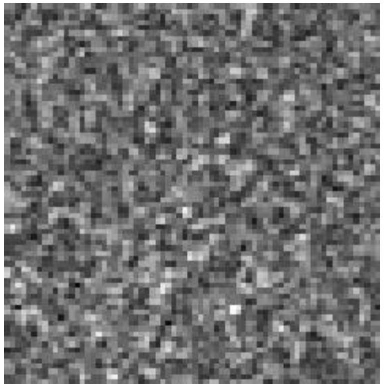
Statistiques et approches par patchs

F. Tupin, C. Deledalle, L.Denis

Ecole d’été Peyresq, juin 2015

- Modèles statistiques pour l'imagerie SAR
- Approches par patchs
- Applications à l'imagerie SAR

Introduction to SAR imagery

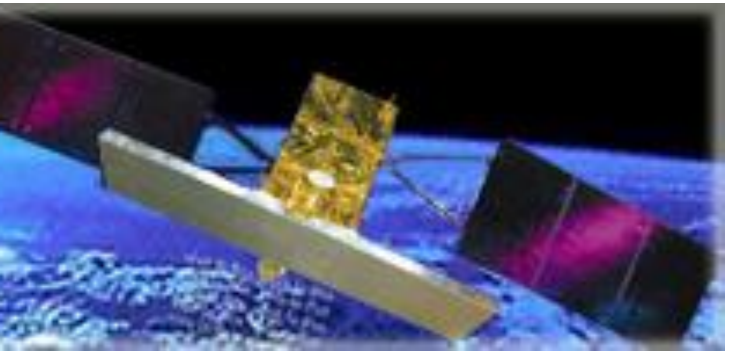


- **Principle: emission of electro-magnetic waves and recording of the backscattered signal**

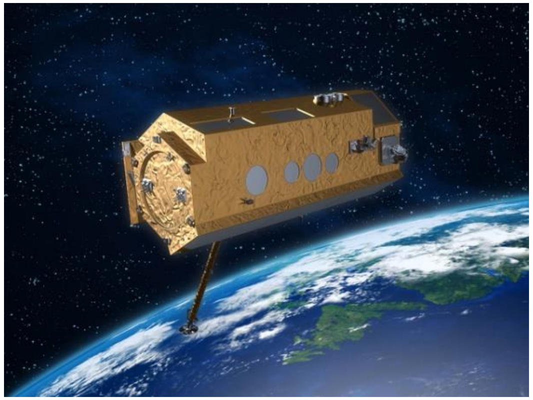
- **Specificities:**
  - All-time system
  - Distance sampling: geometrical distorsions
  - Coherent imagery: speckle noise
  - Interferometry (DTM, ground mvt monitoring,…)
  - Polarimetry

TELECOM ParisTech

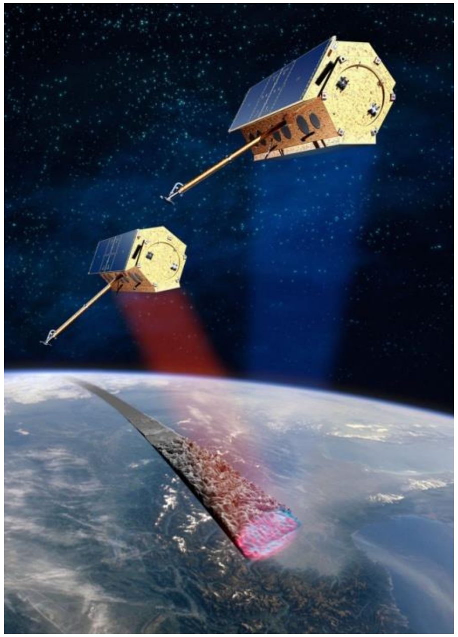
■ **New generation of SAR sensors**



CSK

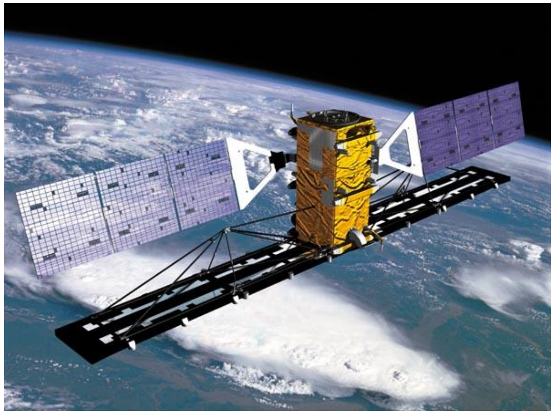


TerraSAR-X



TanDEM-X

Sentinel I



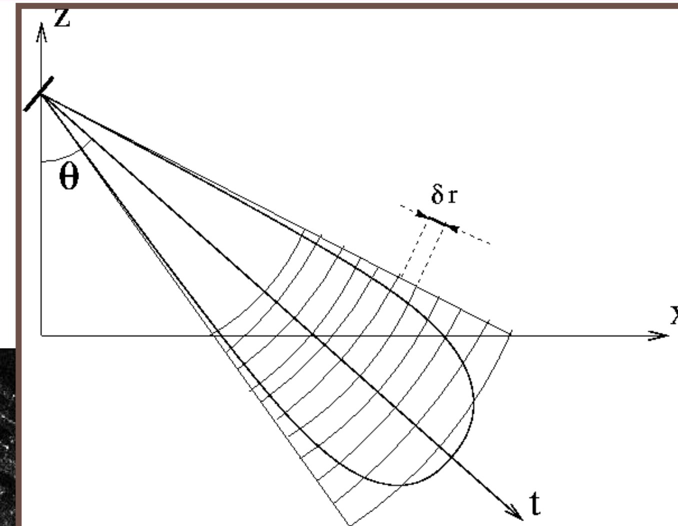
RadarSAT-2

Introduction to SAR imagery

- **Improved resolutions**
  - Spatial resolution
  - Temporal resolution
  - Angular resolution
  - Polarization

TELECOM ParisTech

Introduction to SAR imagery



Difficulties

- Noise is different
- Signal is different
  - Geometric distorsions
  - Sensitivity to corners

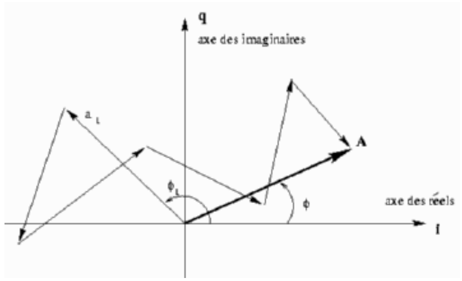
Distance sampling: geometric distorsions

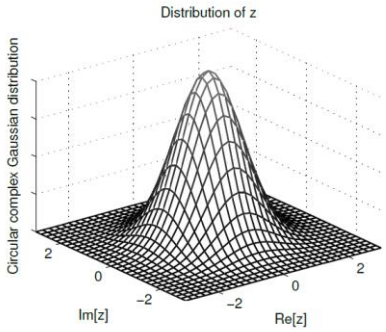
SAR data and statistics

- **Data: complex electro-magnetic field** $z = Ae^{j\varphi}$
(amplitude $A = |z|$ **, intensity** $I = A^2$ **)**

- **Speckle: coherent imagery, interferences**
  - Goodman model (rough surfaces)

$$p(z|\sigma^2) \triangleq p(\mathrm{Re}[z], \mathrm{Im}[z]|\sigma^2) = \frac{1}{\pi\sigma^2}\exp\left(-\frac{|z|^2}{\sigma^2}\right)$$

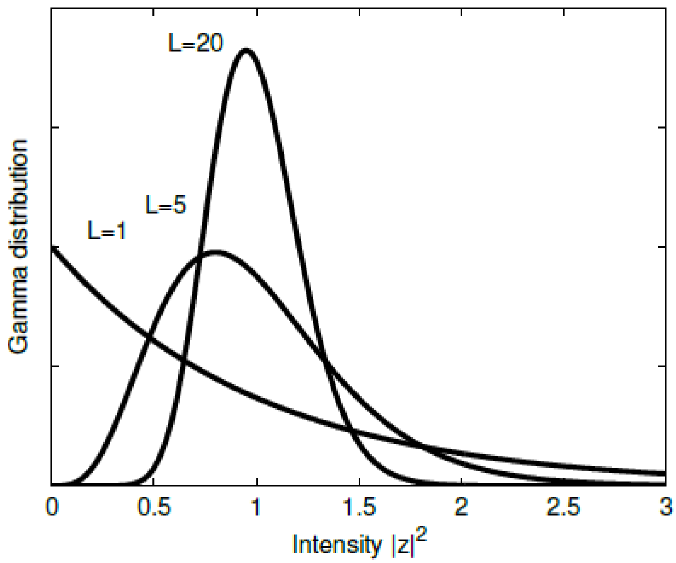


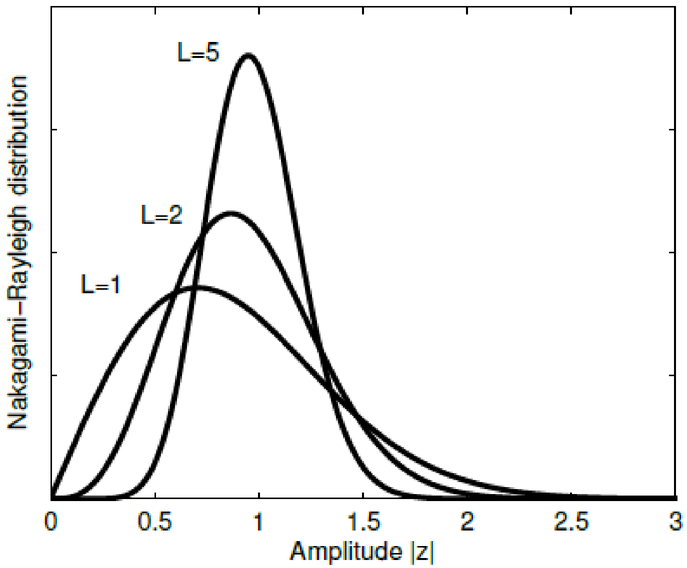


$\sigma^2 = R$

One channel, Goodman model:

- Multi-look images: $I = \frac{1}{L}\sum_{i=1}^{L}|z_i|^2$
- Intensity distribution: Gamma
- Amplitude distributions: Rayleigh-Nakagami





Le multi-vue temporel

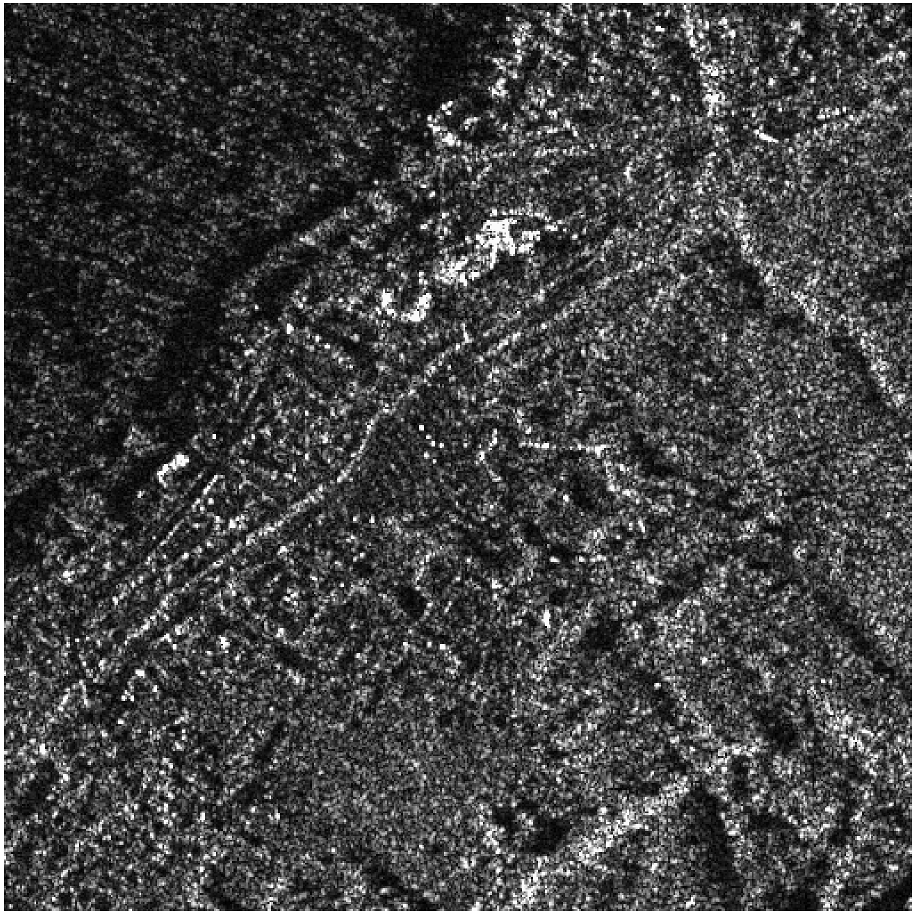
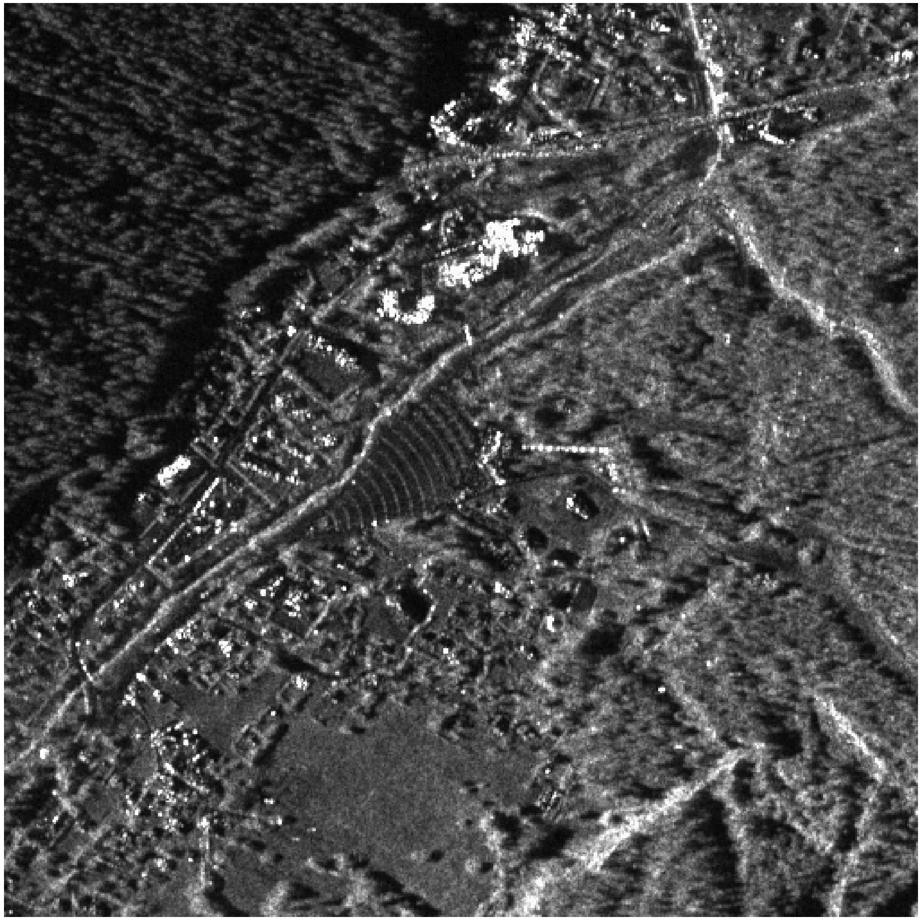
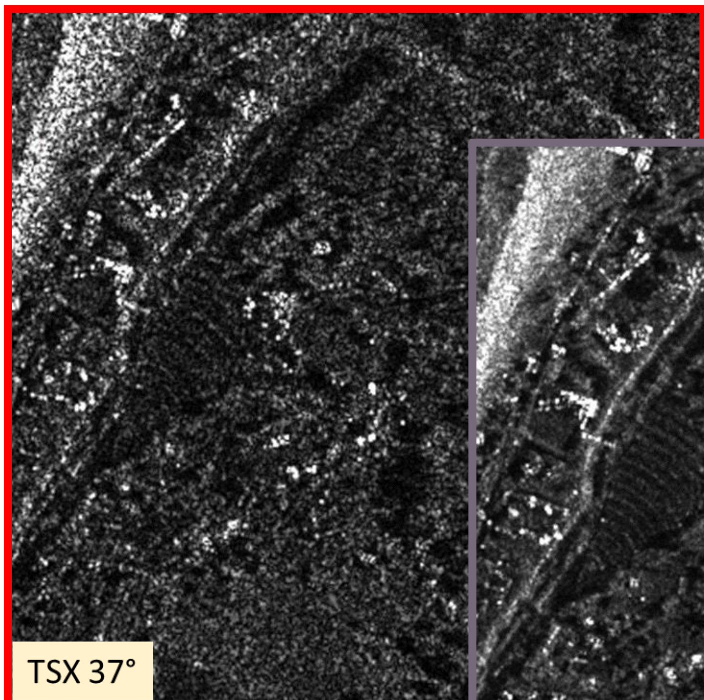


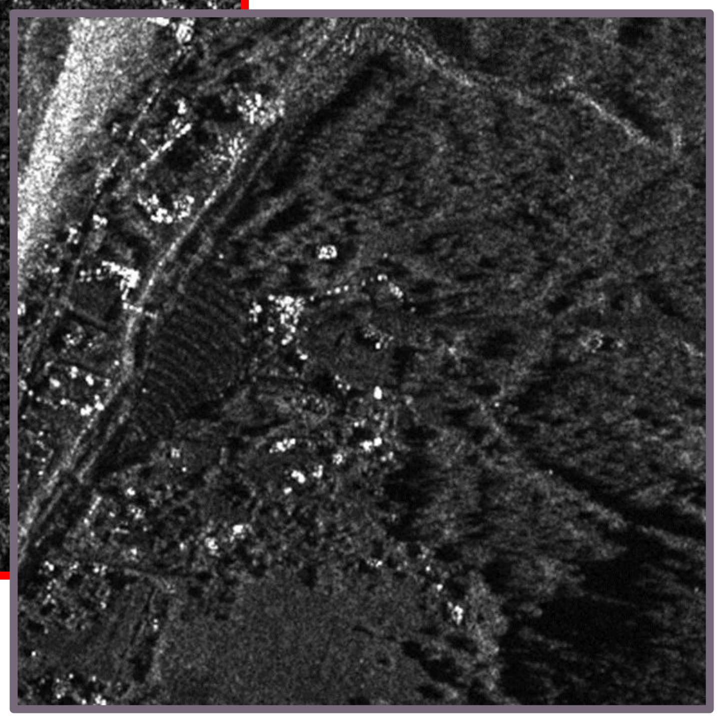
Image Terrasar-X initiale

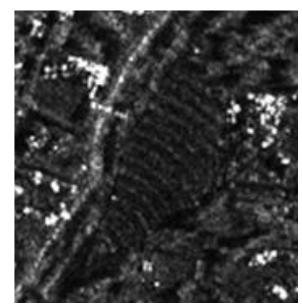


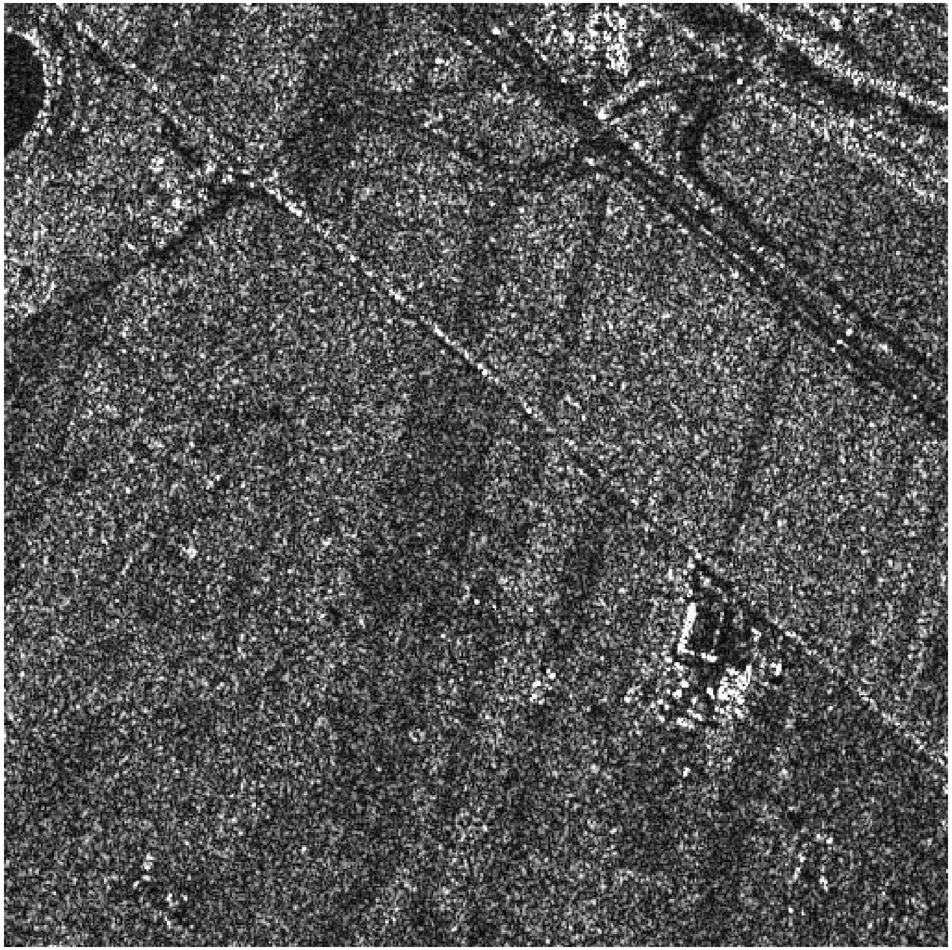
Filtrage multitemporel

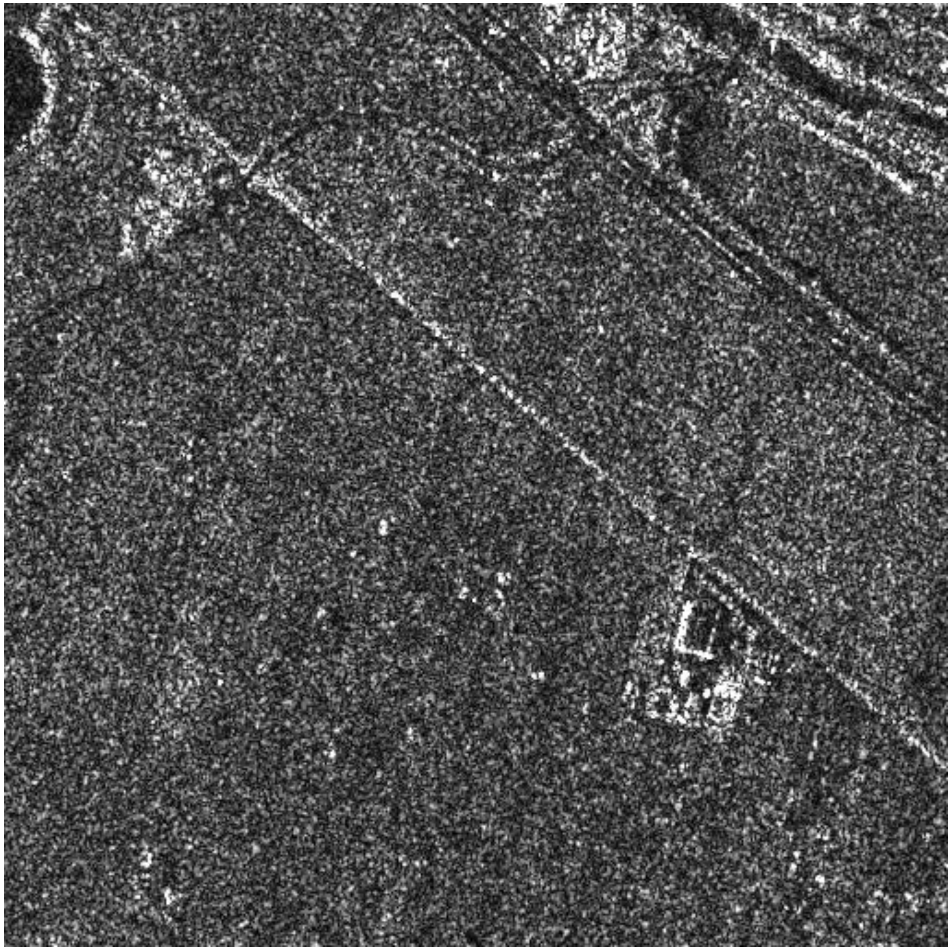
Multi-vues temporel : les super-images

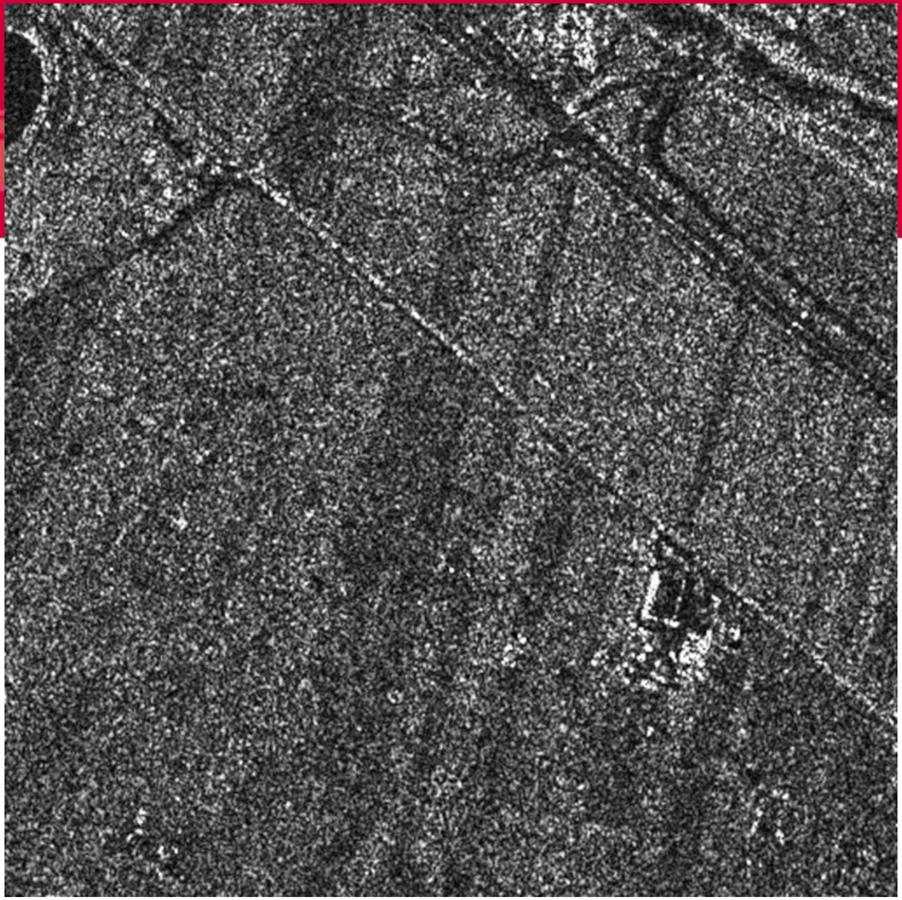


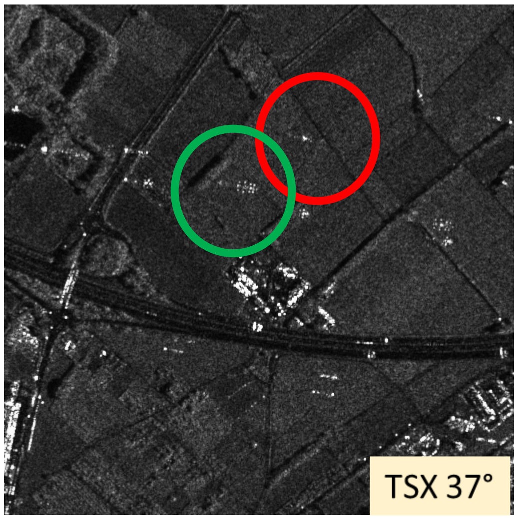


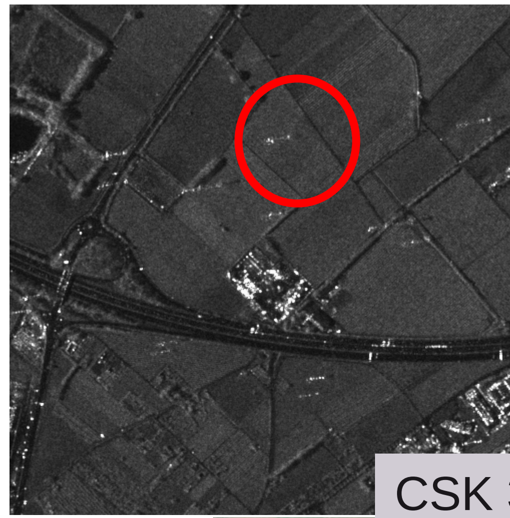




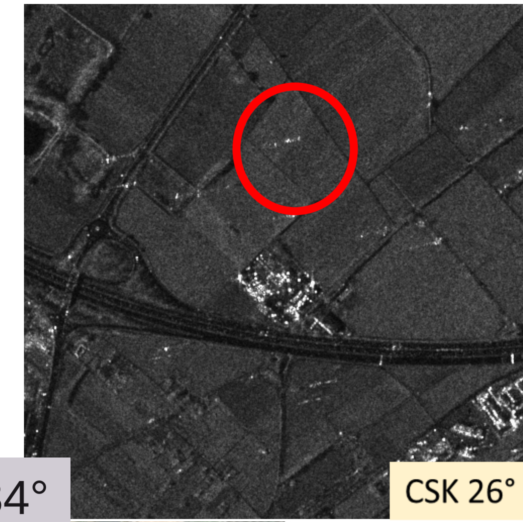


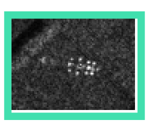


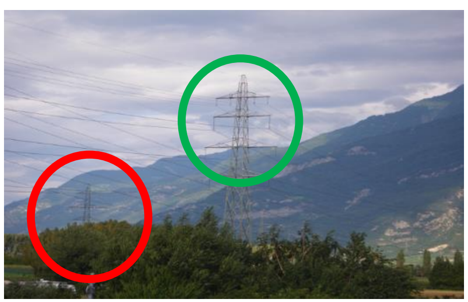


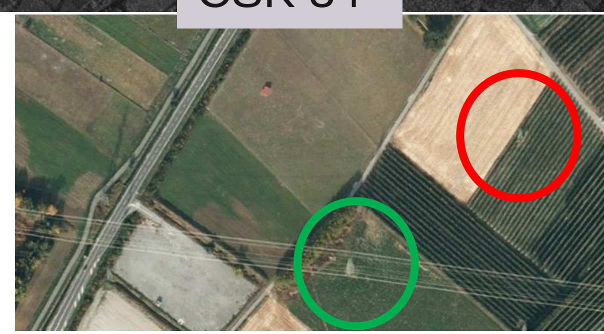


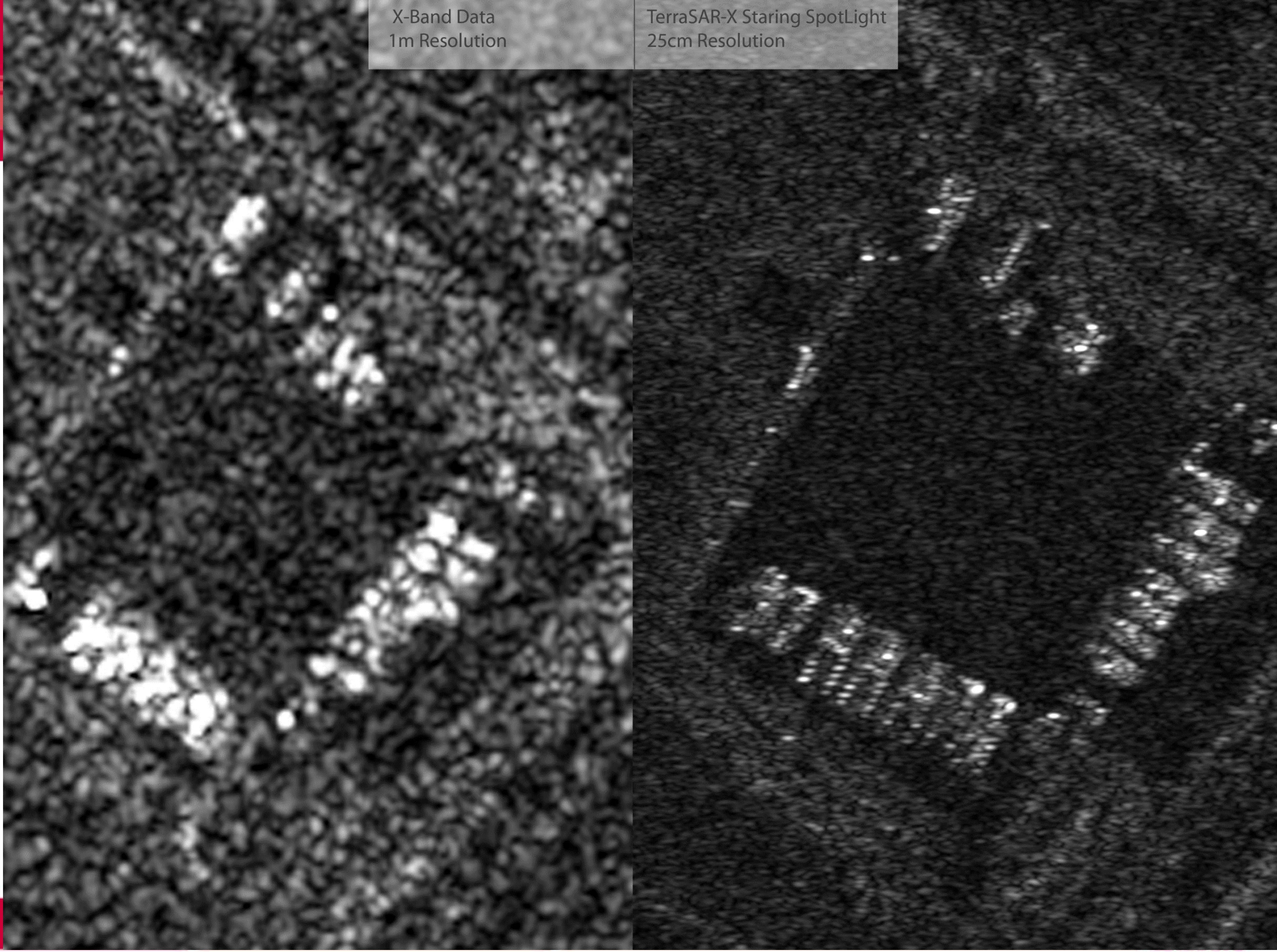
CSK 34°










X-Band Data
1m Resolution
TerraSAR-X Staring SpotLight
25cm Resolution

SAR data and statistics

K channels, Goodman model:

- Vectorial data: $\mathbf{k} = (z_1, \ldots, z_K)$
- Circular complex Gaussian distribution:

$$p(\boldsymbol{k}|\boldsymbol{\Sigma}) = \frac{1}{\pi^K |\boldsymbol{\Sigma}|} \exp\left(-\boldsymbol{k}^\dagger \boldsymbol{\Sigma}^{-1} \boldsymbol{k}\right)$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_1\sigma_k\rho_{1,k} & \cdots & \sigma_1\sigma_K\rho_{1,K} \\ \vdots & \ddots & \vdots & & \vdots \\ \sigma_k\sigma_1\rho_{1,k}^* & \cdots & \sigma_k^2 & \cdots & \sigma_k\sigma_K\rho_{k,K} \\ \vdots & & \vdots & \ddots & \vdots \\ \sigma_K\sigma_1\rho_{1,K}^* & \cdots & \sigma_K\sigma_k\rho_{k,K}^* & \cdots & \sigma_K^2 \end{pmatrix}$$

$$\sigma_k^2 = \mathbb{E}[|z_k|^2] \qquad \rho_{k,l} = \frac{\mathbb{E}[z_k z_l^*]}{\sqrt{\mathbb{E}[|z_k|^2]\mathbb{E}[|z_l|^2]}}$$

SAR data and statistics

- **Multi-look data, Goodman model: Wishart distribution**

$$\boldsymbol{C} = \frac{1}{L}\sum_{i=1}^{L} \boldsymbol{k}_i \boldsymbol{k}_i^{\dagger}$$

$$p(\boldsymbol{C}|\boldsymbol{\Sigma}, L) = \frac{L^{LK}|\boldsymbol{C}|^{L-K}}{\Gamma_K(L)|\boldsymbol{\Sigma}|^L} \exp\left(-L \operatorname{tr}(\boldsymbol{\Sigma}^{-1}\boldsymbol{C})\right)$$

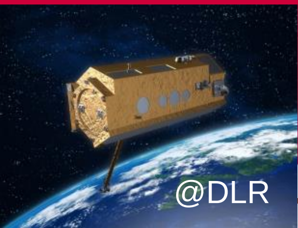
$$I_k = A_k^2 = \frac{1}{L}\sum_{i=1}^{L} |z_{i,k}|^2$$

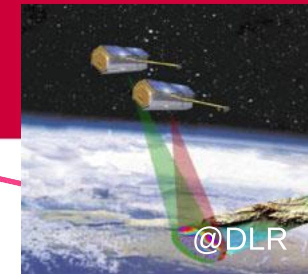
$$d_{k,l} e^{j\phi_{k,l}} = \frac{\sum_{i=1}^{L} z_{i,k} z_{i,l}^*}{\sqrt{\sum_{i=1}^{L} |z_{i,k}|^2 \sum_{i=1}^{L} |z_{i,l}|^2}}$$

coherence ($d_{k,l}$)

phase ($\phi_{k,l}$)

SAR data and statistics





K=1 $z = Ae^{j\varphi}$

Amplitude data (classification, object recognition,…)

K=2 $\mathbf{k} = (z, z')^t$

different incidence angles

Interferometric data: geometric information (elevation, movement)

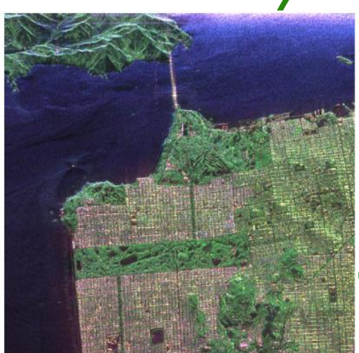
$$\Sigma = \sigma^2 \begin{pmatrix} 1 & De^{j\beta} \\ De^{-j\beta} & 1 \end{pmatrix}$$

K=3 $\boldsymbol{k} = (z_{hh}, z_{vv}, \sqrt{2} z_{hv})^t$

different polarizations

Polarimetric data

Backscattering mechanisms (classification, object recognition,…)

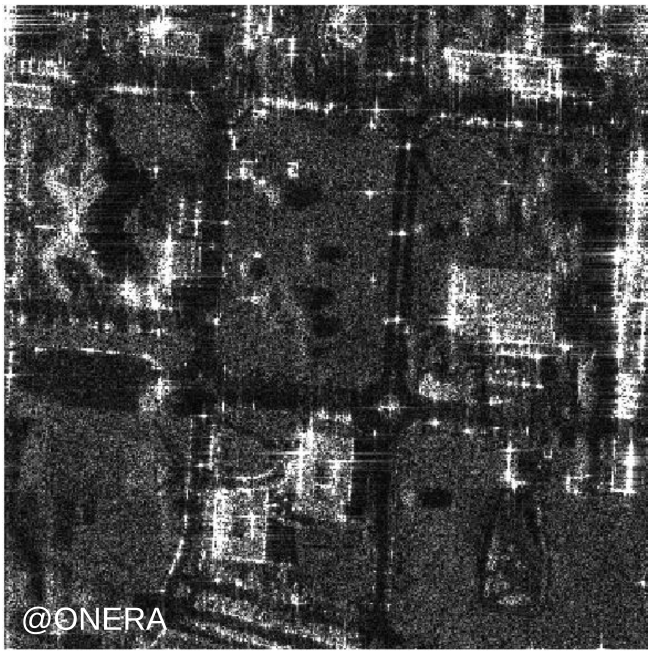


TELECOM ParisTech

- **Modèles statistiques pour l'imagerie SAR**
- **Approches par patchs**
- **Applications à l'imagerie SAR**

Problème d'estimation

Aim: estimate the original signal from noisy observations

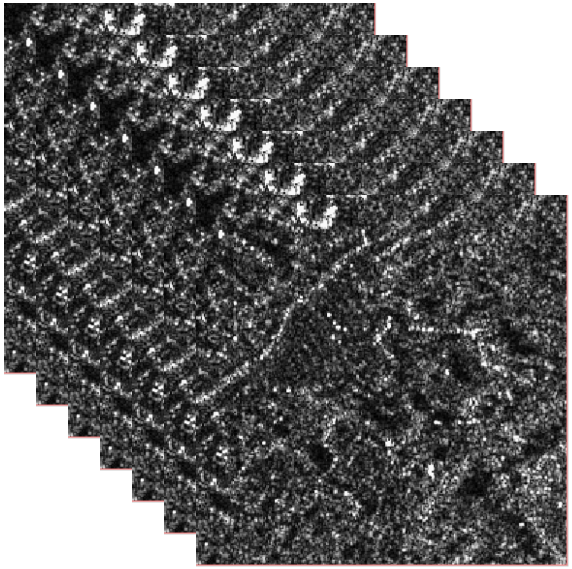


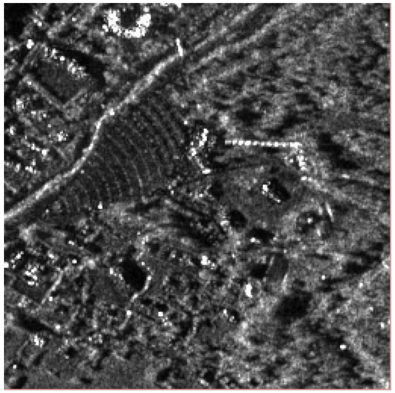
« Noisy » measured signal

Ideal signal

Context: high resolution, urban areas

Denoising and « averaging »



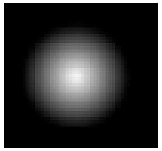


- Averaging of many noisy values: estimation of the « true » reflectivity
- …only if the selected values are coming from the same underlying noise-free value…

How can we select them on the image?

Selection based filtering

- **Local selection :**
  - Selection of the closest pixels (mean filter)



  - Selection and weighting by distance of the closest pixels: gaussian filter

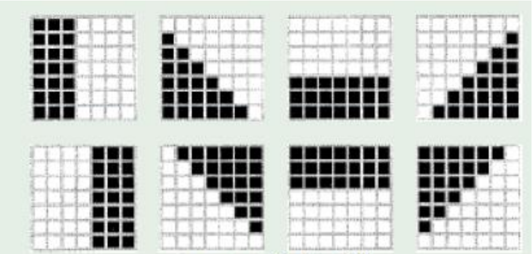


image extracted from [Lee, 1981]

  - Selection of the most homogeneous sub-window (splitting in 4 directions)

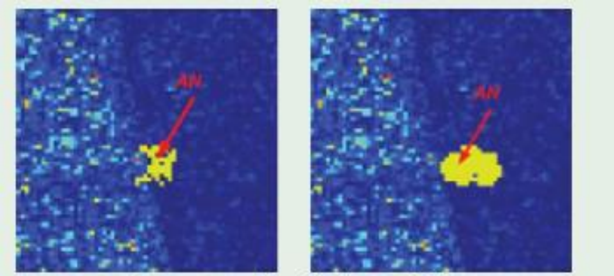


image extracted from [Vasile et al., 2006]

  - Selection with a region growing algorithm (adaptive neighborhood)

Selection-based filtering

- **Non-local approaches:**
  - Relaxing locality and connexity constraints for pixel selection: selection based on similarity

$$\hat{u}_s = \sum_{t \in \Omega} w(s,t) v_t$$

u_s searched noise-free value
$\hat{u}_s$ estimated noise-free value
v_s observed noisy value

Selection-based filtering

- **Non-local approaches:**
  - Relaxing locality and connexity constraints for pixel selection: selection based on similarity [Yaroslavsky, 85]

$$\hat{u}_s = \sum_{t \in \Omega} w(s,t) v_t \qquad w(s,t) = \exp\left(-\frac{d(v_s, v_t)}{h^2}\right)$$

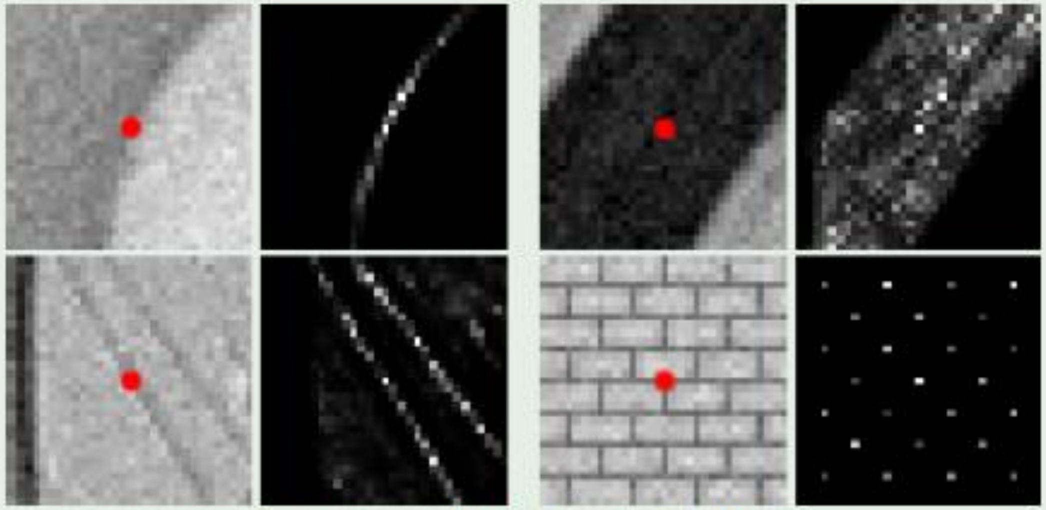


image extracted from [Buades et al., 2005]

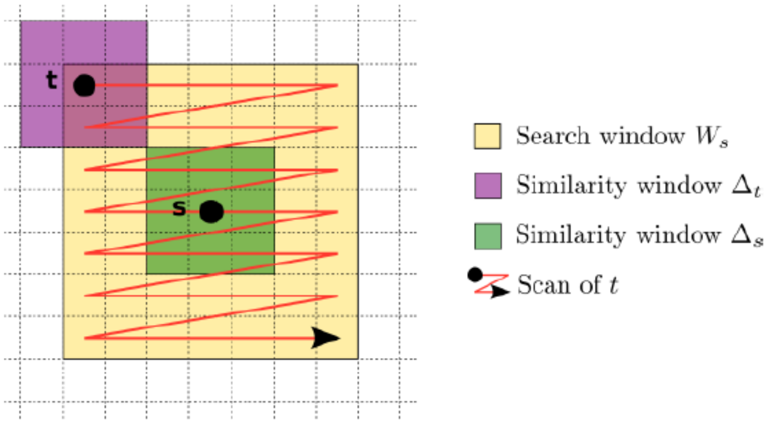
Selection-based filtering

Non-local approaches:

- Relaxing of locality and connexity constraints for pixel selection: selection based on similarity [Yaroslavsky, 85]

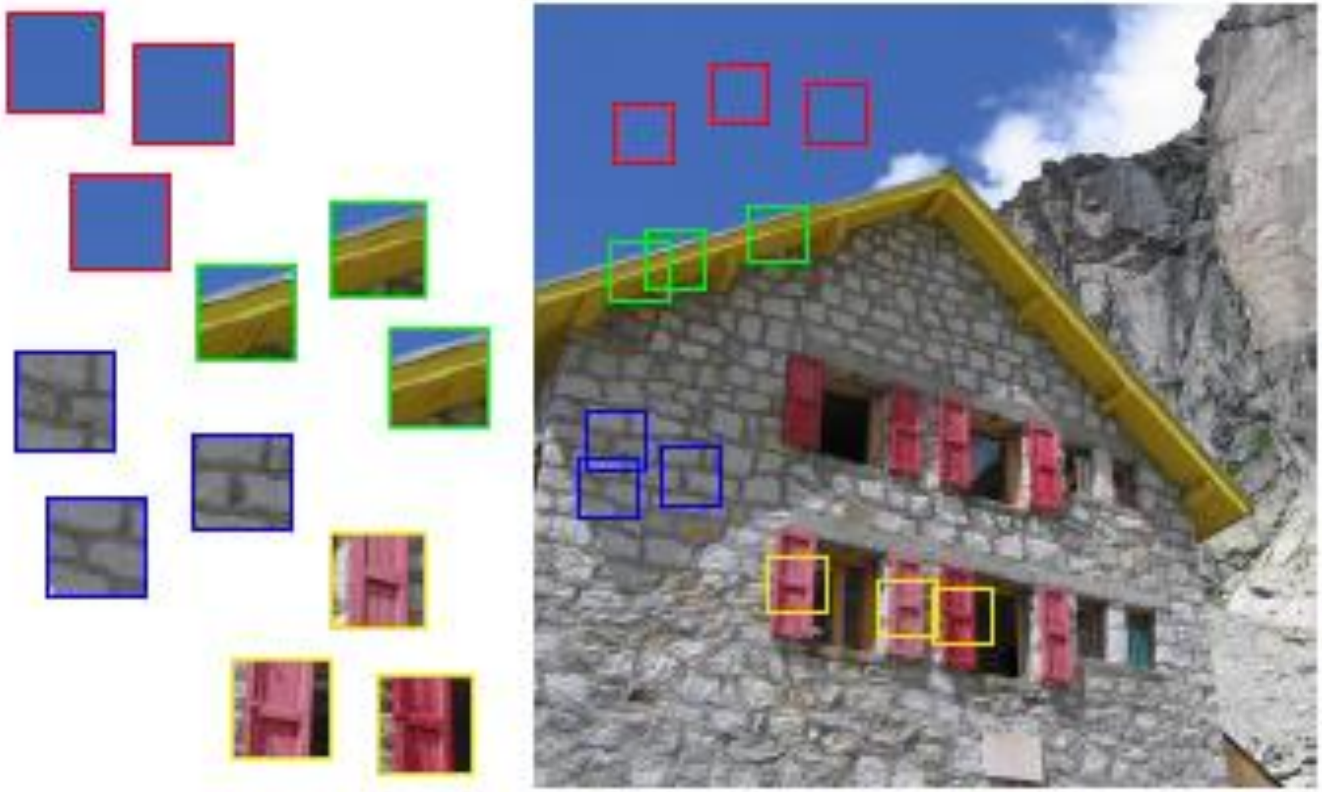
$$\hat{u}_s = \sum_{t \in \Omega} w(s,t) v_t \qquad w(s,t) = \exp\left(-\frac{d(v_s, v_t)}{h^2}\right)$$

- Similarity of pixels = similarity of patches [Buades, 05]



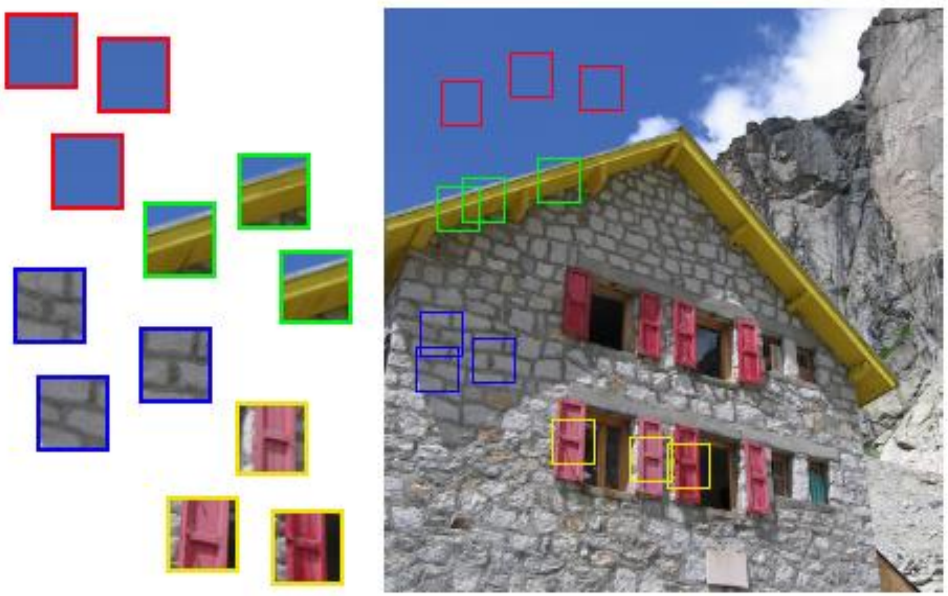
Non-local approaches

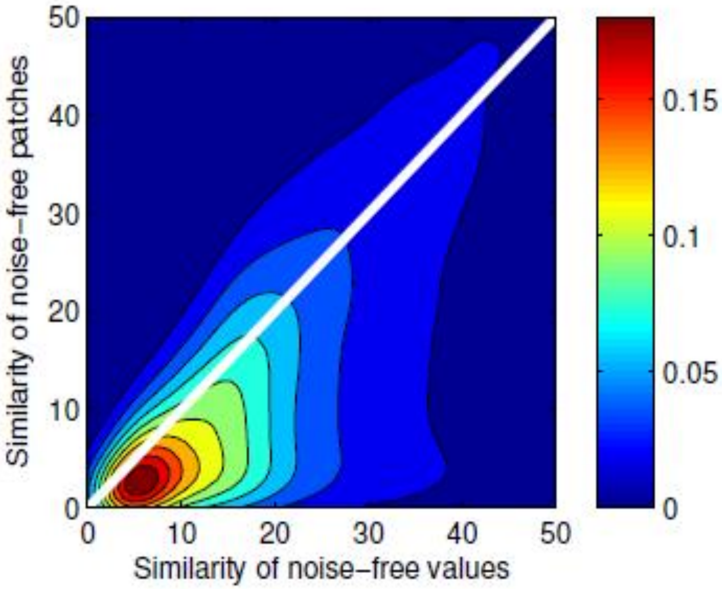
H1 : Hypothesis of redundancy in images



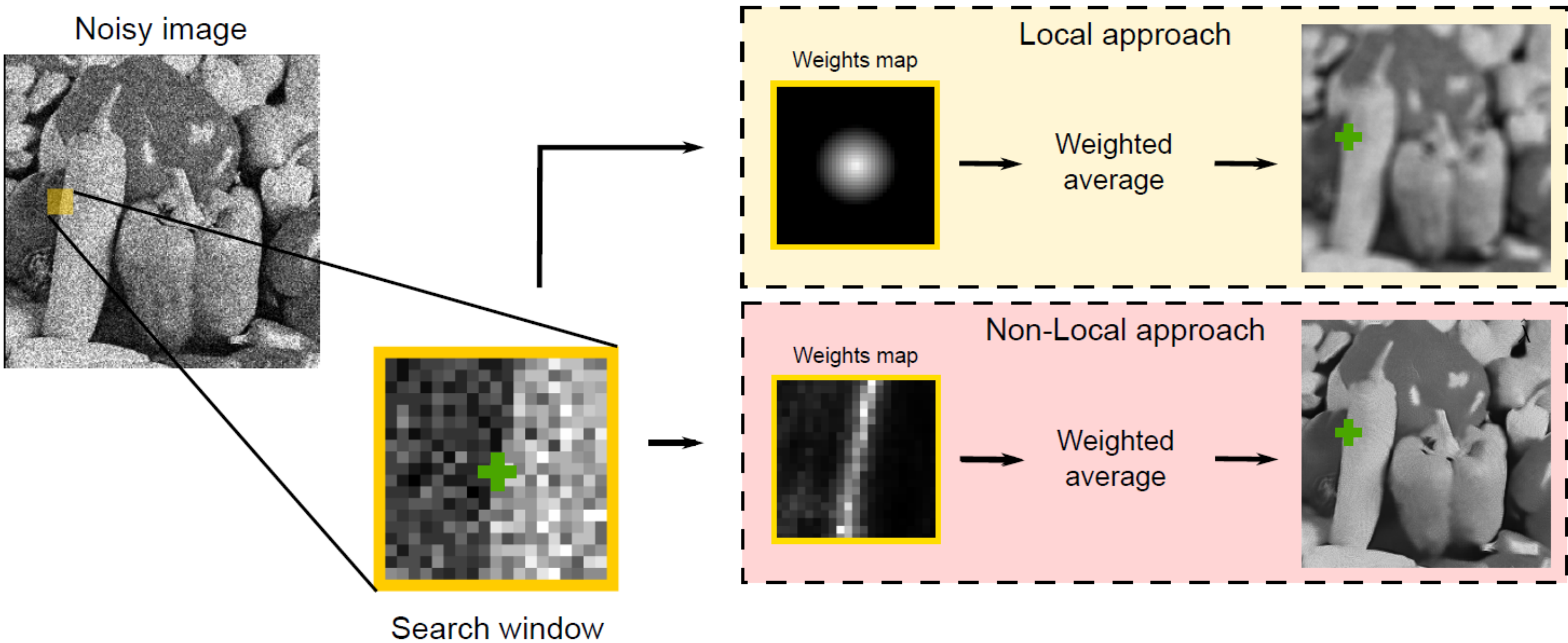
Non-local approaches

H2 : similarity between patches ➡ similarity of central pixels

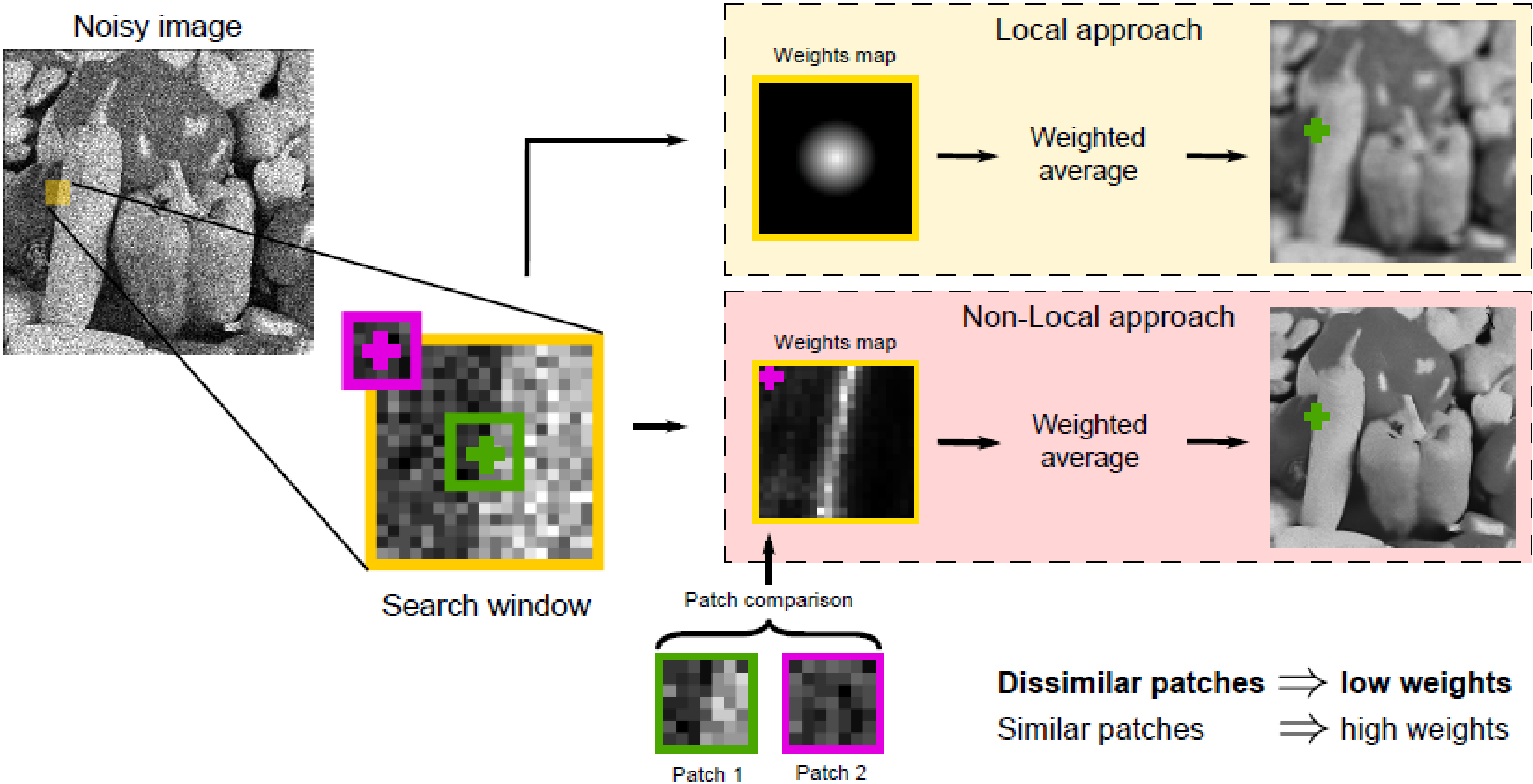




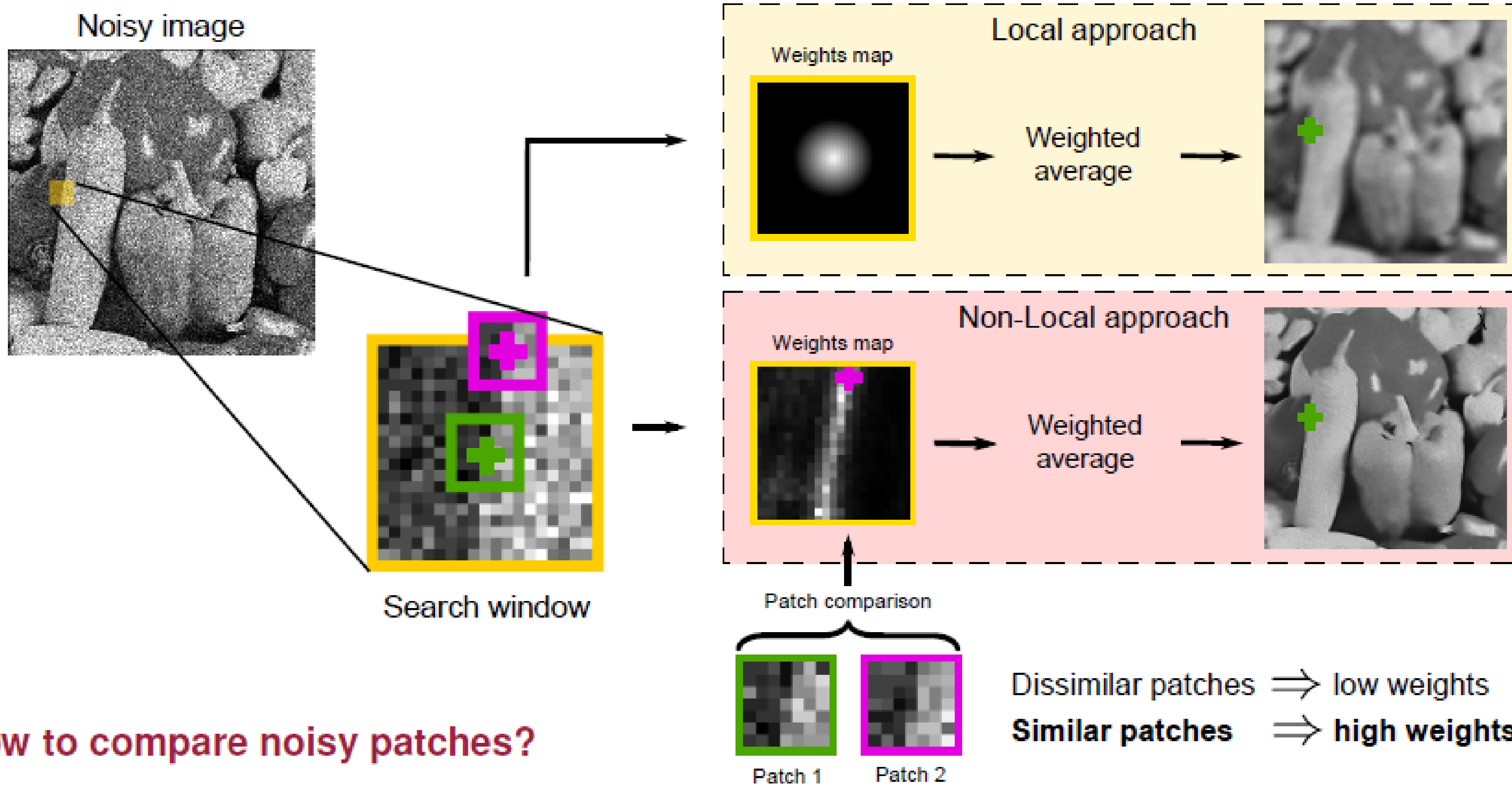
Local / non-local



Non-locality and patches



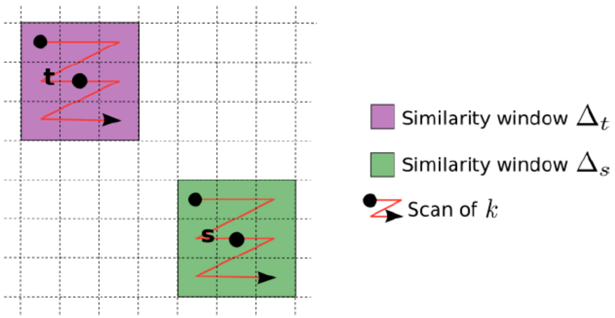
Non-locality and patches



How to compare noisy patches?

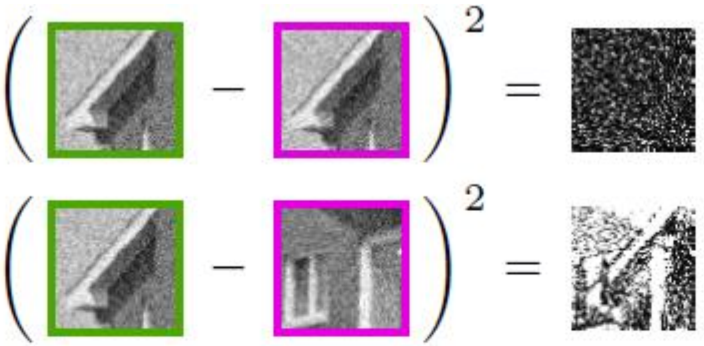
How to compare noisy patches ?

- **Buades et al. (2005)**
  - Euclidean distance between patches
  - Implicit assumption of AWGN



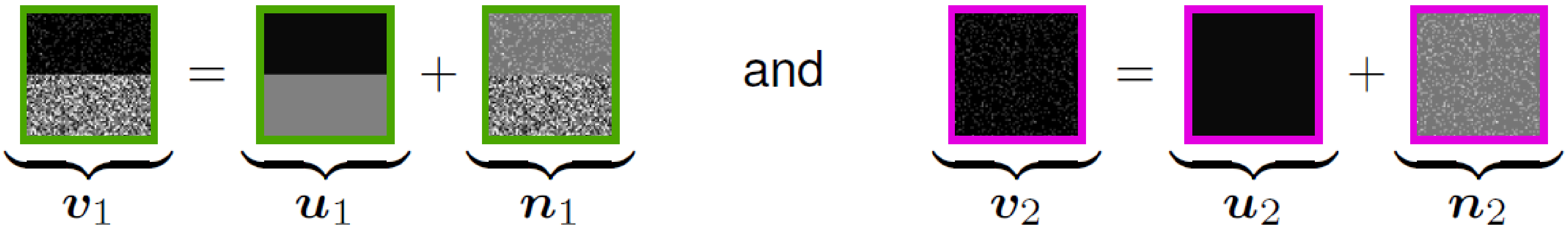
when $u_1 = u_2$: $\left(\cdot - \cdot \right)^2 =$ is low $\Rightarrow$ decide "similar"

when $u_1 \neq u_2$: $\left(\cdot - \cdot \right)^2 =$ is high $\Rightarrow$ decide "dissimilar"



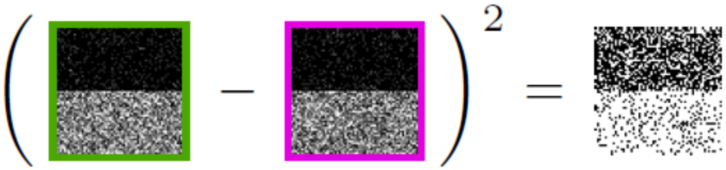
How to compare noisy patches ?

- **Example of signal dependant-noise:**

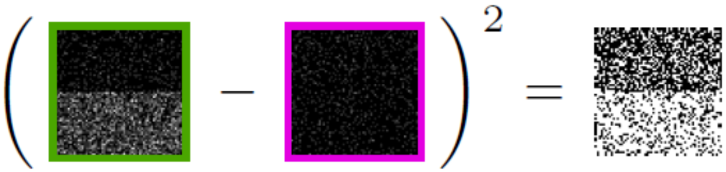


- Limits of the euclidean distance:

when $u_1 = u_2$:



when $u_1 \neq u_2$:



How to compare noisy patches ?

Noisy image
(gaussian noise)

Denoised (« oracle »
Driven by noise-free
Image content)

Denoised
(driven by noisy
Image content)

How to compare noisy patches ?

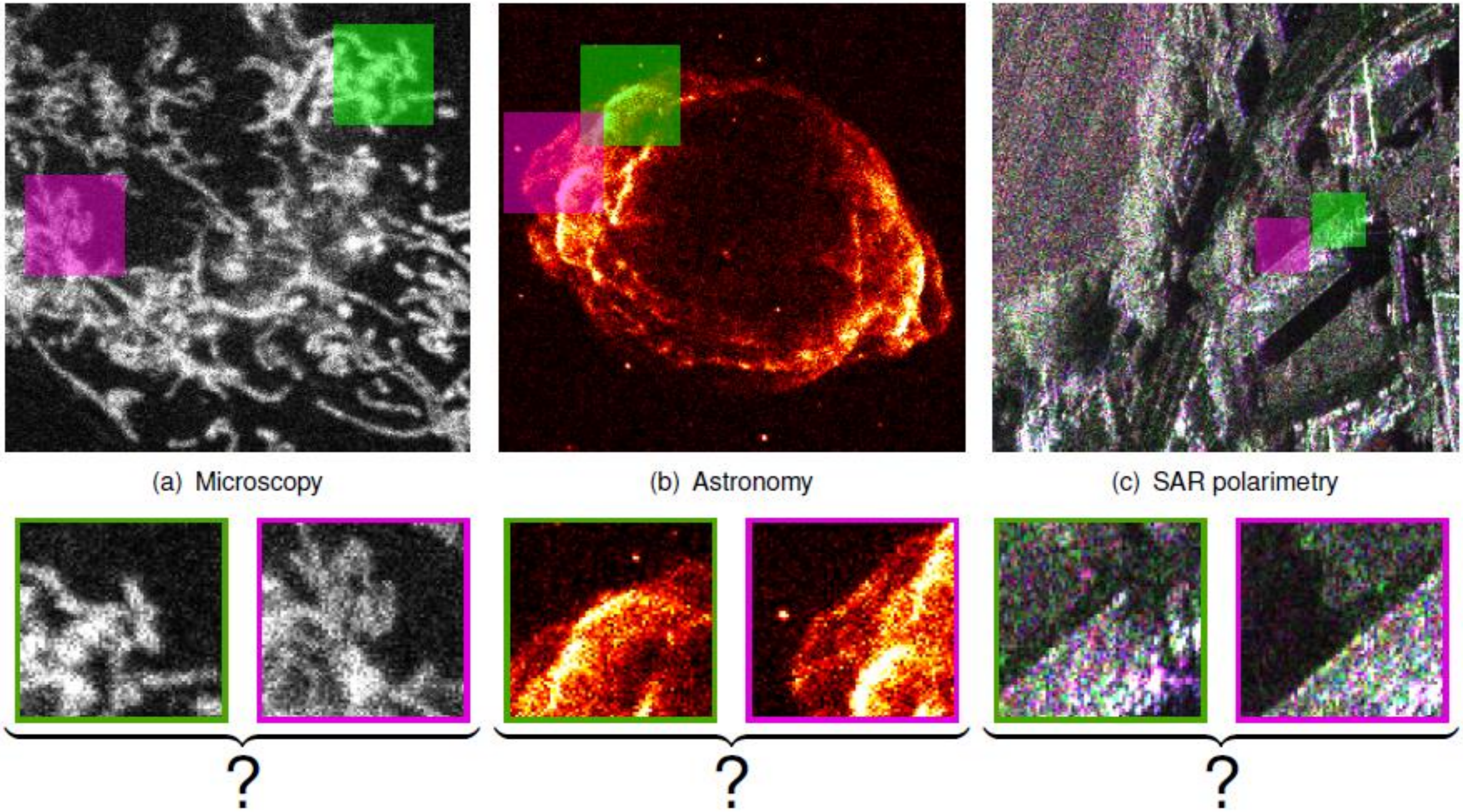
Noisy image
(Poisson noise
Signal dependent noise)

Denoised (« oracle »
driven by noise-free
Image content)

Denoised
(driven by noisy
Image content)

Noise distribution has to be taken into account

How to compare noisy patches ?



How to take into account the noise model?

Talk overview

- Modèles statistiques pour l’imagerie SAR
- Approches par patchs
- Applications à l’imagerie SAR

A probabilistic framework

- **Principle: adaptation of the NL-means to any kind of (known) noise distribution**
  - Estimation step:

    Weighted average is replaced by weighted maximum likelihood estimation

$$\hat{u}(x) = \arg\max_{t} \sum_{x'} w(x, x') \log p(v(x')|t)$$

  - Detection of similar patches:

    Weight definition is defined in a detection framework by *hypothesis testing*

Similarity definition

- **Similarity is defined by an hypothesis test:**

$$\mathcal{H}_0 : u_1 = u_2 \equiv u_{12} \qquad \text{(null hypothesis)}$$
$$\mathcal{H}_1 : u_1 \neq u_2 \qquad \text{(alternative hypothesis)}$$

- **Performance measured by:**

$$P_{FA} = \mathbb{P}(\text{decide “dissimilar”} \mid u_{12}, \mathcal{H}_0) \qquad \text{(false-alarm rate)}$$
$$P_D = \mathbb{P}(\text{decide “dissimilar”} \mid u_1, u_2, \mathcal{H}_1) \qquad \text{(detection rate)}$$

- **The likelihood ratio test maximizes PD**

$$L(v_1, v_2) = \frac{p(v_1, v_2 \mid u_{12}, \mathcal{H}_0)}{p(v_1, v_2 \mid u_1, u_2, \mathcal{H}_1)}$$

Similarity definition

- **Unknown values are replaced by ML estimates (GLR):**

$$\frac{\sup_t p(v_1, v_2 \mid u_{12} = t, \mathcal{H}_0)}{\sup_{t_1, t_2} p(v_1, v_2 \mid u_1 = t_1, u_2 = t_2, \mathcal{H}_1)}$$

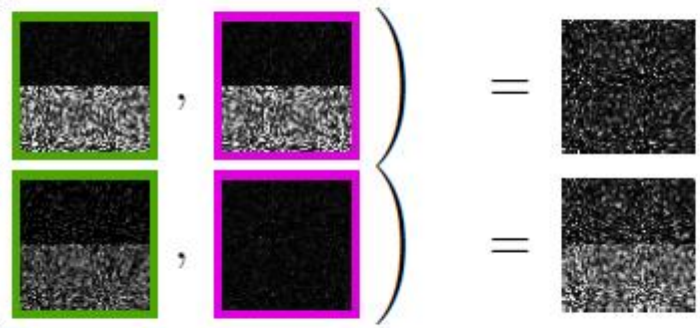
$$\frac{p(v_1 \mid u_1 = \hat{t}_{12})\, p(v_2 \mid u_2 = \hat{t}_{12})}{p(v_1 \mid u_1 = \hat{t}_1)\, p(v_2 \mid u_2 = \hat{t}_2)}$$

- **Study of this criterion**

GLR

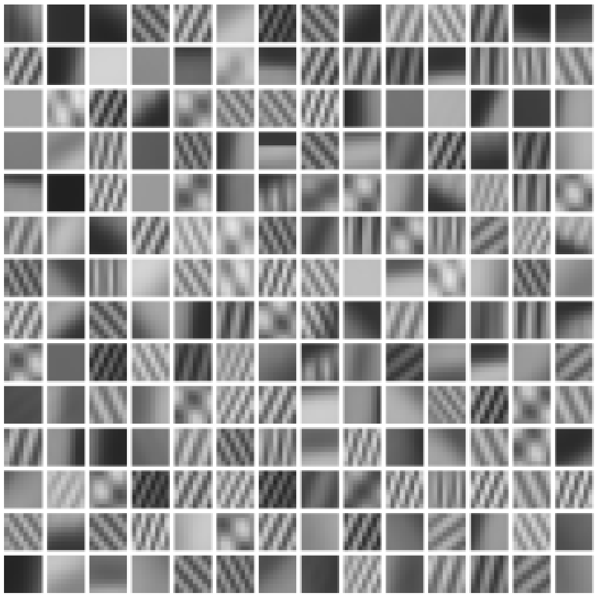
when $u_1 = u_2$: $-\log GLR$ (,) =

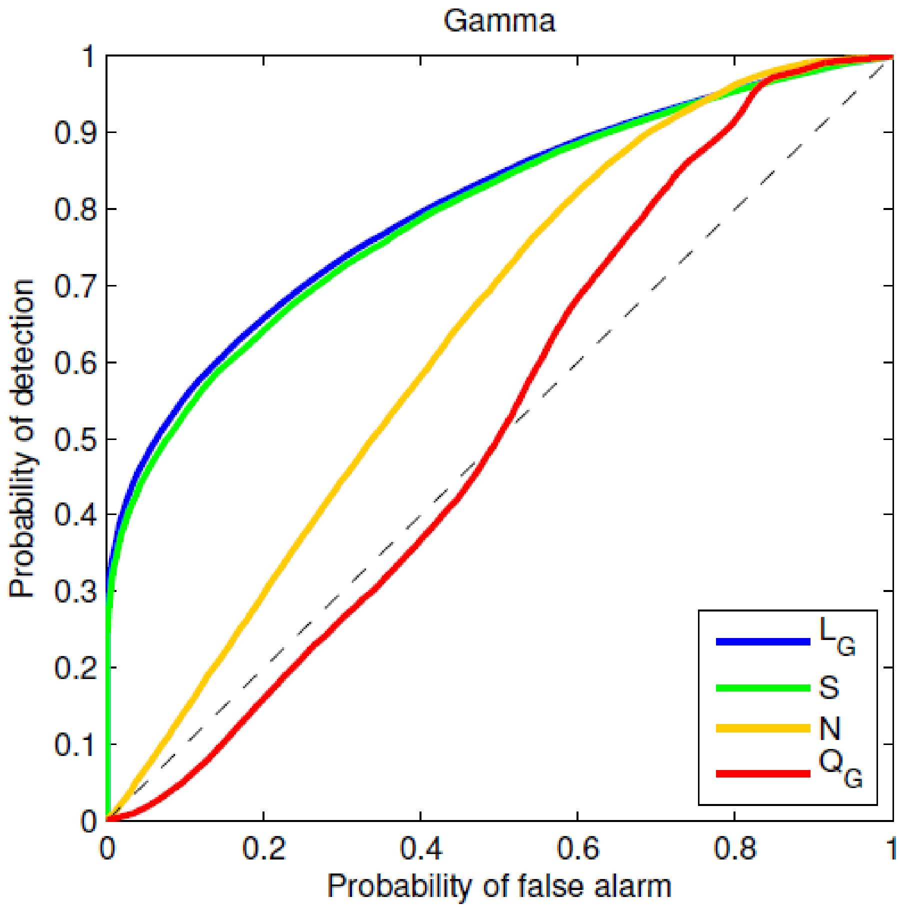
when $u_1 \neq u_2$: $-\log GLR$ (,) =



TELECOM ParisTech

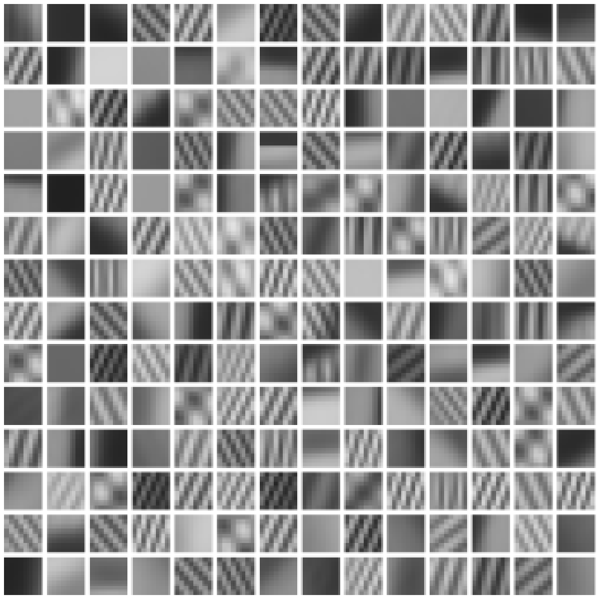
Evaluation of similarity criterion

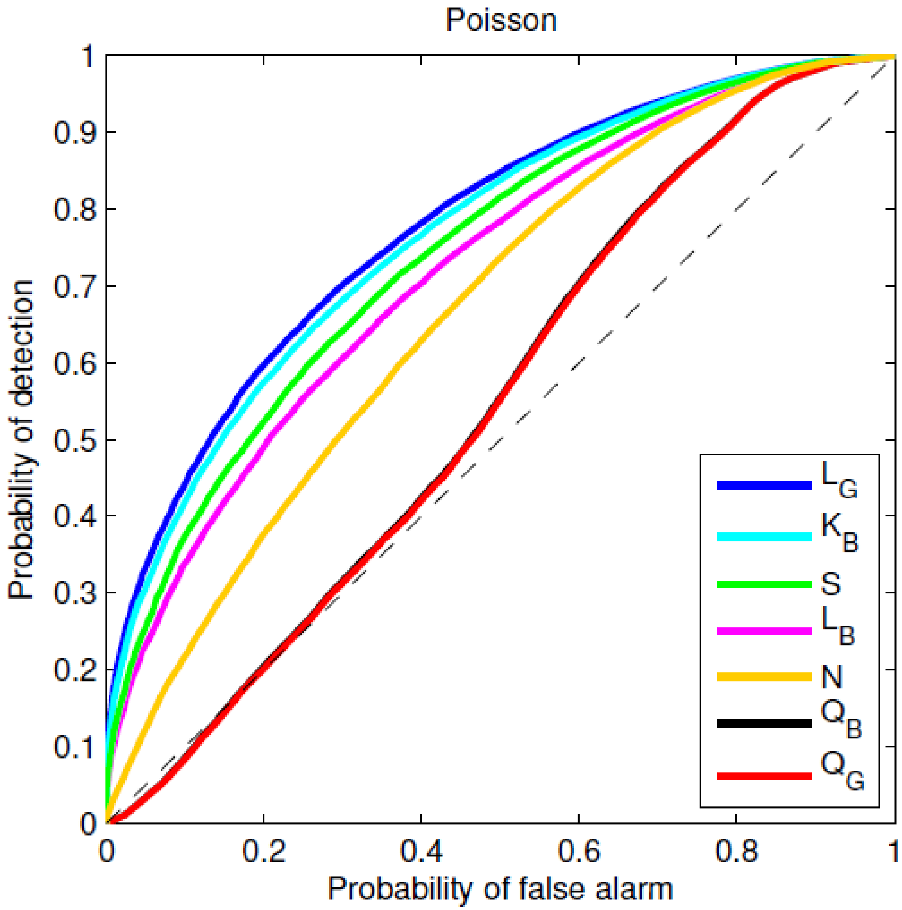




- Generalized likelihood ratio
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood [Alter et al., 2006]
- Mutual information kernel [Seeger, 2002]
- Bayesian likelihood ratio [Minka, 1998, Minka, 2000]
- Bayesian joint likelihood [Yianilos, 1995, Matsushita and Lin, 2007]

Evaluation of similarity criterion



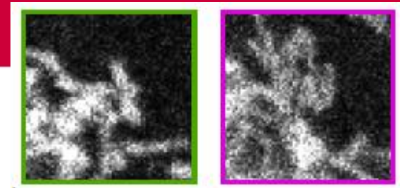


- Generalized likelihood ratio [Alter et al., 2006]
- Variance stabilization
- Euclidean distance
- Maximum joint likelihood
- Mutual information kernel [Seeger, 2002]
- Bayesian likelihood ratio [Minka, 1998, Minka, 2000]
- Bayesian joint likelihood [Yianilos, 1995, Matsushita and Lin, 2007]

Limits of similarity criterion

- **Limits of GLR:**
  - In case of strong noise, low contrast features are badly discriminated
  - Influence of patch size and search window size
- **Solutions**
  - 1. Use of prefiltering to improve the weight estimation: iterative version (GLR + Kullback-Leibler weights)
  - 2. Use of pre-filtering and a set of parameters and then combine them in a spatially adaptive aggregation

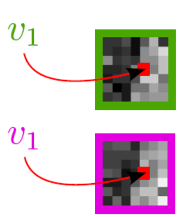
(1) Iterative version
Similarity definition - refinement

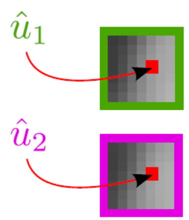


$$\frac{\mathbb{P}(\mathcal{H}_0 \,|\, \square_1, \square_2)}{\mathbb{P}(\mathcal{H}_1 \,|\, \square_1, \square_2)} = \frac{p(\square_1, \square_2 \,|\, \mathcal{H}_0)}{p(\square_1, \square_2 \,|\, \mathcal{H}_1)} \times \frac{\mathbb{P}(\mathcal{H}_0)}{\mathbb{P}(\mathcal{H}_1)}$$

Computed on noisy data
using noise distribution and GLR

Computed on noise-free data
using an iterative scheme
and symmetrical KL divergence

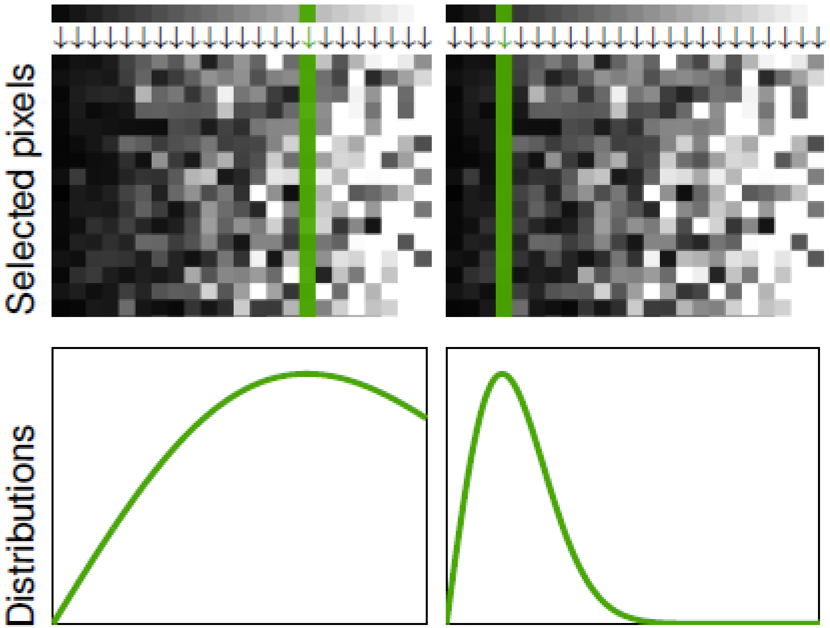




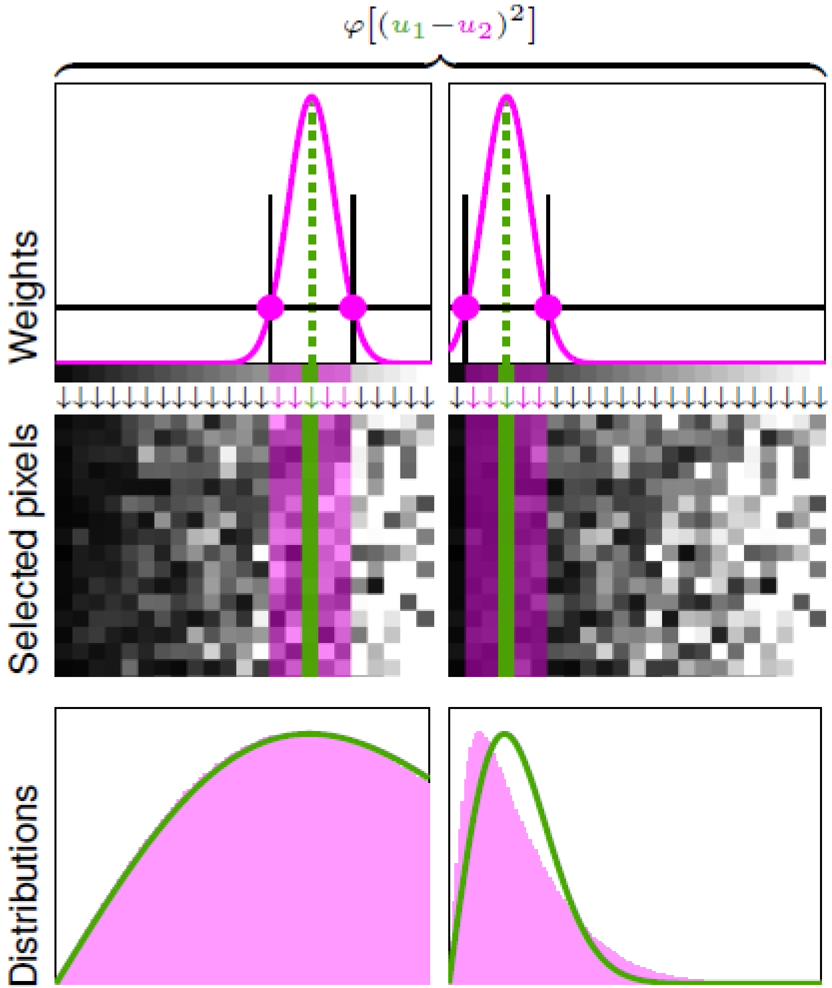
$$\mathcal{D}_{KL}(\hat{u}_1 \| \hat{u}_2)$$

TELECOM ParisTech

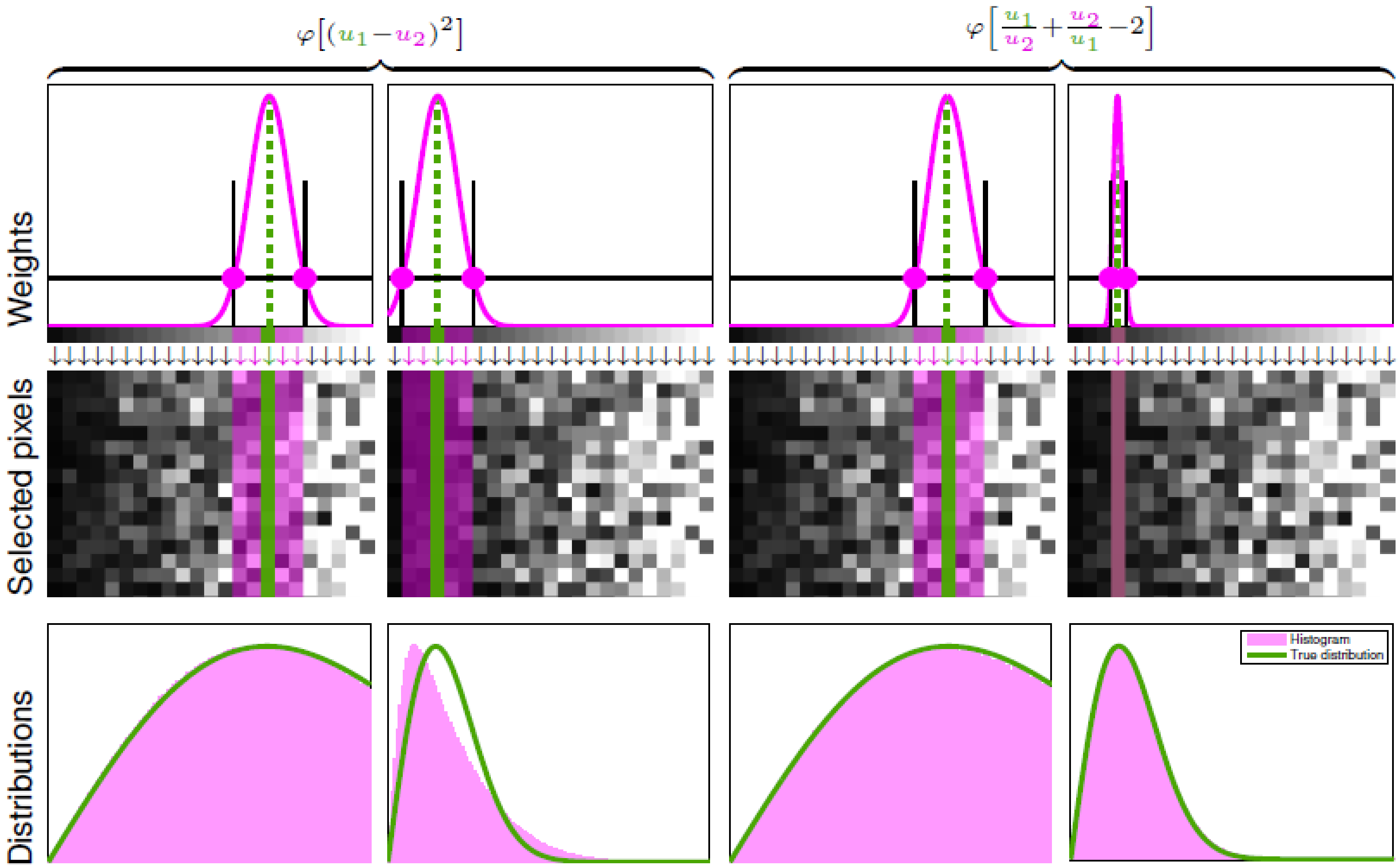
(1) Iterative version- Weight refinement



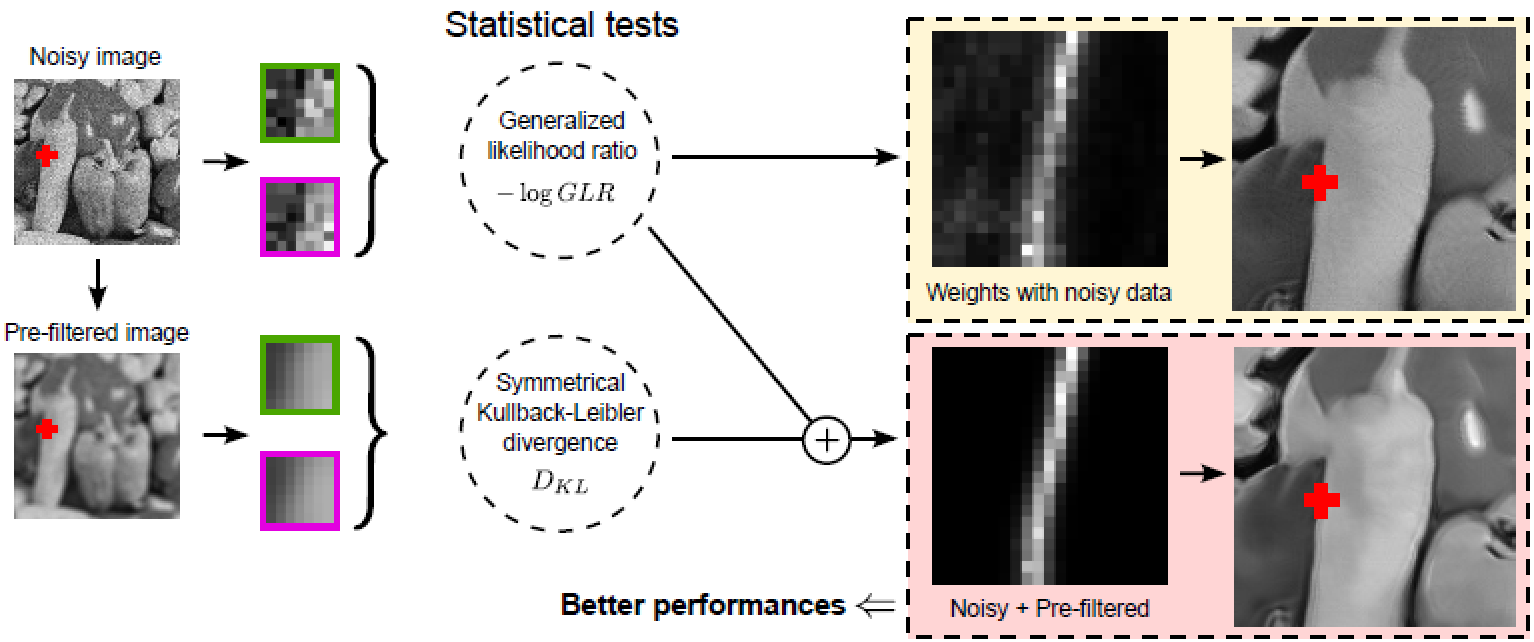
(1) Iterative version - Weight refinement



(1) Iterative verion - Weight refinement

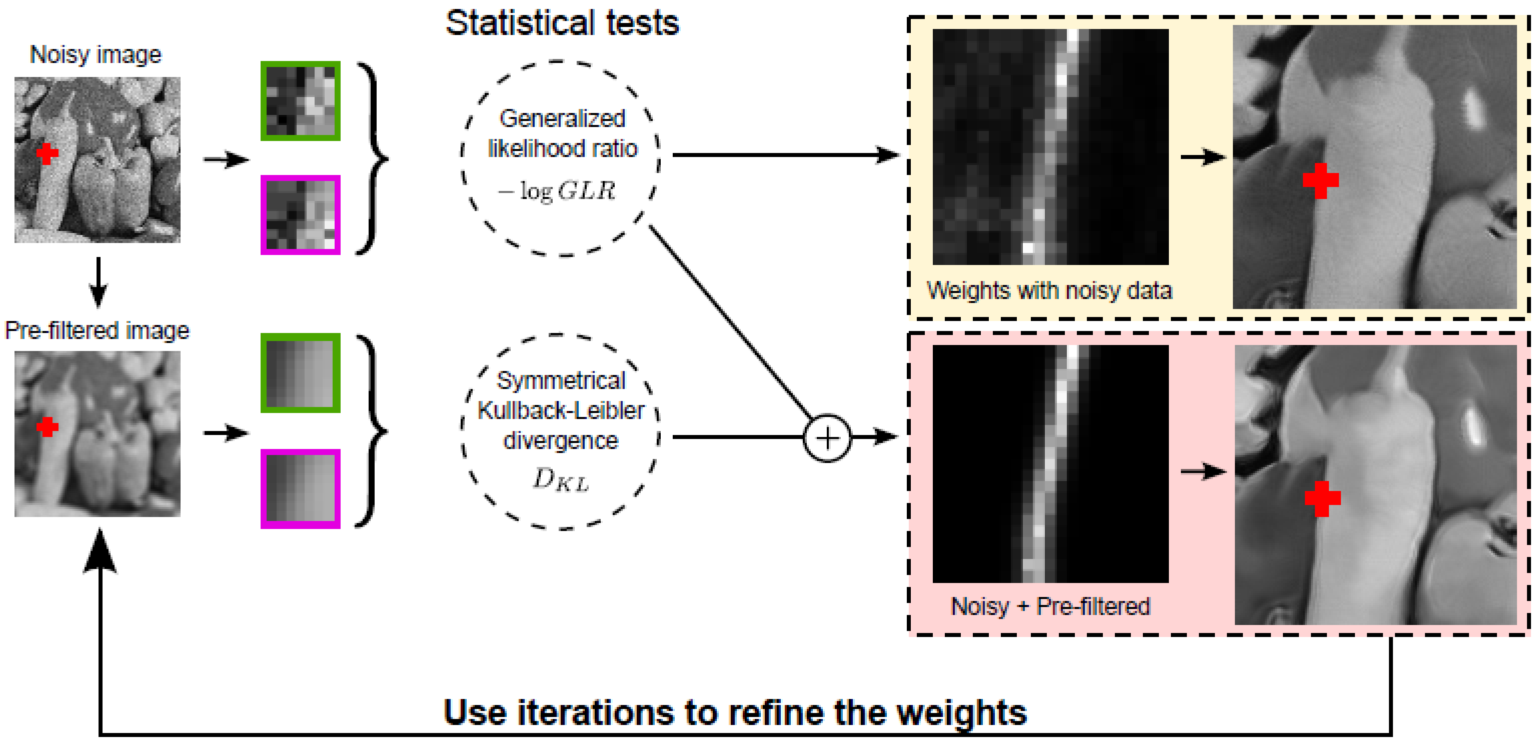


(1) Iterative verion - Global scheme



TELECOM ParisTech

(1) Iterative version - Global scheme



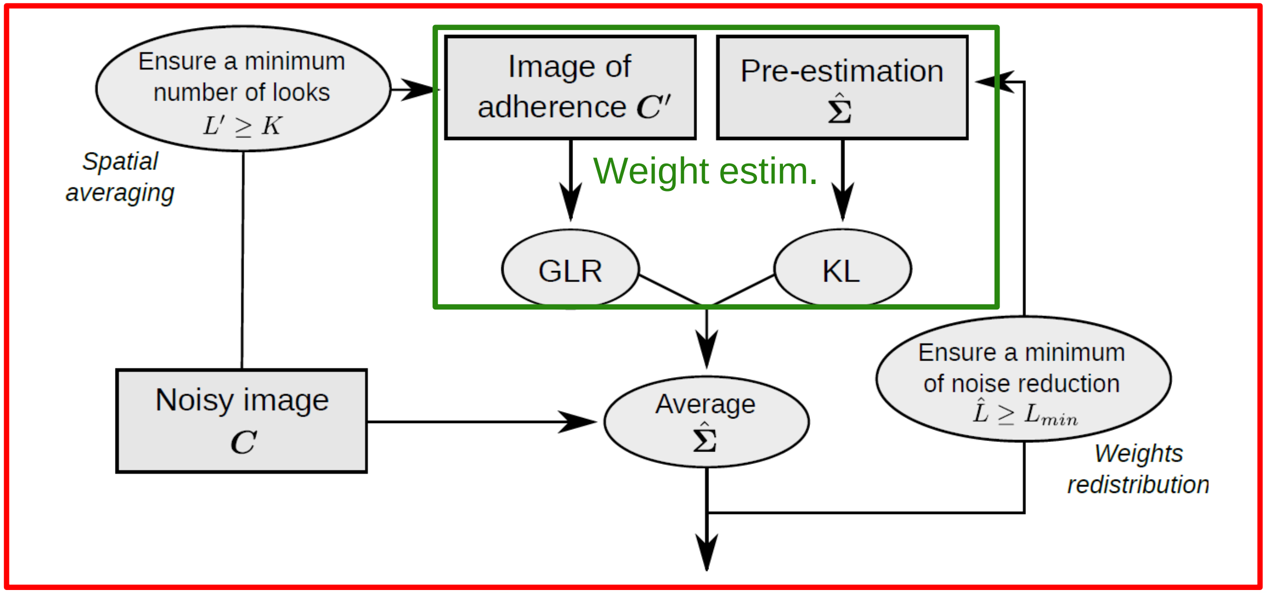
Limits: number of parameters (W, p, number of iterations)

NL-SAR – general formulation

- Parameter of interest: $\boldsymbol{\Sigma}(x)$ an $K \times K$ complex covariance matrix
- Observations: $\boldsymbol{C}(x)$ an $K \times K$ empirical covariance matrix s.t.:

$$p(\boldsymbol{C}|\boldsymbol{\Sigma}, L) = \frac{L^{LK}|\boldsymbol{C}|^{L-K}}{\Gamma_K(L)|\boldsymbol{\Sigma}|^L} \exp\left(-L \operatorname{tr}(\boldsymbol{\Sigma}^{-1}\boldsymbol{C})\right) \qquad \text{(Wishart distribution)}$$

- To denoise: to search for an estimate $\hat{\boldsymbol{\Sigma}}(x)$ of $\boldsymbol{\Sigma}(x)$



NL-SAR – general formulation

- Similarity between noisy patches:

$$-\log GLR(C_1, C_2) = 2L \log \left(\frac{|C_1 + C_2|}{\sqrt{|C_1||C_2|}} \right) - 2LK \log 2$$

- Similarity between noise-free patches:

$$\mathcal{D}_{KL}(\hat{\Sigma}_1 \| \hat{\Sigma}_2) = L\mathrm{tr}\left(\hat{\Sigma}_1{}^{-1}\hat{\Sigma}_2 + \hat{\Sigma}_2{}^{-1}\hat{\Sigma}_1 \right) - 2LK.$$

Example : amplitude images

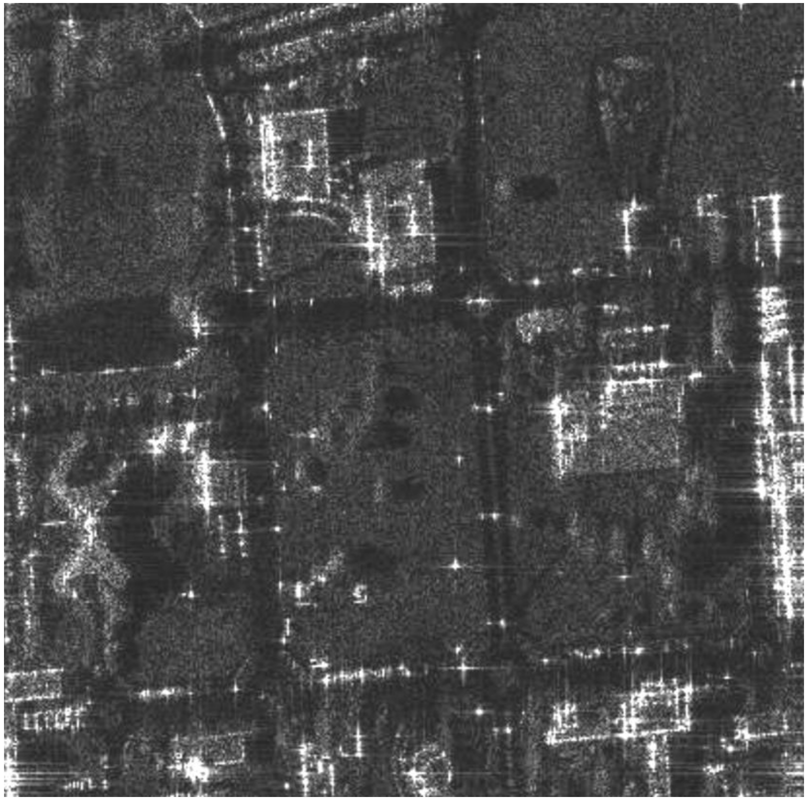
- Similarity between noisy patches:

$$-\log GLR(v_1, v_2) = 2 \log \left(\frac{v_1}{v_2} + \frac{v_1}{v_2} \right) - 2 \log 2$$

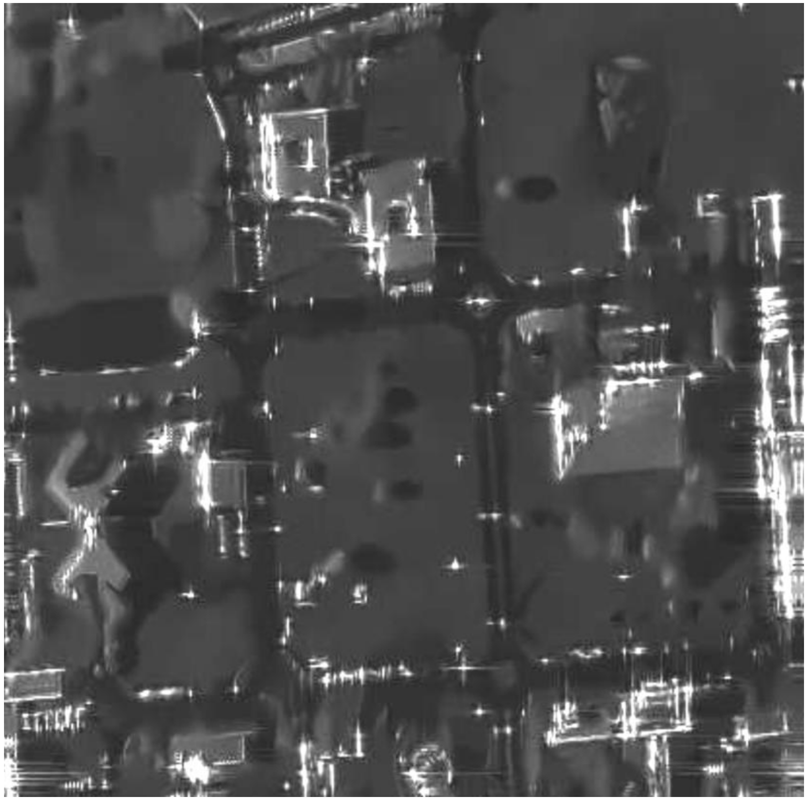
- Similarity between noise-free patches:

$$\mathcal{D}_{KL}(\hat{u}_1 \| \hat{u}_2) = \frac{\hat{u}_1}{\hat{u}_2} + \frac{\hat{u}_2}{\hat{u}_1} - 2$$

NL-SAR – Results (amplitude)

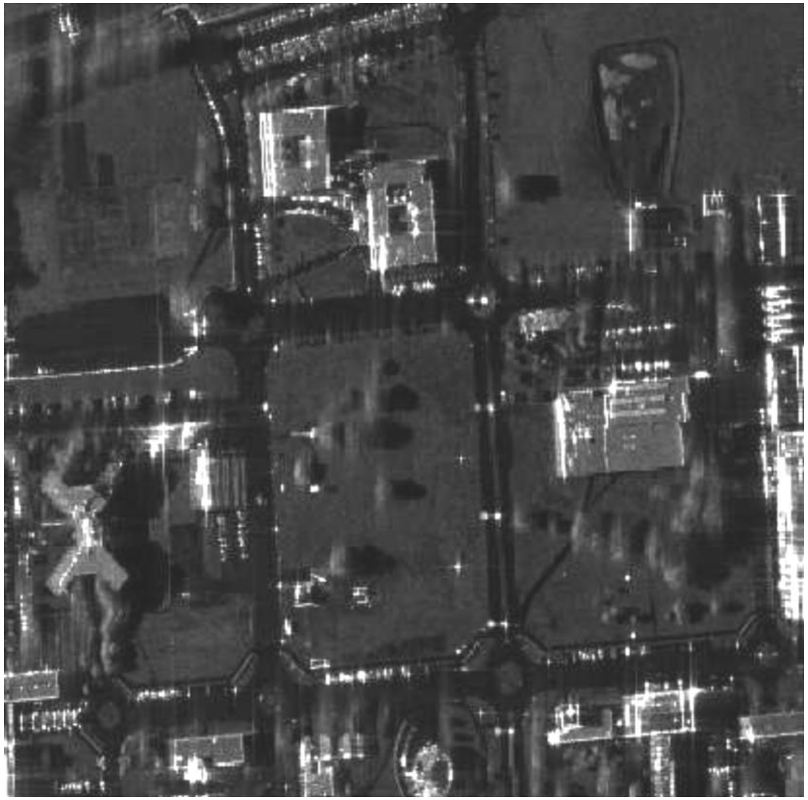


Original 1-look SAR image @ONERA

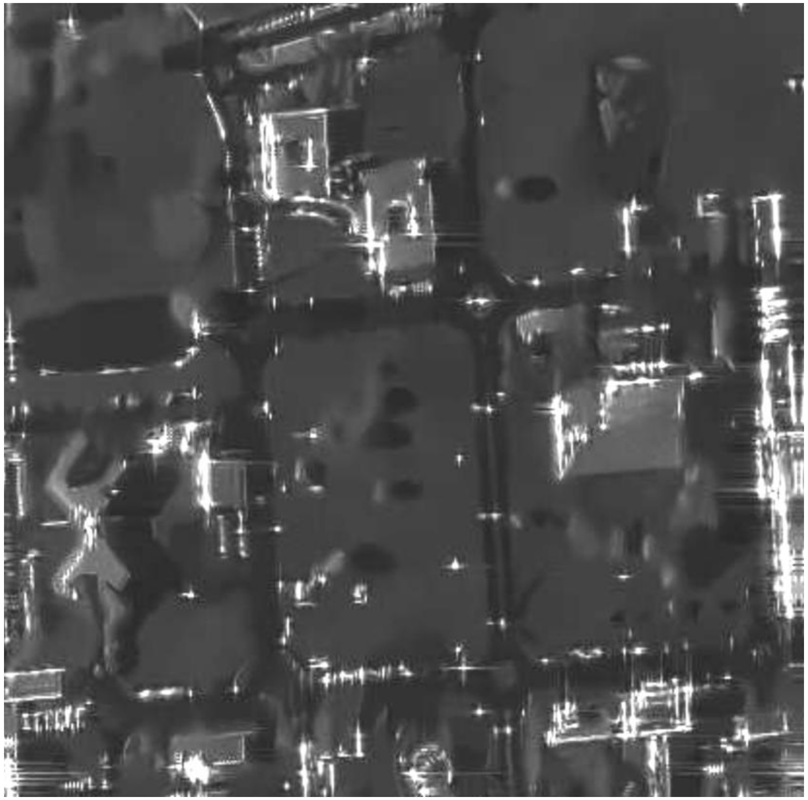


NL-SAR denoising

NL-SAR – Results (amplitude)



100-looks SAR image @CNES/ONERA

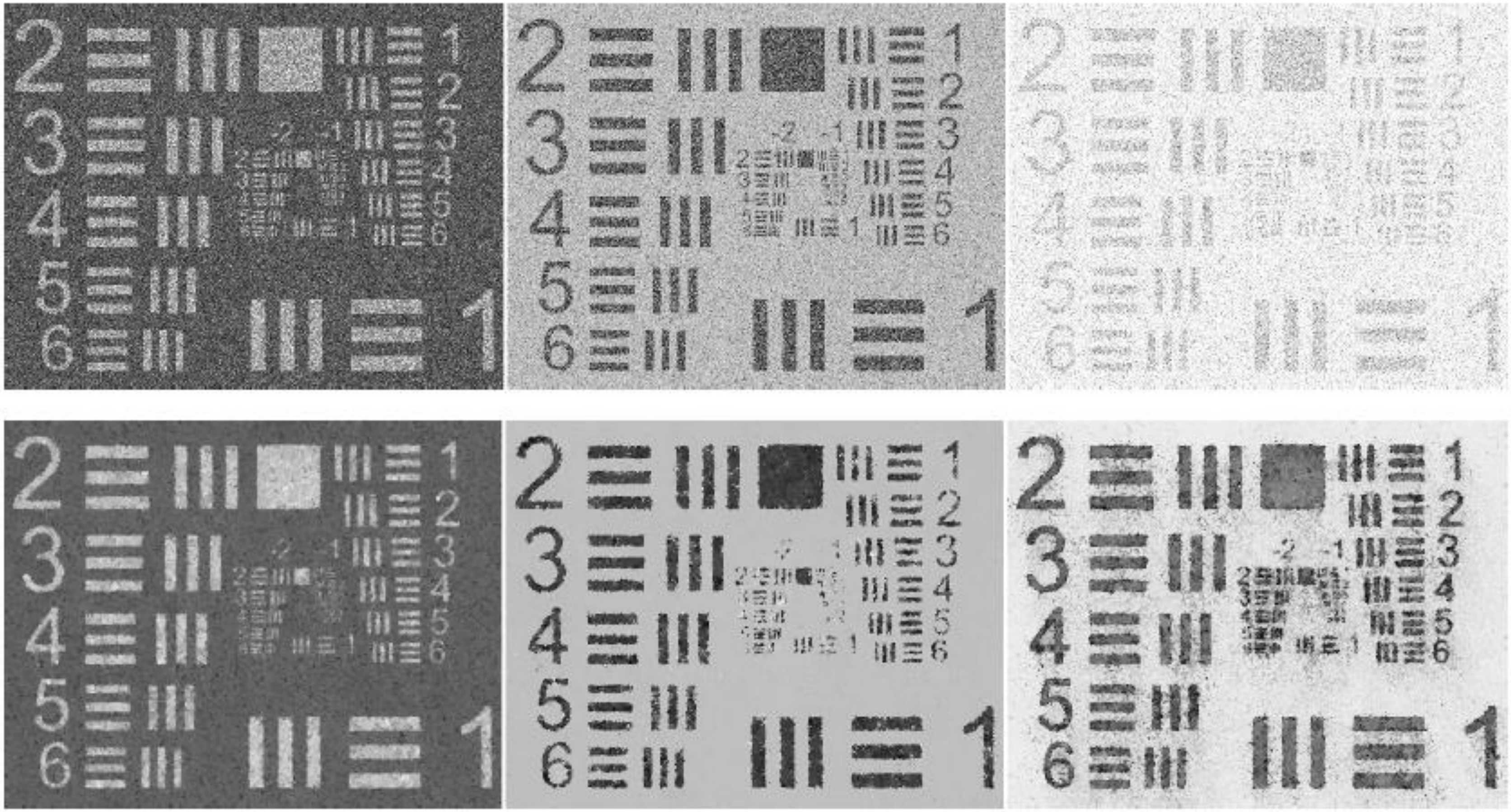


NL-SAR denoising

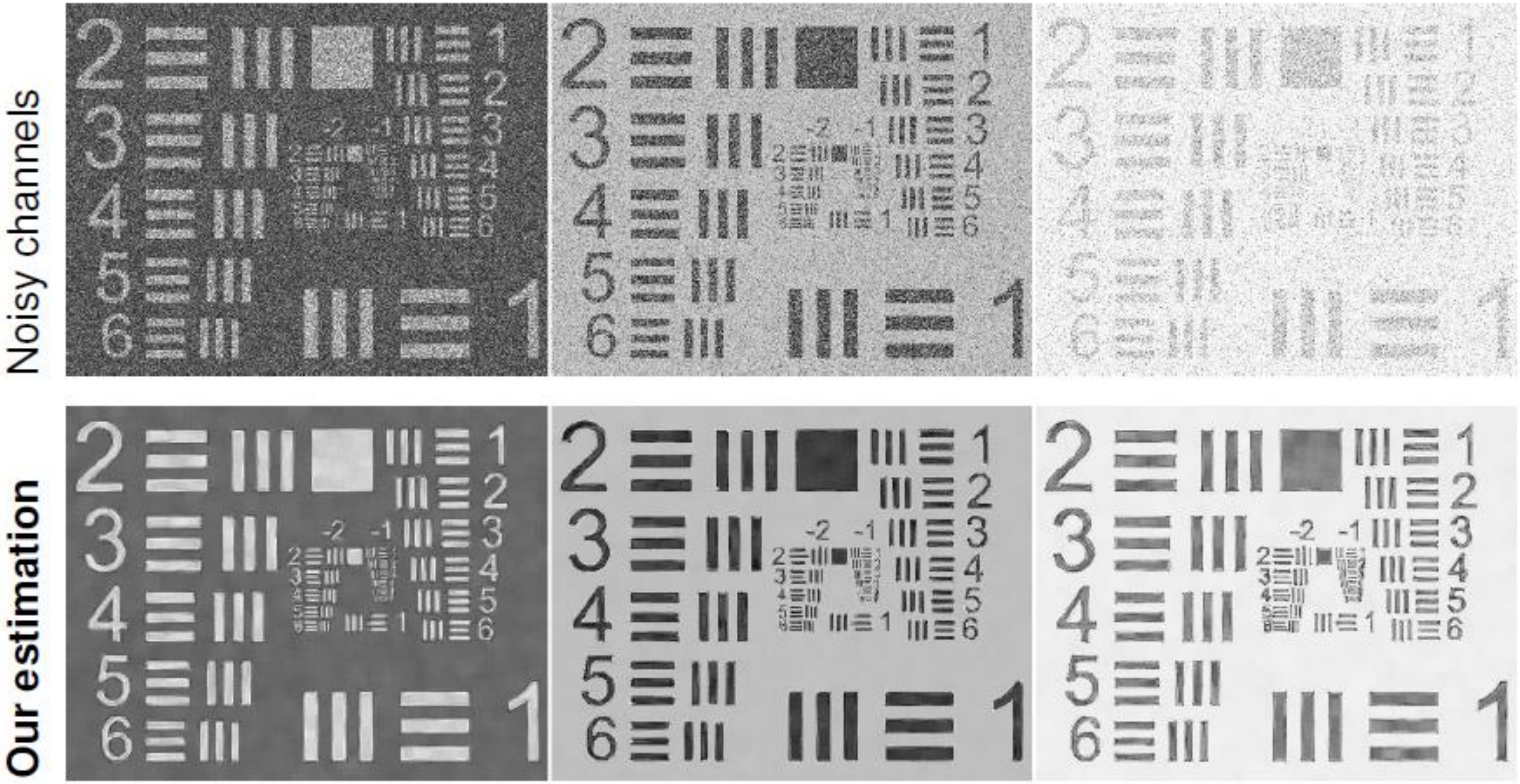
NL-SAR – Results (interferometry)

Noisy channels

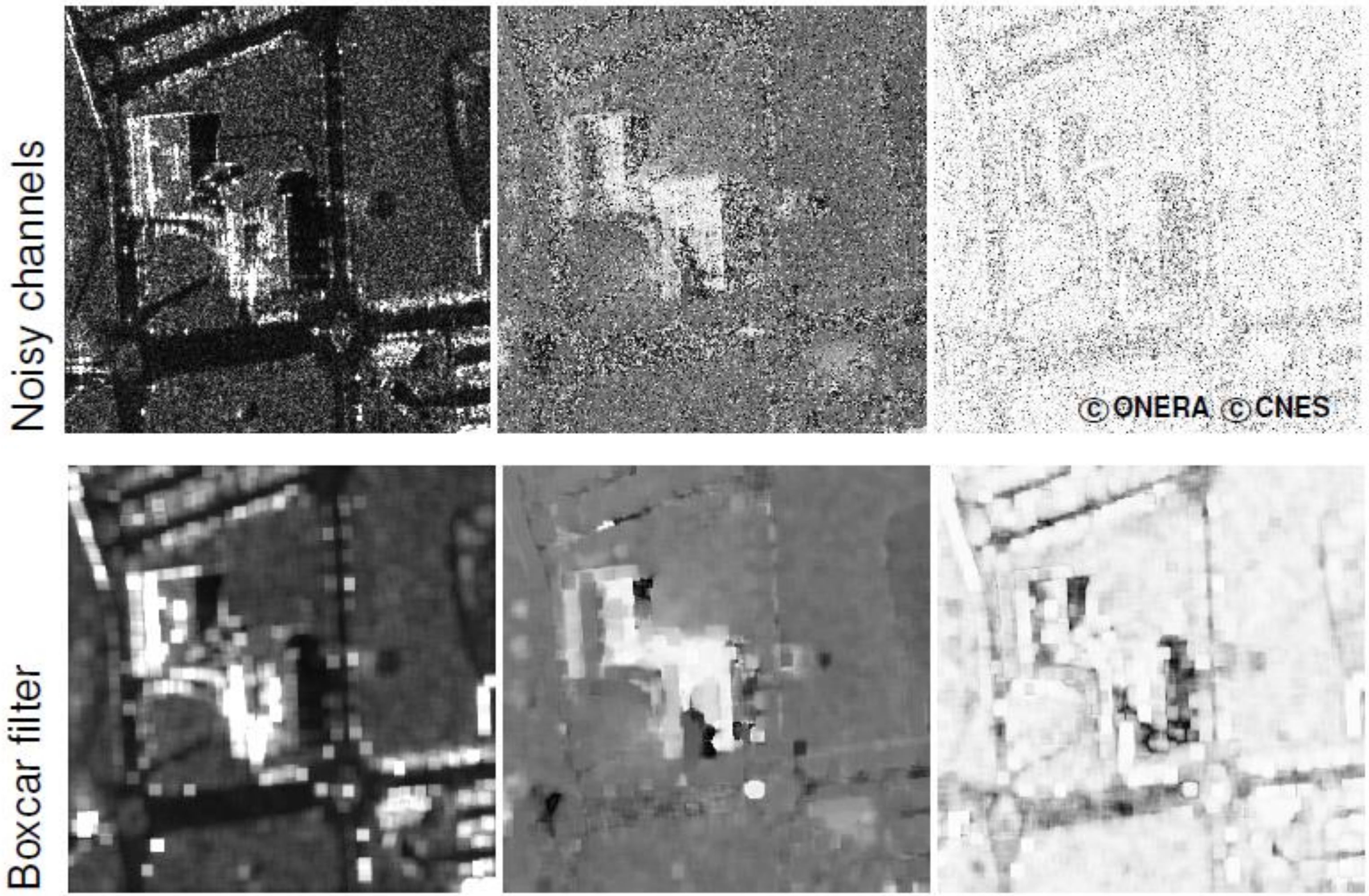
[Vasile et al., 2006]



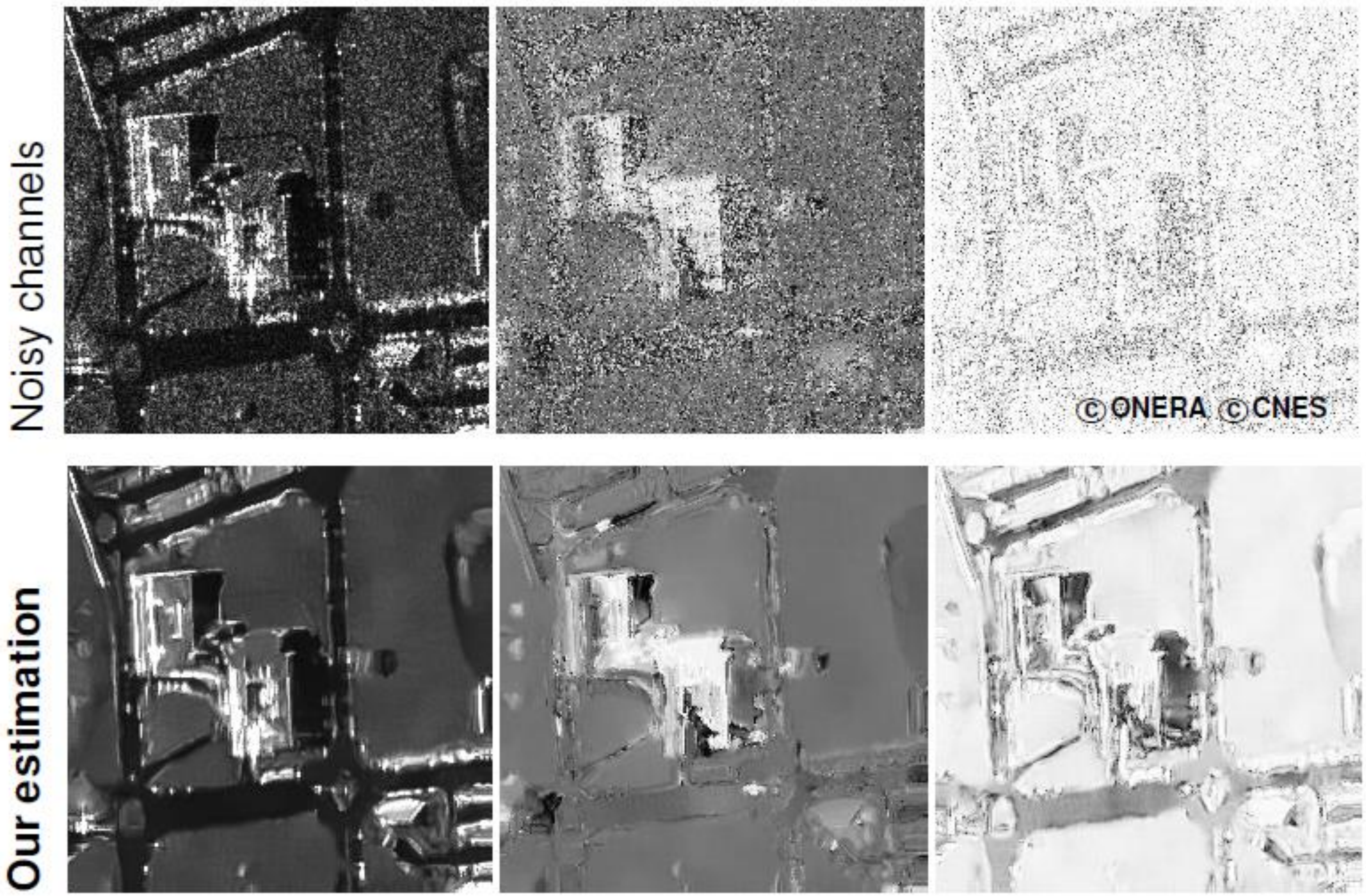
NL-SAR – Results (interferometry)



NL-SAR – Results (interferometry)



NL-SAR – Results (interferometry)



NL-SAR – Results (polarimetry)

(a) Noise-free (b) Noisy (c) Boxcar 7×7

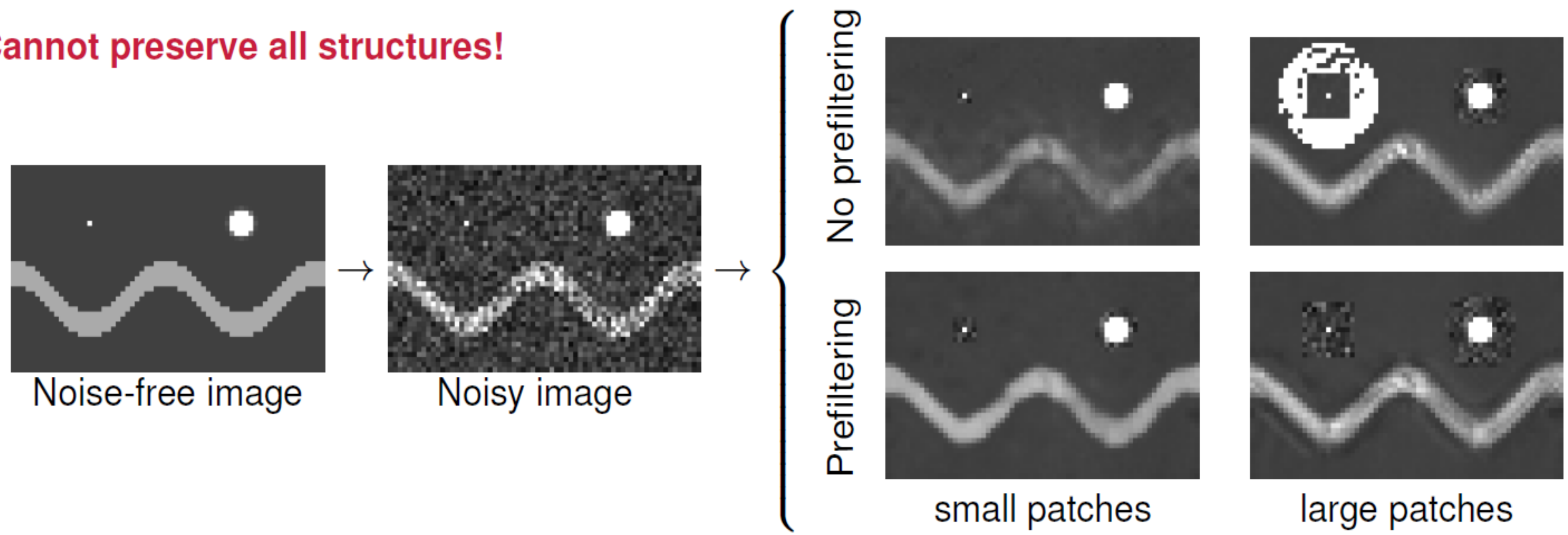
(d) Lee (e) IDAN (f) **NL-PolSAR**

$$\underbrace{|HH + VV|, |HH - VV|, |HV|}_{RVB}$$

(2) Spatially adaptive aggregation

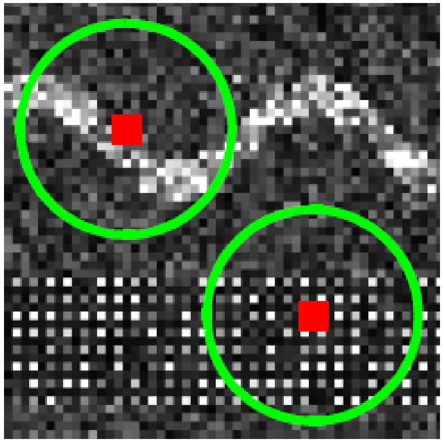
- **Many parameters:**
  - Search window size (rare patch, influence of small weights)
  - Patch size (rare patch effect, noise halo)
  - Number of iterations / pre-filtering strength (bias / variance)
- **Antagonist criteria: no best global tuning**
  - Quality of the estimation / amount of filtering

× **Cannot preserve all structures!**

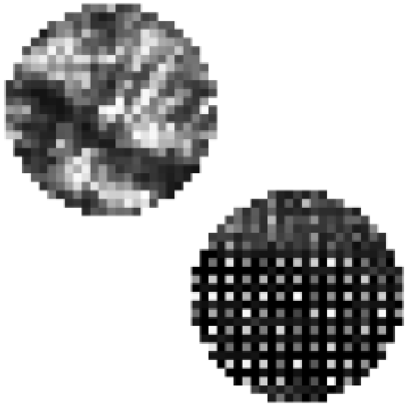


Influence of pre-filtering

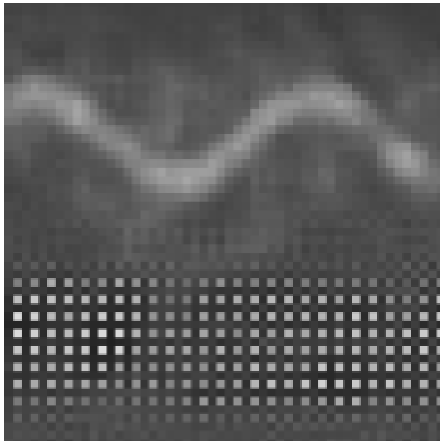
Noisy image



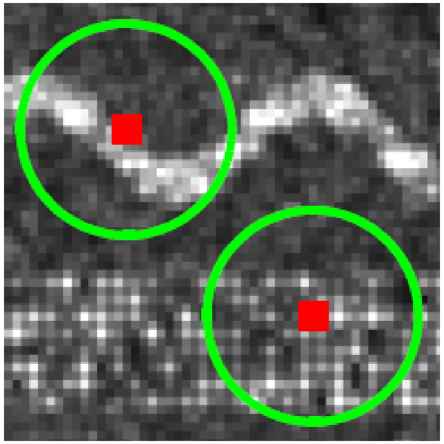
Weights without prefiltering



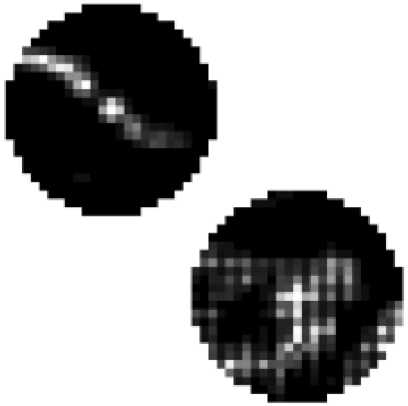
Result without prefiletring



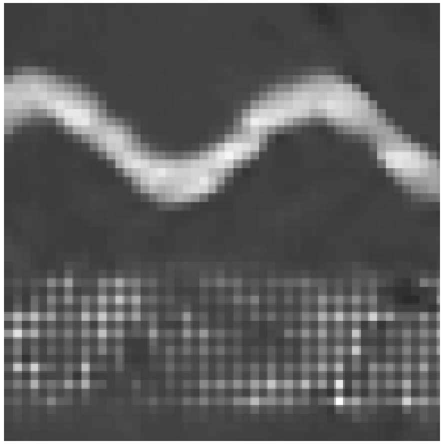
Prefiletred image



Weights with prefiltering



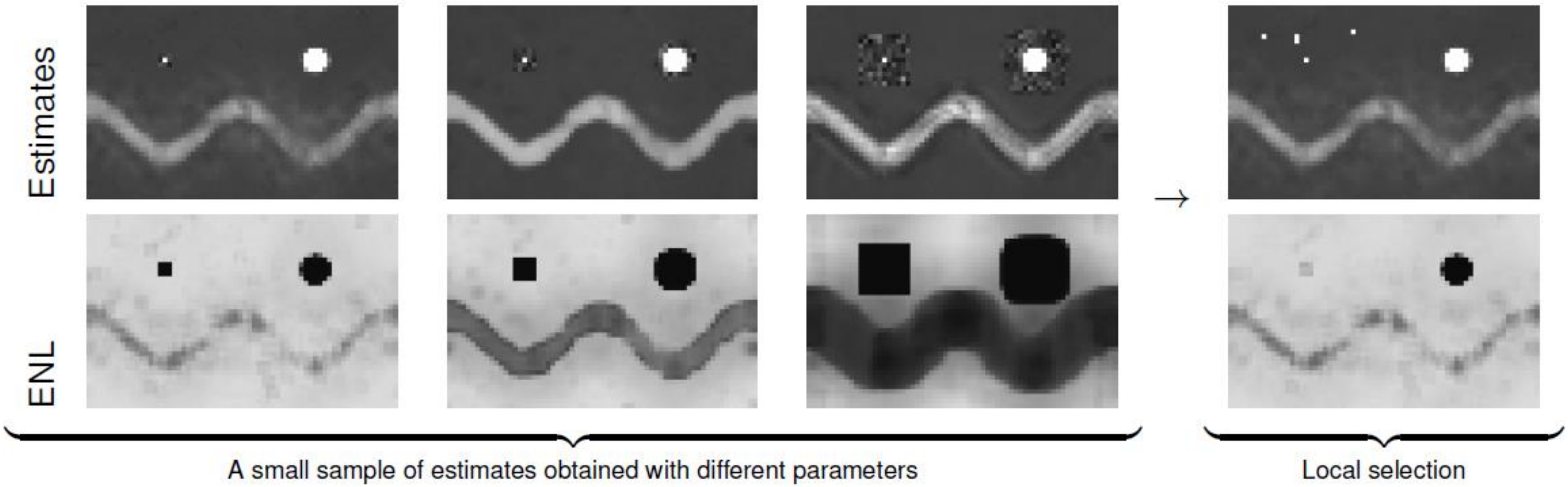
Result with prefiletring



(2) Spatially adaptative aggregation

- **Aggregation:**
  - Compute several estimates with different parameters
  - Select the estimate with the best smoothing

$$\hat{L}^{\mathrm{NL}}(x) = \frac{\left(\sum_{x'} w(x,x')\right)^2}{\sum_{x'} w(x,x')^2}$$

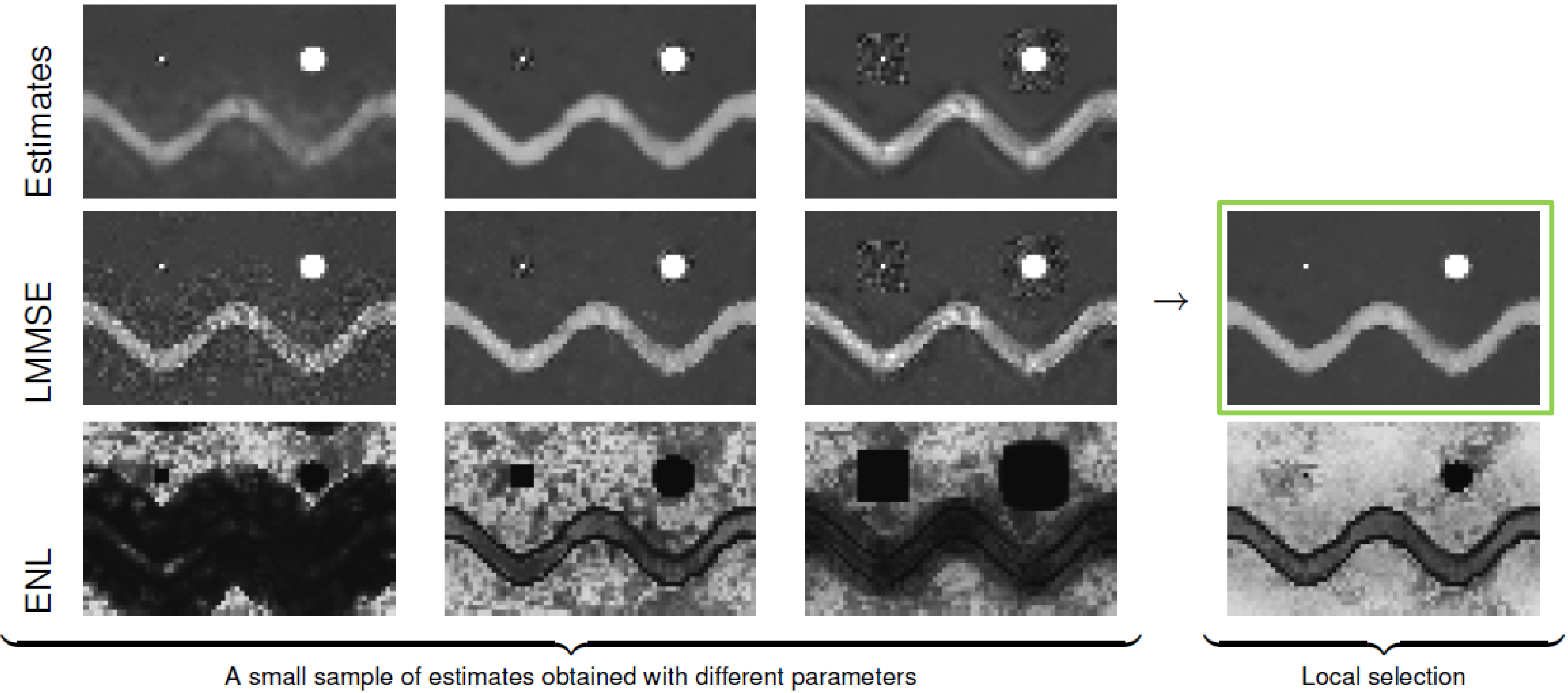


Strong blurring: only takes into account estimation variance but not the bias

TELECOM ParisTech

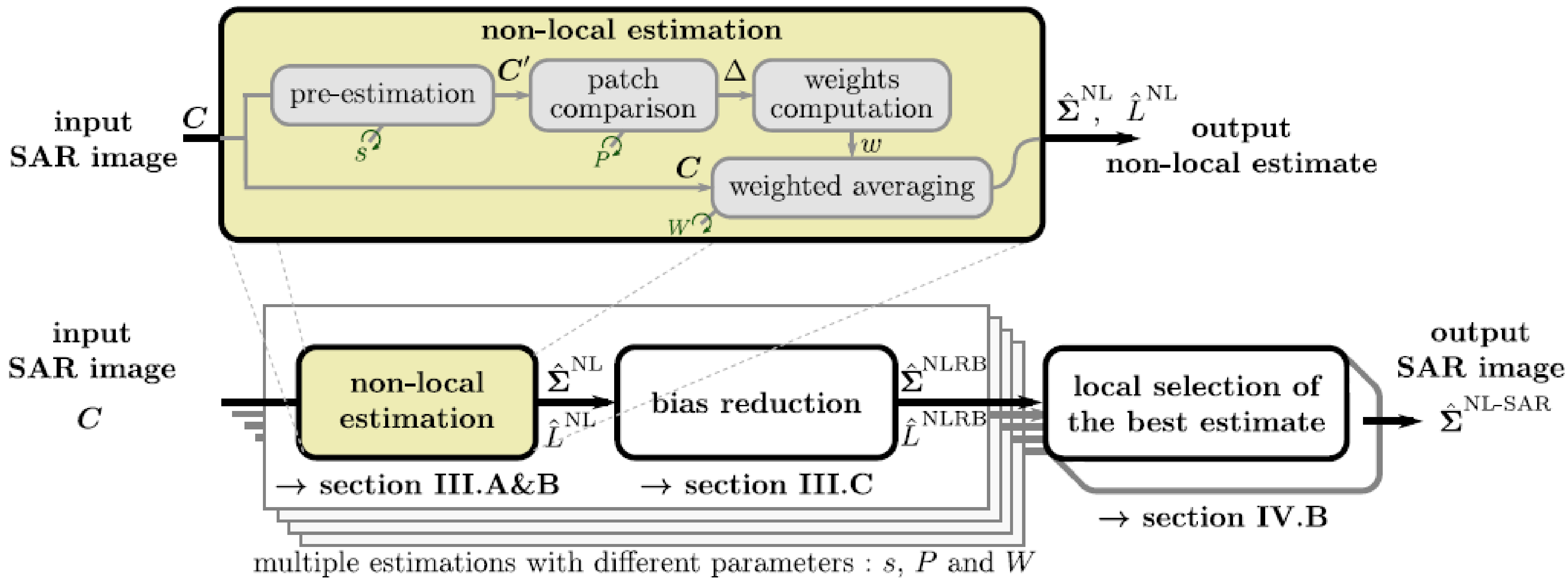
(2) Spatially adaptive aggregation

- **Before aggregation:**
  - Apply bias reduction for each estimation
  - Select the bias reduced estimate with the best smoothing

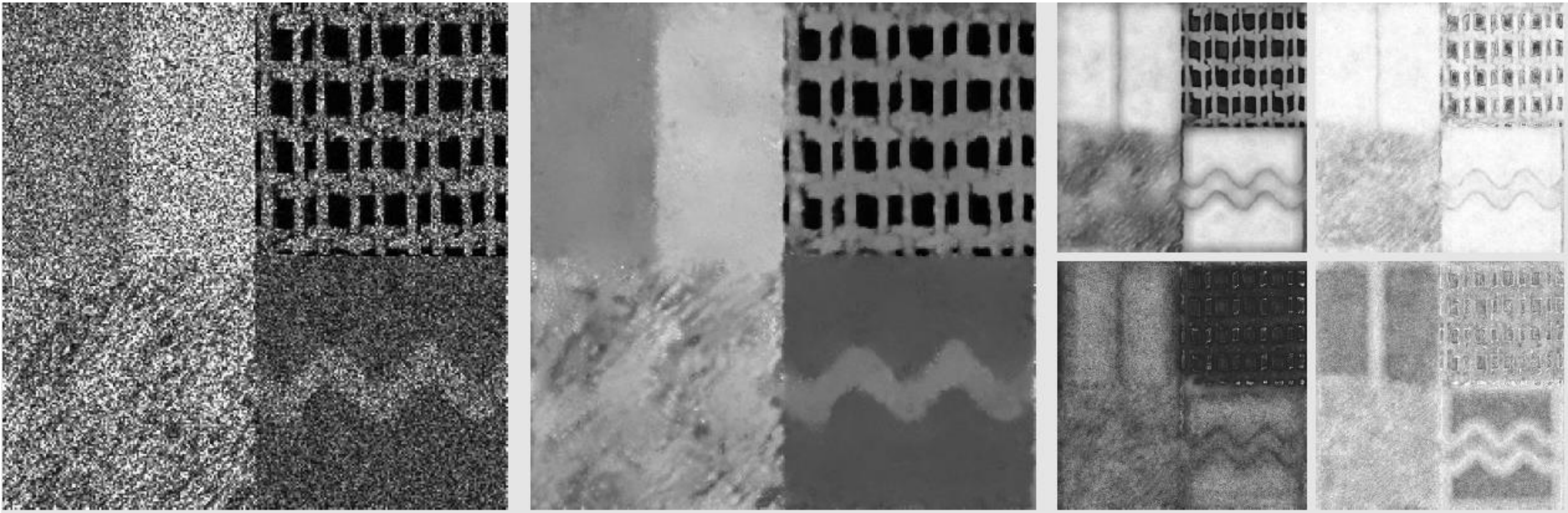


(2) Spatially adaptive aggregation

- **General scheme:**



Example of spatially adaptive aggregation



(a) (b) (c)

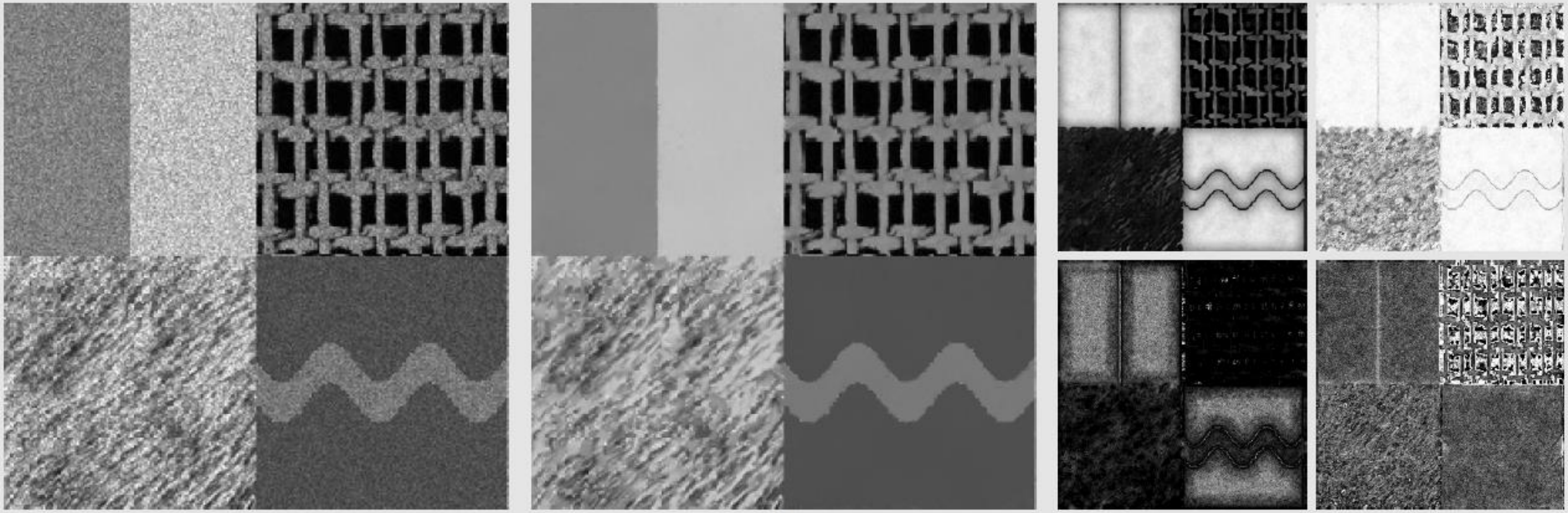
(a) Noisy image.

(b) Result of the adaptive approach.

(c) From left to right, top to bottom:

- smoothing strength (range: $[0, 20 \times 20]$),
- search window sizes (range: $[0, 20 \times 20]$),
- the patch size (range: $[3 \times 3, 11 \times 11]$),
- prefiltering strength (range: $[1, 3]$).

Example of spatially adaptive aggregation



(a) (b) (c)

(a) Noisy image.

(b) Result of the adaptive approach.

(c) From left to right, top to bottom:

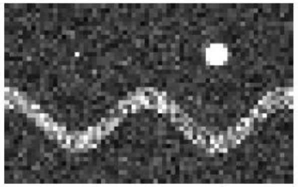
- smoothing strength (range: $[0, 20 \times 20]$),
- search window sizes (range: $[0, 20 \times 20]$),
- the patch size (range: $[3 \times 3, 11 \times 11]$),
- prefiltering strength (range: $[1, 3]$).

TELECOM ParisTech

NL-SAR – general formulation

- **Observation:**

$$\begin{cases} \mathbf{k} \\ \boldsymbol{C}(x) = \dfrac{1}{L}\displaystyle\sum_{t=1}^{L} \boldsymbol{k}^{(t)} \boldsymbol{k}^{(t)\dagger} \end{cases}$$



Observation $I = |z|^2 = \boldsymbol{C}$

- **To be estimated:** $\boldsymbol{\Sigma} = \mathbb{E}\{\boldsymbol{k}\boldsymbol{k}^{\dagger}\}$

Speckle-free image $\sigma^2 = \boldsymbol{\Sigma}$

?

- **(Pre-estimation: local gaussian filtering C')**

- **Weights definition:**

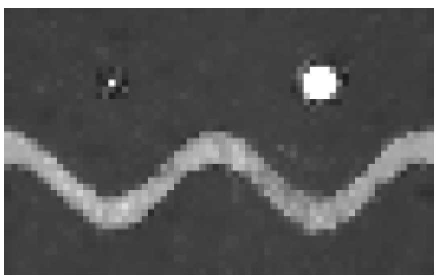
$$\begin{cases} \mathcal{L}_G(\boldsymbol{C}'_1, \boldsymbol{C}'_1) = \dfrac{|\boldsymbol{C}'_1|^{L'} \cdot |\boldsymbol{C}'_2|^{L'}}{|\frac{1}{2}(\boldsymbol{C}'_1 + \boldsymbol{C}'_2)|^{2L'}} \\ \Delta(x, x') = \displaystyle\sum - \log \mathcal{L}_G\left[\boldsymbol{C}'(x+\tau), \boldsymbol{C}'(x'+\tau)\right] \\ w(x, x') = \exp\left[-\dfrac{\Delta(x, x')}{h}\right] \end{cases}$$

NL-SAR – general formulation

- **Weighted ML estimation:**

$$\widehat{\mathrm{Var}\left[I_j\right]}^{\mathrm{NL}}(x) = \frac{\sum_{x'} w(x,x') I_j(x')^2}{\sum_{x'} w(x,x')} - \hat{I}_j^{\mathrm{NL}}(x)^2$$

- **De-biasing:**



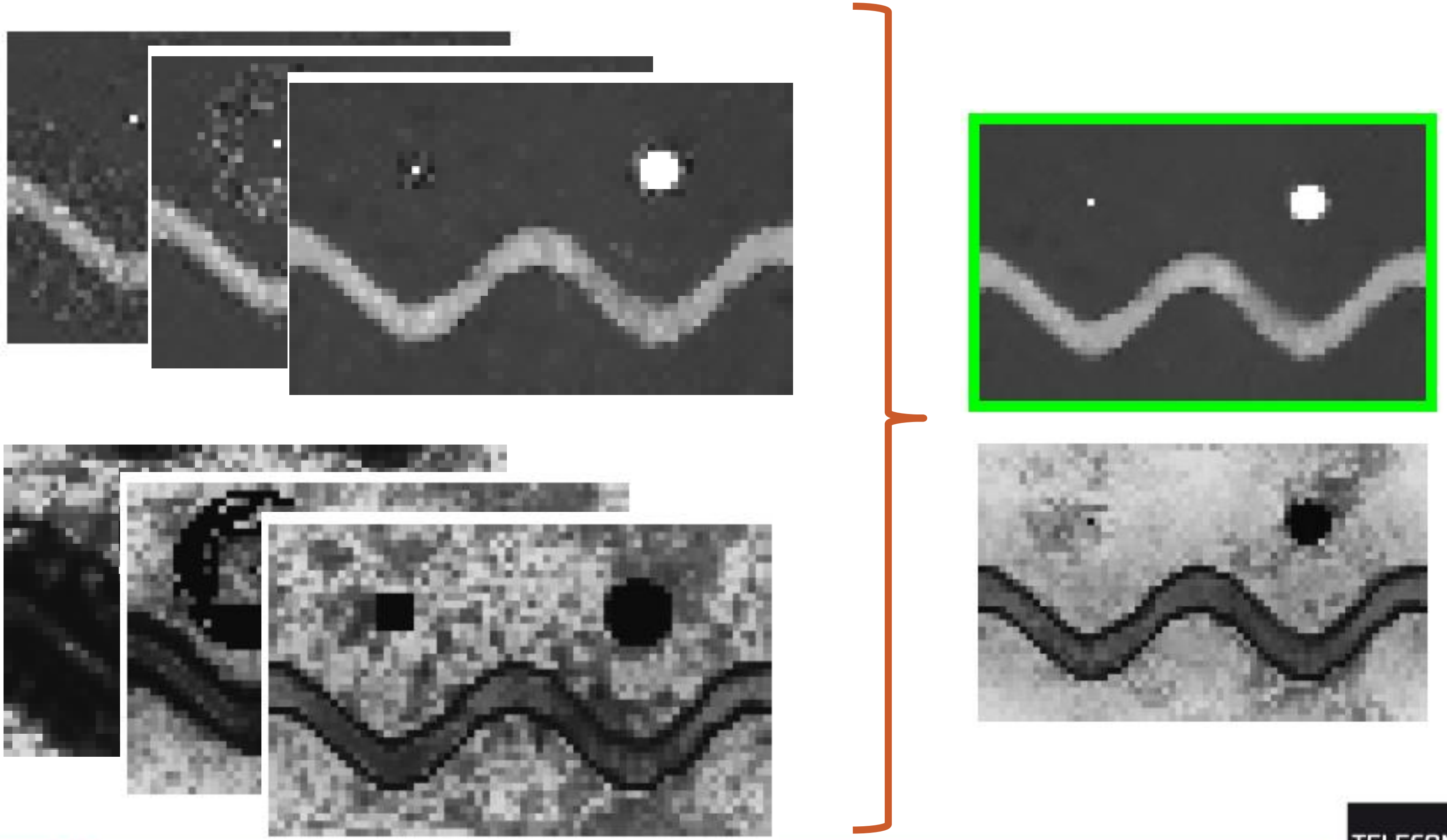
$$\begin{cases} \hat{\mathbf{\Sigma}}^{\mathrm{NLRB}}(x) = \hat{\mathbf{\Sigma}}^{\mathrm{NL}}(x) + \alpha \left[\mathbf{C}(x) - \hat{\mathbf{\Sigma}}^{\mathrm{NL}}(x) \right] \\ \alpha^{\mathrm{NLRB}} = \max_j \left[\max \left(0, \frac{\widehat{\mathrm{Var}\left[I_j\right]}^{\mathrm{NL}}(x) - \hat{I}_j^{\mathrm{NL}}(x)^2/L}{\widehat{\mathrm{Var}\left[I_j\right]}^{\mathrm{NL}}(x)} \right) \right] \end{cases}$$

- **Smoothing strength:**

$$\begin{cases} \hat{L}^{\mathrm{NL}}(x) = \frac{\left(\sum_{x'} w(x,x')\right)^2}{\sum_{x'} w(x,x')^2} \\ \hat{L}^{\mathrm{NLRB}}(x) = \frac{\hat{L}^{\mathrm{NL}}(x)}{(1-\alpha)^2 + \left(\alpha^2 + \frac{2\alpha(1-\alpha)}{\sum_{x'} w(x,x')}\right) \hat{L}^{\mathrm{NL}}(x)} \end{cases}$$

NL-SAR – general formulation

- **Agglomeration:**



Results – synthetic data

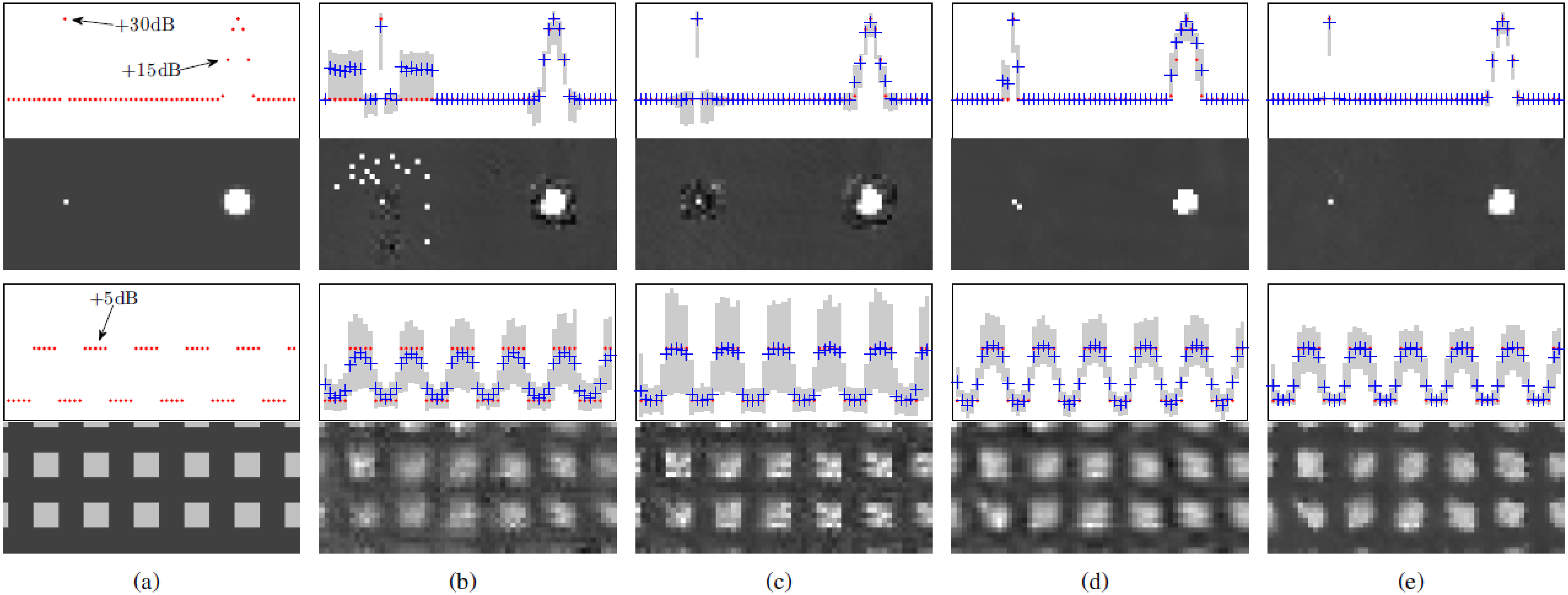
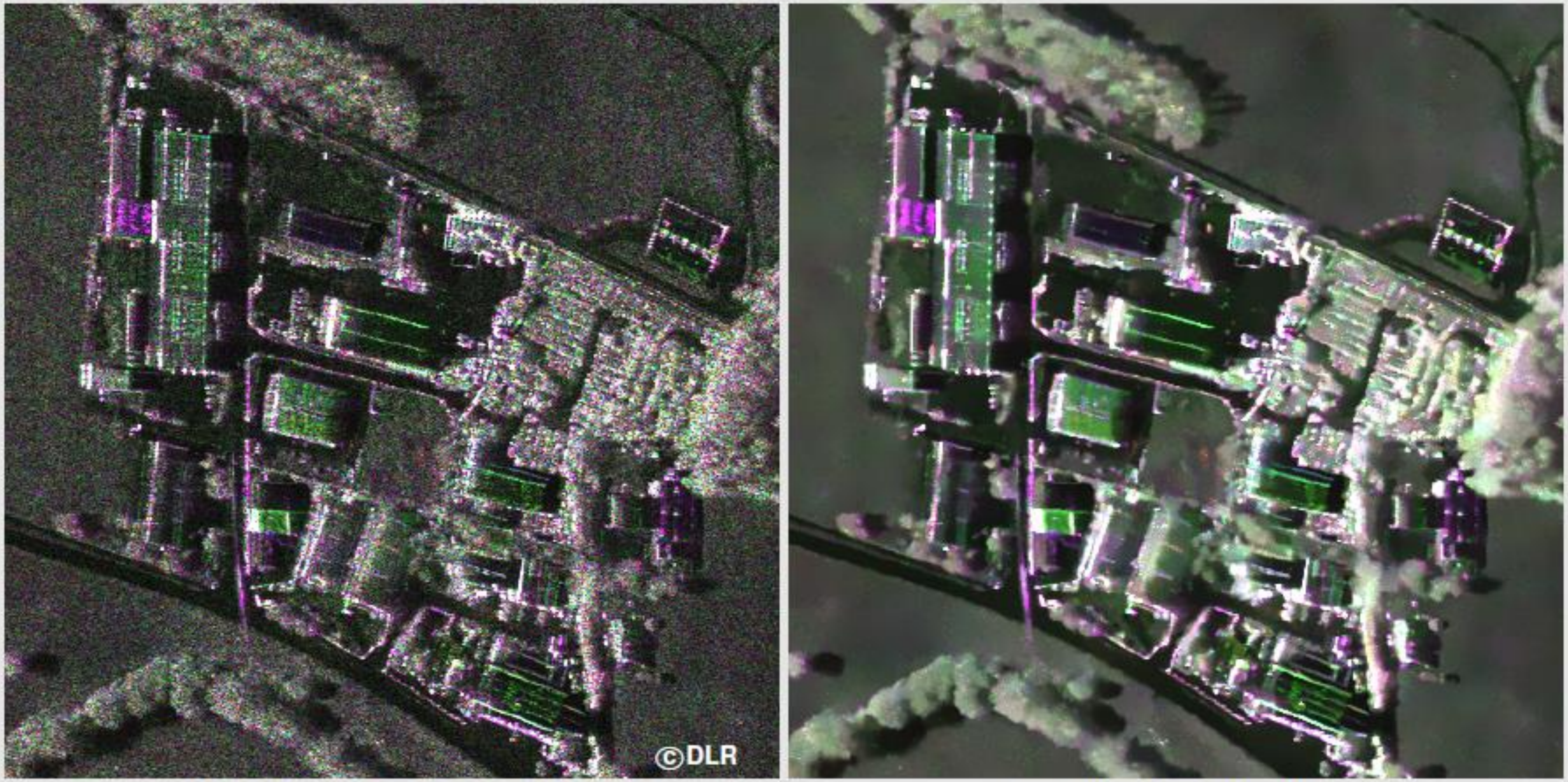


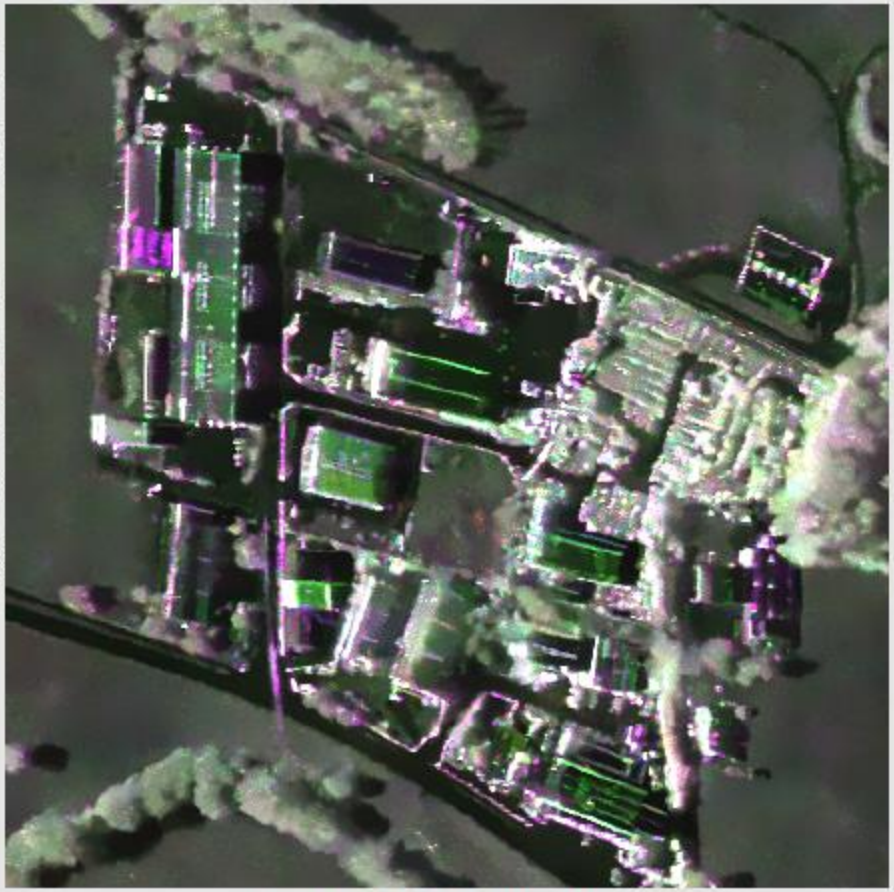
Fig. 6. Bias-variance characterization of the several speckle reduction methods for single look SAR intensity denoising: (a) underlying signal, (b) Pretest non-local filter [37], (c) PPB-it [34], (d) SAR-BM3D [38], (e) NL-SAR proposed in this paper. Two types of structures are analyzed: bright targets (upper half) and repeating squares (lower half). Grayscale images show the output of each denoising method for a given noisy realization. Above each grayscale image, line profiles corresponding to the expectation (blue crosses) and 0.98% confidence intervals (gray area) of each estimator are drawn. Line profile intensities on the top row are drawn in log-scale to adapt to the high dynamic range.

Polarimetry – NL-SAR results

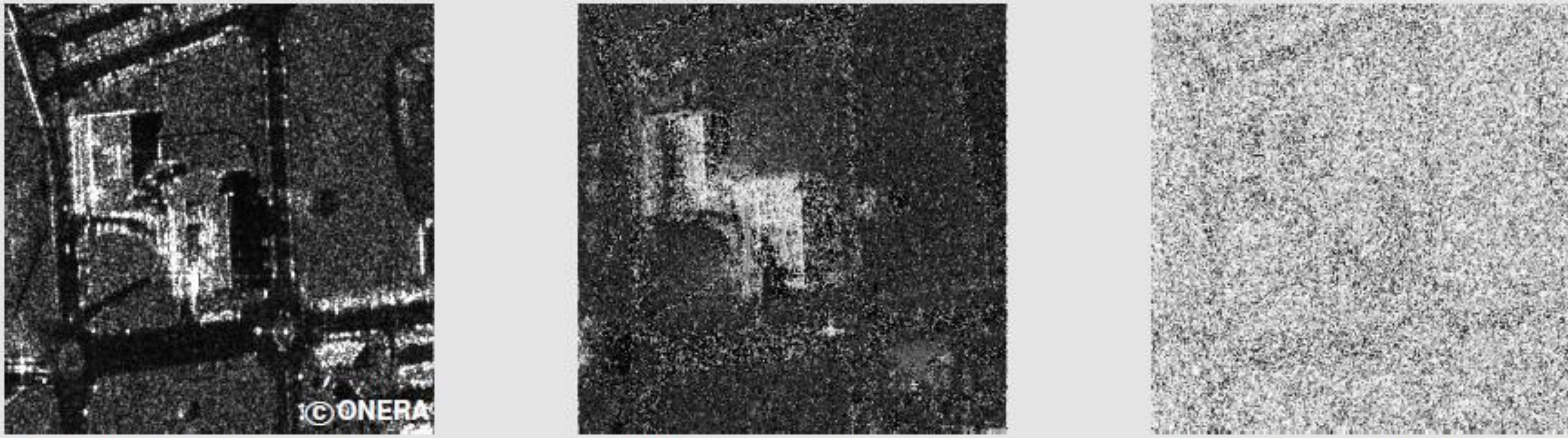


(a) High-resolution S-band SAR image (b) **Adaptive estimation**

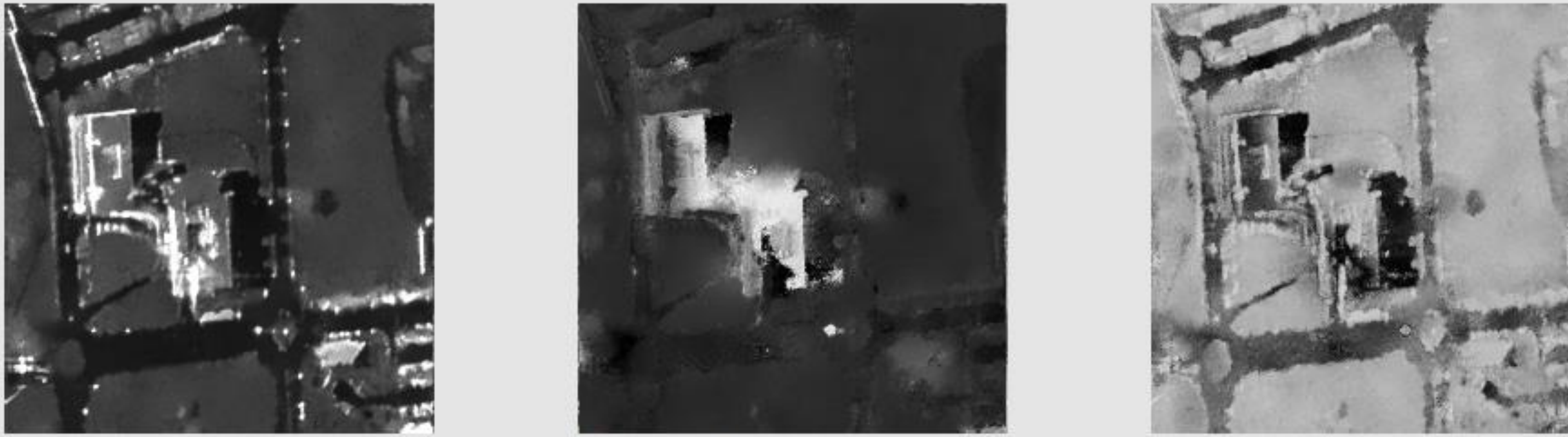
(a) **Smoothing strength**



(b) **Adaptive estimation**



(a) High resolution SAR image (b) Interferometric phase (c) Interferometric coherence



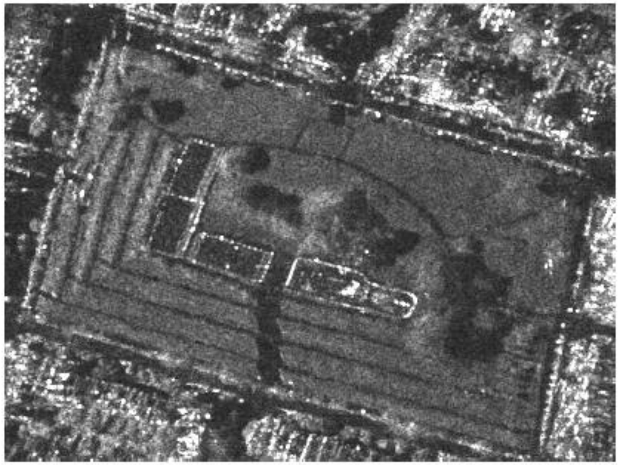
(d) (Joint) Adaptive estimation

(c) *Saint-Gervais-les-Bains* images. From left to right: noisy image, denoising result by multi-looking, denoising results by 1-PPB on single image.

(d) *Saint-Gervais-les-Bains* images. From left to right: 3D-ANF, denoising result by T-PPB on temporal images, denoising results by 2S-PPB on temporal images.

(a) y_{t1}, One of the noisy images (1-$look$)

(b) Temporal average without miss-registration estimation



(c) Temporal average with miss-registration estimation

(d) Temporal PPB (T-PPB) using 6 un-registered SAR images

(e) The proposed 2S-PPB (with miss-registration estimation) using 6 un-registered SAR images

(f) The proposed 2S-PPB using 6 well-registered SAR images

Conclusion

- **Non-local approaches for SAR data**
  - Integration of the data acquisition models (data distributions)
  - General formulation
  - Spatial and temporal adaptation

- **Perspectives**
  - New statistical models ?
  - Understanding of radar signal: dictionaries, complex spectrum

Acknowledgements to data providers: ONERA, CNES, DLR (project MTH0232)
ASI (project 2145 – 1780)