



Inverse problems in functional brain imaging

Identification of the hemodynamic response in fMRI

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1: CEA/NeuroSpin/LNAO

2: IFR49

NeuroSpin



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GDR -ISIS Summer school -Peyresq



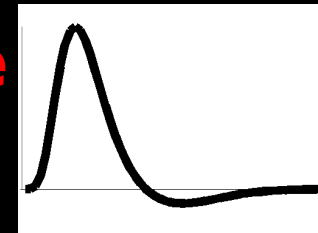
Brain dynamics in BOLD fMRI

Probe brain dynamics
non-invasively



stimulus →

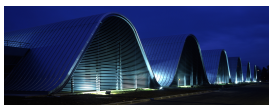
**hemodynamic response
function (HRF)**



parametric HRF [Friston et al, 1994; Glover et al, 1999]
non-parametric HRF [Goutte et al, 2000; Marrelec et al,
2003]

non-stationary linear model [Donnet et al, 2006]
Balloon model [Buxton et al, 1998; Friston, 2000;
Buxton et al, 2004]

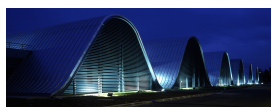
↓
BOLD fMRI



Brain dynamics in fMRI

Why is it important?

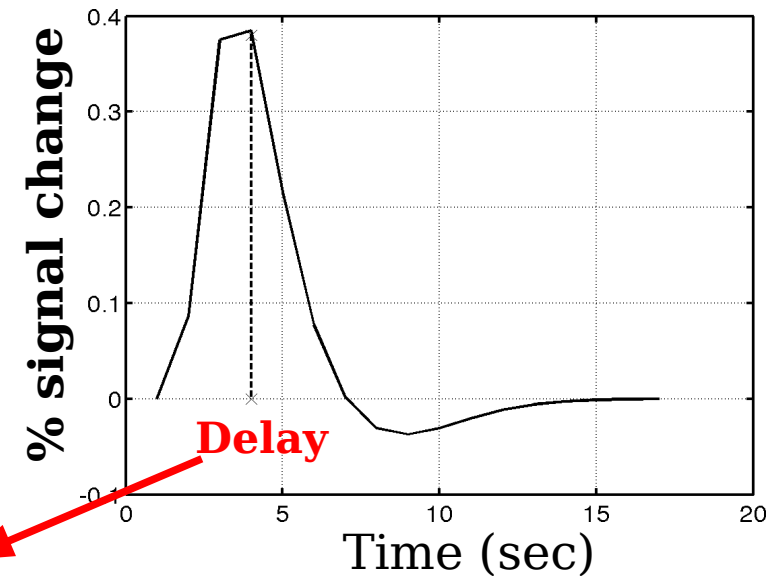
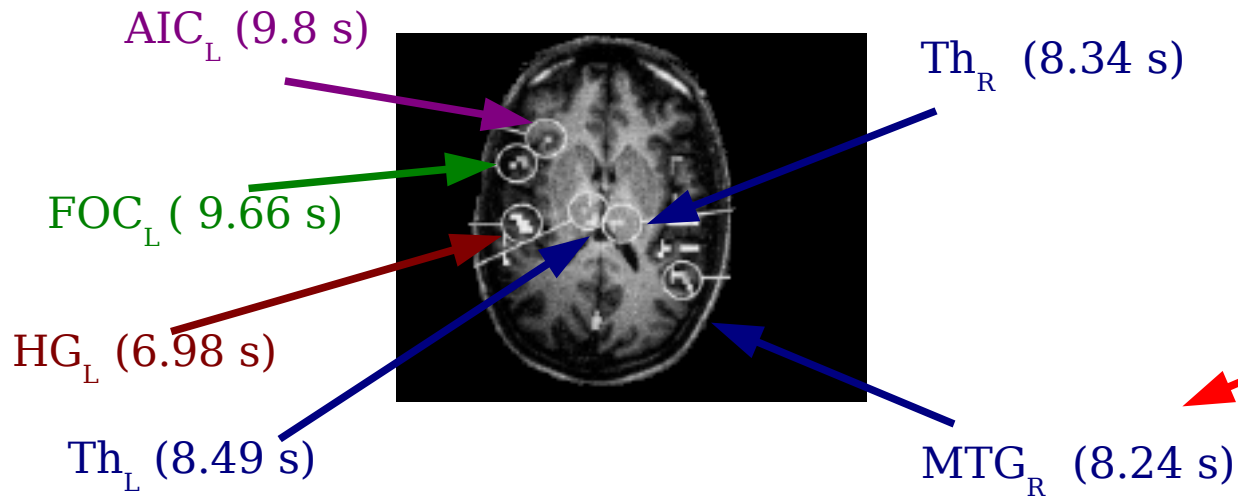
- Elucidate neural code:
 - Extract temporal information (magnitude, delay, width)
 - Study variability between conditions or tasks
 - Study non-linear or non-stationary effects
- Reflect subject's strategy or performance
 - Between subject variability
- Complementary analysis of electromagnetic modalities



Extract temporal information

- Understanding the chronology of activations in single trial fMRI experiments
- Inferring the causality of underlying neural processes

[Kruggel & von Cramon, 1999]

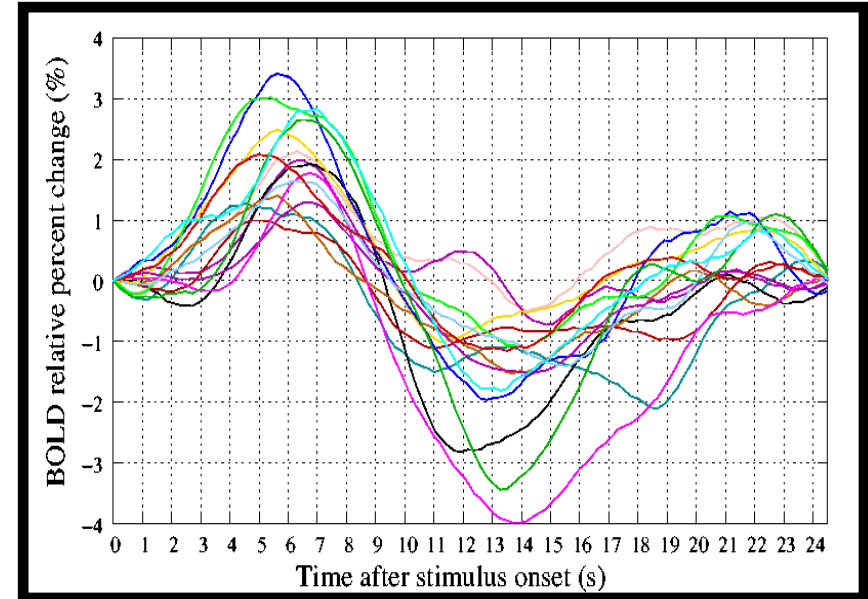
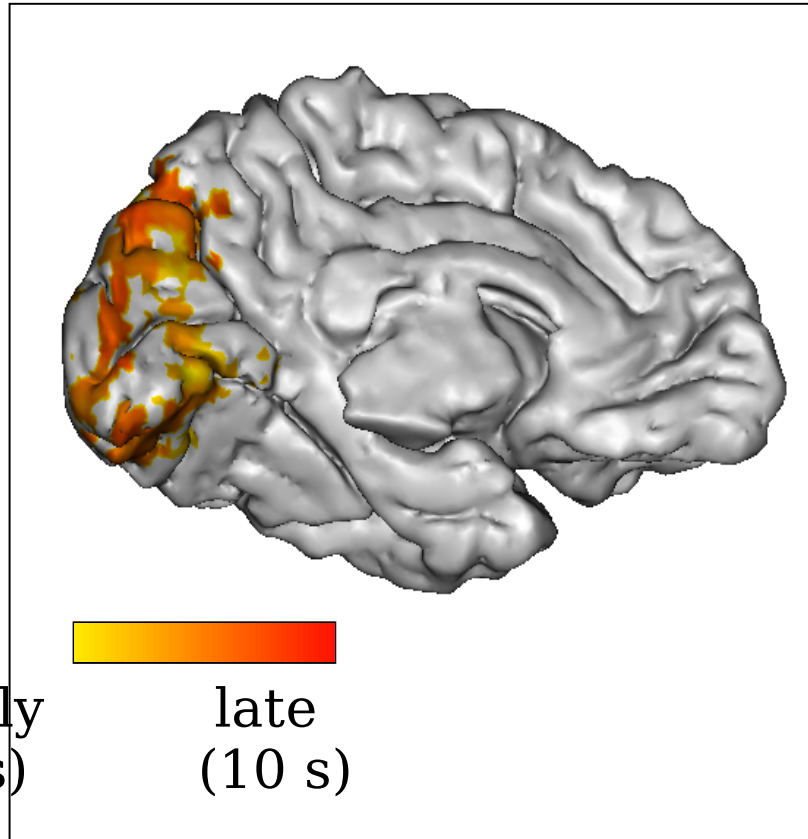


Delays (sec): $HG_L \sim HG_R < MTG_R \sim Th_R \sim Th_L < FOC_L$

[Saad et al 2001; Liao et al, 2002; Henson et al, 2002]



Delay mapping



Comparison of the HRF time to peak between voxels

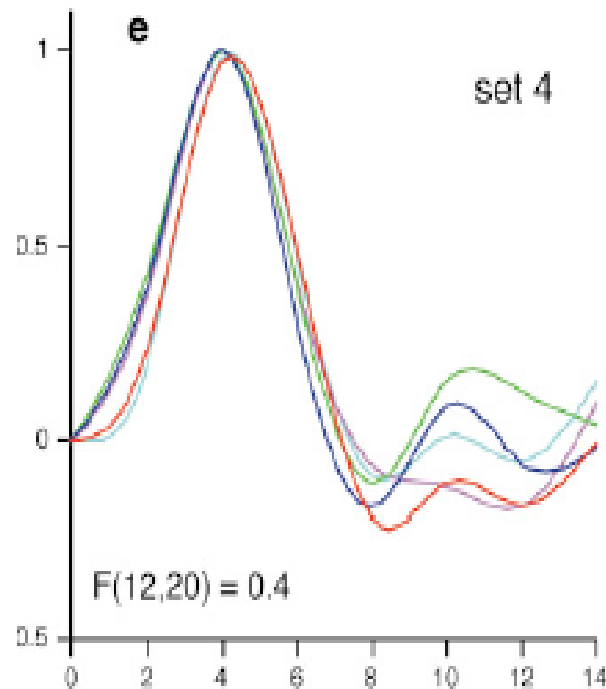
[Rabrait, Ciuciu et al, ISMRM '06]



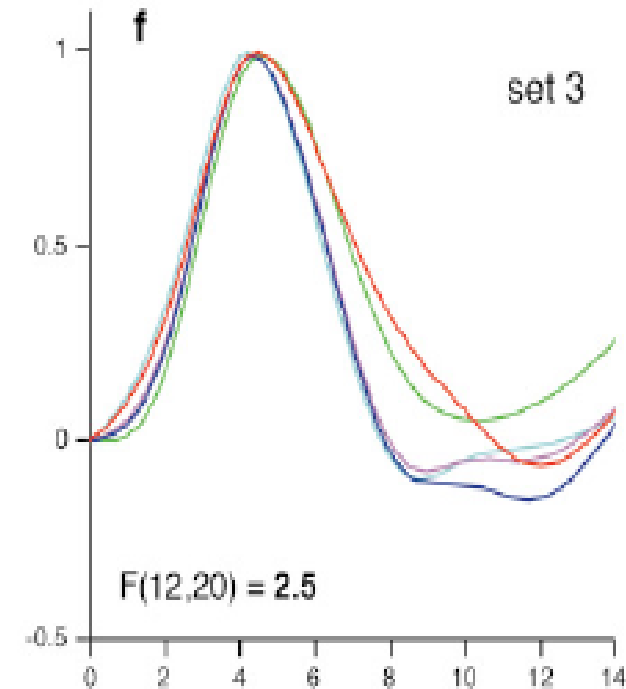
Sources of variability

- Subject, session

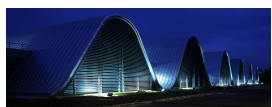
[Aguirre et al, NIM 1998]



Subject 1
less variable

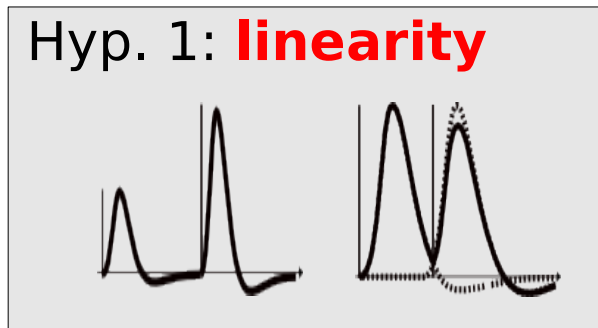


Subject 2
most variable

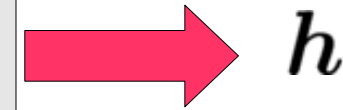
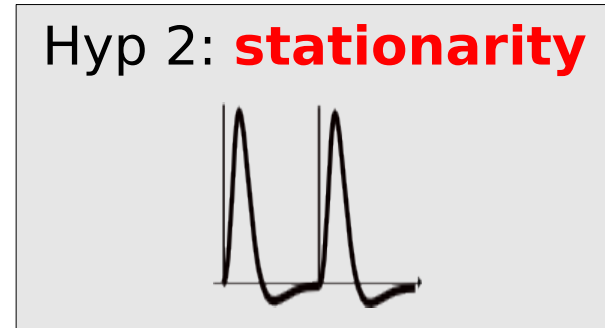


FIR modelling

- Explicit assumptions on the BOLD response

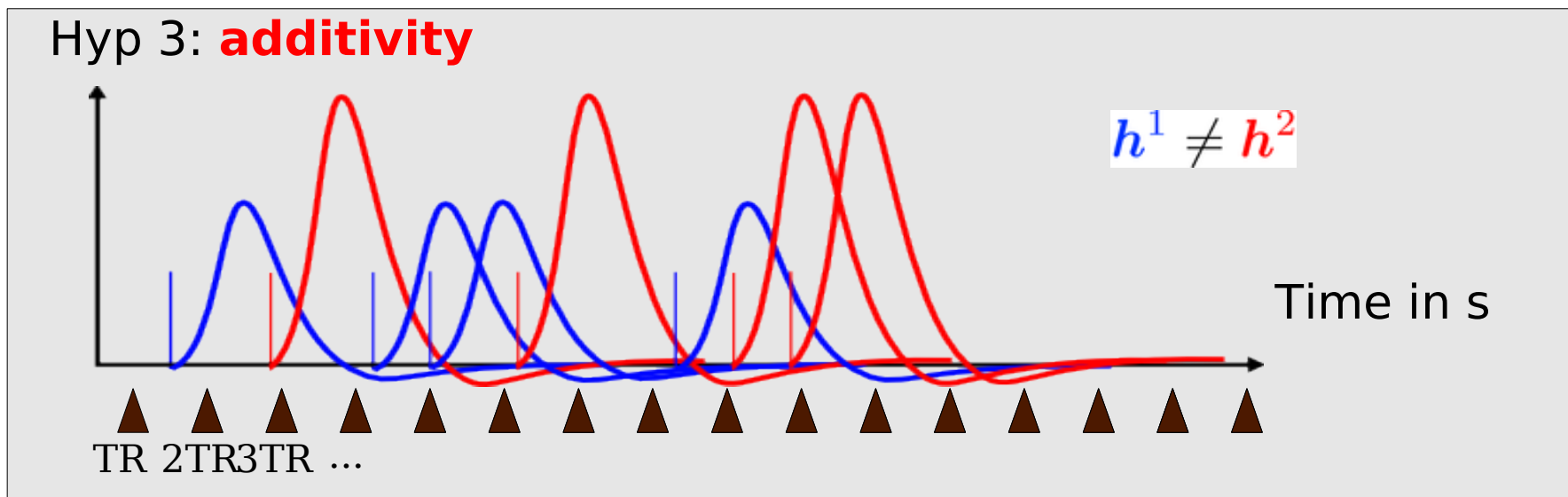


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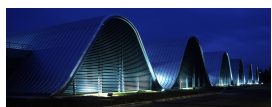
 h

Convolution kernel

[Aguirre et al, 1998; McGonigle et al, 2000; Smith et al, 2005]



[Ciuciu et al, 2003; Makni et al, 2008]



FIR modelling

- Design matrix for estimating the evoked BOLD response

$$\begin{bmatrix} y_j(t_1) \\ y_j(t_2) \\ y_j(t_3) \\ y_j(t_4) \\ y_j(t_5) \\ \vdots \\ \vdots \\ y_j(t_9) \\ y_j(t_{10}) \\ y_j(t_{11}) \\ y_j(t_{12}) \\ y_j(t_{13}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_j^1(0) \\ h_j^1(\Delta t) \\ \vdots \\ h_j^1(4\Delta t) \\ h_j^2(0) \\ h_j^2(\Delta t) \\ \vdots \\ h_j^2(4\Delta t) \\ m_j \end{bmatrix} + \begin{bmatrix} \varepsilon_j(t_1) \\ \varepsilon_j(t_2) \\ \varepsilon_j(t_3) \\ \varepsilon_j(t_4) \\ \varepsilon_j(t_5) \\ \vdots \\ \vdots \\ \varepsilon_j(t_9) \\ \varepsilon_j(t_{10}) \\ \varepsilon_j(t_{11}) \\ \varepsilon_j(t_{12}) \\ \varepsilon_j(t_{13}) \end{bmatrix}$$

($\Delta t = \text{TR}$)

$$\mathbf{y}_j = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}] \begin{bmatrix} h_j^1 \\ h_j^2 \\ m_j \end{bmatrix} + \boldsymbol{\varepsilon}_j$$



$$= \sum_{m=1}^M \mathbf{X}^m \mathbf{h}^m + m_j \mathbf{1} + \boldsymbol{\varepsilon}_j$$



FIR modelling

- Design matrix for estimating the evoked BOLD response

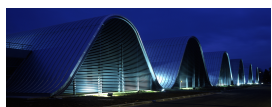
$$\begin{bmatrix} y_j(t_1) \\ y_j(t_2) \\ y_j(t_3) \\ y_j(t_4) \\ y_j(t_5) \\ \vdots \\ \vdots \\ y_j(t_9) \\ y_j(t_{10}) \\ y_j(t_{11}) \\ y_j(t_{12}) \\ y_j(t_{13}) \end{bmatrix} = \begin{bmatrix} \text{[Pulse 1]} & \text{[Pulse 2]} & \text{[Pulse 3]} & \text{[Pulse 4]} & \text{[Pulse 5]} & \text{[Pulse 6]} & \text{[Pulse 7]} & \text{[Pulse 8]} & \text{[Pulse 9]} & \text{[Pulse 10]} & \text{[Pulse 11]} & \text{[Pulse 12]} & \text{[Pulse 13]} \end{bmatrix} \begin{bmatrix} h_j^1(0) \\ h_j^1(\Delta t) \\ \vdots \\ h_j^1(4\Delta t) \\ h_j^2(0) \\ h_j^2(\Delta t) \\ \vdots \\ h_j^2(4\Delta t) \\ m_j \end{bmatrix} + \begin{bmatrix} \varepsilon_j(t_1) \\ \varepsilon_j(t_2) \\ \varepsilon_j(t_3) \\ \varepsilon_j(t_4) \\ \varepsilon_j(t_5) \\ \vdots \\ \vdots \\ \varepsilon_j(t_9) \\ \varepsilon_j(t_{10}) \\ \varepsilon_j(t_{11}) \\ \varepsilon_j(t_{12}) \\ \varepsilon_j(t_{13}) \end{bmatrix}$$

($\Delta t = \text{TR}$)

$$\mathbf{y}_j = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}] \begin{bmatrix} h_j^1 \\ h_j^2 \\ m_j \end{bmatrix} + \varepsilon_j$$



$$= \sum_{m=1}^M \mathbf{X}^m \mathbf{h}^m + m_j \mathbf{1} + \varepsilon_j$$



FIR fitting procedure

- Least squares solution (white Gaussian noise)

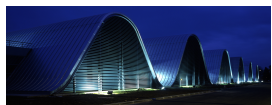
$$\rightarrow \begin{bmatrix} \hat{\mathbf{h}}_j^{\text{OLS}} \\ \hat{m}_j^{\text{OLS}} \end{bmatrix} = (\mathbb{X}^t \mathbb{X})^{-1} \mathbb{X}^t \mathbf{y}_j$$

$$\text{with } \mathbb{X} = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}]$$

- Maximum likelihood solution:
 - Noise structure modelling & estimation

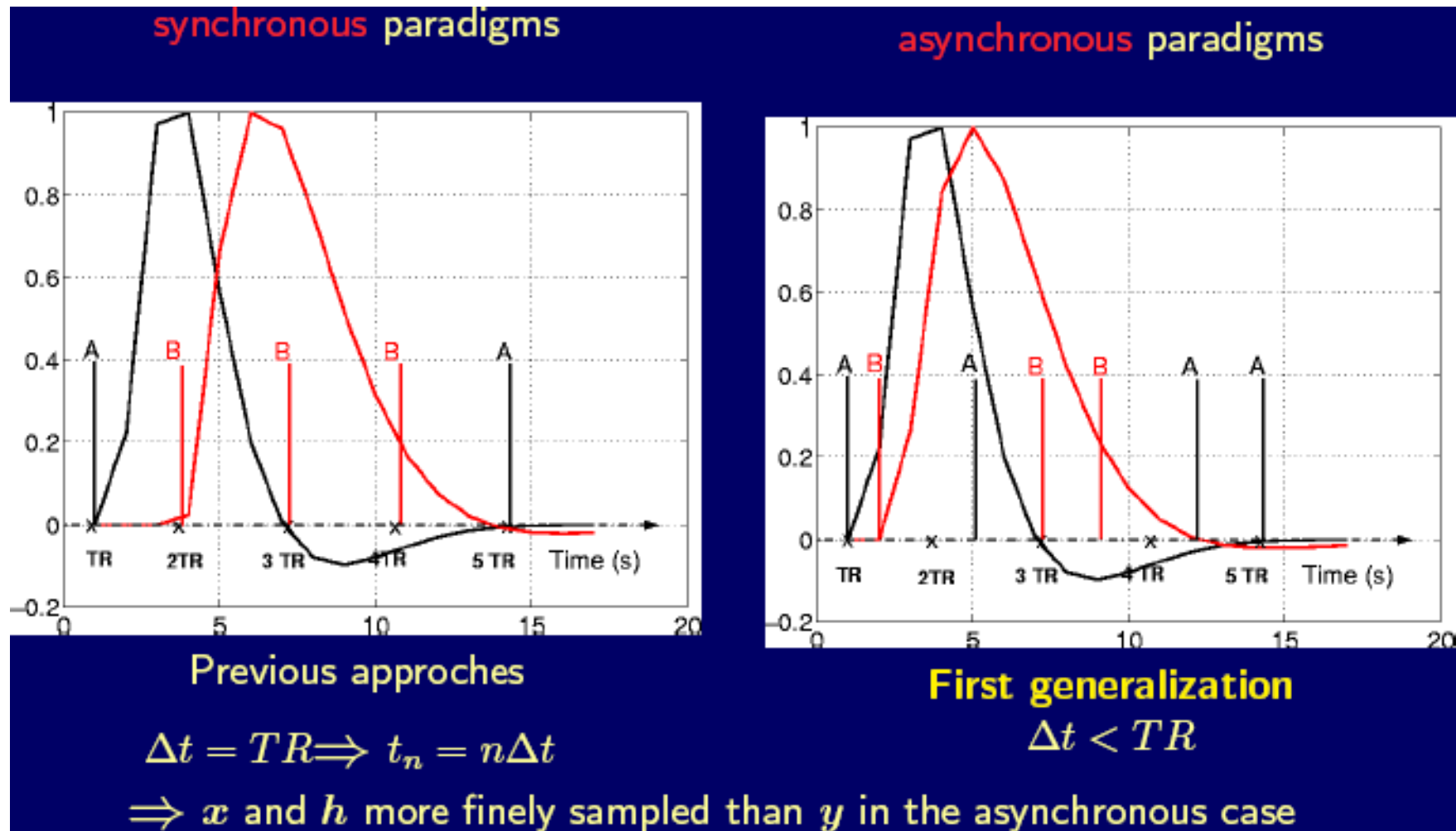
$$\rightarrow \begin{bmatrix} \hat{\mathbf{h}}_j^{\text{ML}} \\ \hat{m}_j^{\text{ML}} \end{bmatrix} = (\mathbb{X}^t \Sigma_j^{-1} \mathbb{X})^{-1} \mathbb{X}^t \Sigma_j^{-1} \mathbf{y}_j$$

$$\text{with } \mathbb{X} = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}]$$



Actual fMRI experiments

- “Asynchronous” paradigms (jittering)



over-sampled FIR model

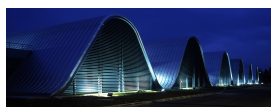
- Design matrix for estimating a single HRF
 - 3 events

$$\begin{bmatrix} y_j(t_1) \\ y_j(t_2) \\ y_j(t_3) \\ \vdots \\ \vdots \\ \vdots \\ y_j(t_{11}) \\ y_j(t_{14}) \\ y_j(t_{15}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} h_j^1(0) \\ h_j^1(\Delta t) \\ h_j^1(4\Delta t) \\ \vdots \\ \vdots \\ h_j^1(7\Delta t) \\ h_j^1(8\Delta t) \\ h_j^1(9\Delta t) \\ m_j \end{bmatrix} + \begin{bmatrix} \varepsilon_j(t_1) \\ \varepsilon_j(t_2) \\ \varepsilon_j(t_3) \\ \vdots \\ \vdots \\ \vdots \\ \varepsilon_j(t_{11}) \\ \varepsilon_j(t_{14}) \\ \varepsilon_j(t_{15}) \end{bmatrix} \quad (\Delta t < TR)$$



$$\mathbf{y}_j = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}] \begin{bmatrix} h_j^1 \\ h_j^2 \\ m_j \end{bmatrix} + \varepsilon_j$$

$$= \sum_{m=1}^M \mathbf{X}^m \mathbf{h}^m + m_j \mathbf{1} + \varepsilon_j$$



over-sampled FIR model

- Design matrix for estimating a single HRF
 - 3 events

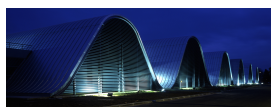
$$\begin{bmatrix} y_j(t_1) \\ y_j(t_2) \\ y_j(t_3) \\ \vdots \\ \vdots \\ \vdots \\ y_j(t_{11}) \\ y_j(t_{14}) \\ y_j(t_{15}) \end{bmatrix} = \begin{bmatrix} \text{[Matrix with 10 columns and 15 rows]} \end{bmatrix} \begin{bmatrix} h_j^1(0) \\ h_j^1(\Delta t) \\ h_j^1(4\Delta t) \\ \vdots \\ \vdots \\ h_j^1(7\Delta t) \\ h_j^1(8\Delta t) \\ h_j^1(9\Delta t) \\ m_j \end{bmatrix} + \begin{bmatrix} \varepsilon_j(t_1) \\ \varepsilon_j(t_2) \\ \varepsilon_j(t_3) \\ \vdots \\ \vdots \\ \vdots \\ \varepsilon_j(t_{11}) \\ \varepsilon_j(t_{14}) \\ \varepsilon_j(t_{15}) \end{bmatrix}$$

($\Delta t < \text{TR}$)

\rightarrow

$$\mathbf{y}_j = [\mathbf{X}^1 \mid \mathbf{X}^2 \mid \mathbf{1}] \begin{bmatrix} h_j^1 \\ h_j^2 \\ m_j \end{bmatrix} + \varepsilon_j$$

$$= \sum_{m=1}^M \mathbf{X}^m \mathbf{h}^m + m_j \mathbf{1} + \varepsilon_j$$



Actual fMRI experiments

- Possible extension to multisession datasets
 - HRF fixed across sessions
 - Session-varying low frequency fluctuations
 - Session-dependent noise statistics

➔ **Fixed effect** model:

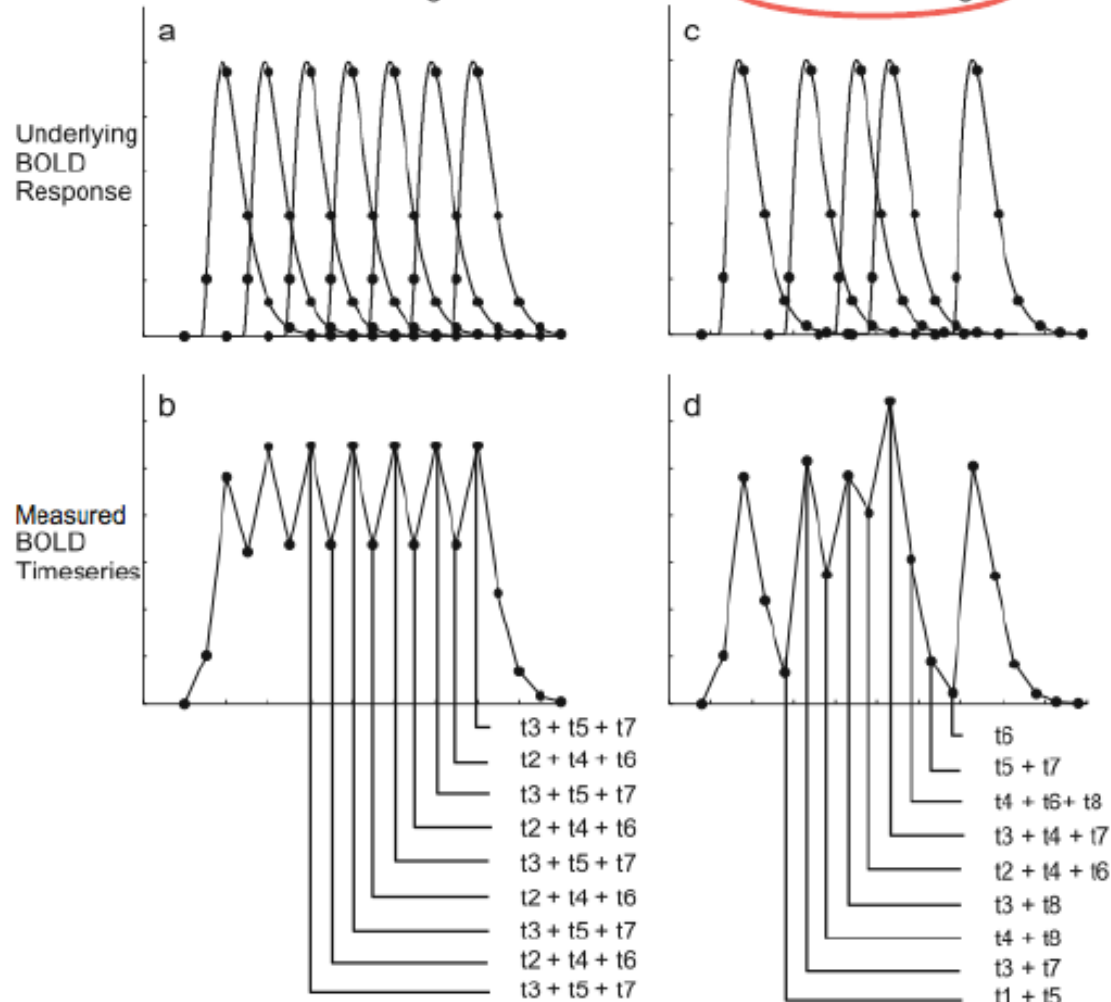
$$\forall s, \quad \mathbf{y}_j^{(s)} = \sum_{m=1}^M \mathbf{X}_s^m \mathbf{h}^m + \mathbf{P}_s \boldsymbol{\ell}_j^{(s)} + \boldsymbol{\varepsilon}_j^{(s)}$$

- Noise assumptions
 - homoscedasticity: $\sigma_{\boldsymbol{\varepsilon}^{(s)}}^2 = \sigma^2, \forall s$
 - heteroscedasticity: $\sigma_{\boldsymbol{\varepsilon}^{(s)}}^2 \neq \sigma_{\boldsymbol{\varepsilon}^{(t)}}^2$ for $s \neq t$
- Alternative: **Random effect** model
 - Session dependent HRF
 - Test HRF mean over sessions

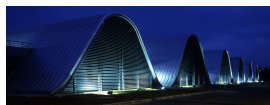
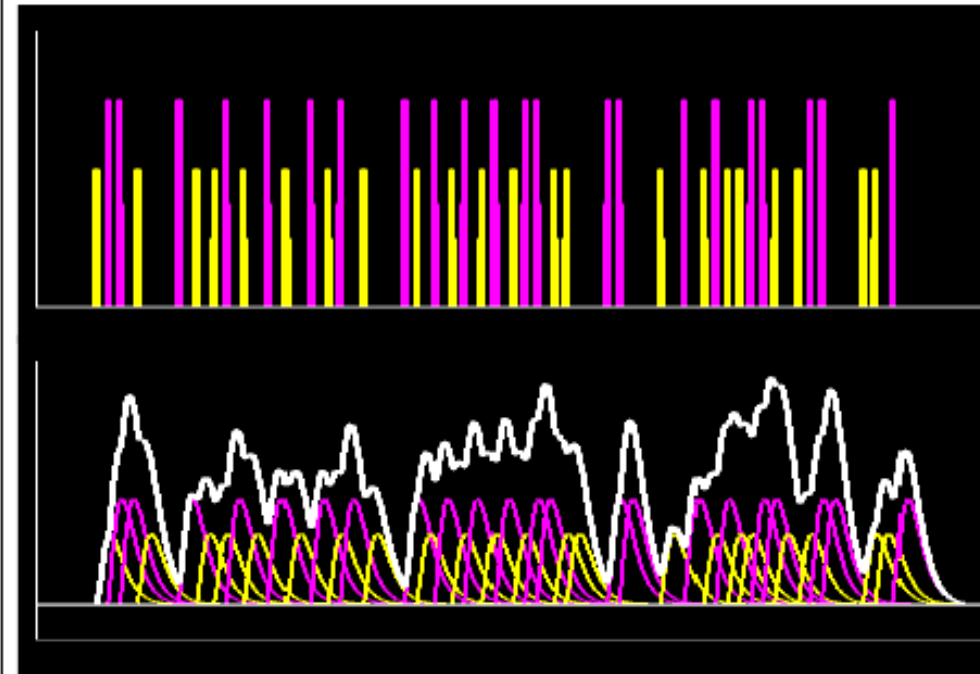


FIR estimation efficiency

Fixed ISI Design



Best estimation efficiency:
jittered ISI and
randomised event order



Bayesian inference

$$\underbrace{p(\text{HRF} \mid \text{data})}_{\text{Posterior distribution}} = \frac{\overbrace{p(\text{data} \mid \text{HRF})}^{\text{likelihood}} \overbrace{p(\text{HRF})}^{\text{Prior distribution}}}{\underbrace{p(\text{data})}_{\text{evidence}}}$$



Bayes' rule

$$p(\text{HRF} \mid \text{data}) = \frac{\overset{\text{likelihood}}{p(\text{data} \mid \text{HRF})} p(\text{HRF})}{p(\text{data})}$$

How the data are generated from the HRF?
Forward modeling

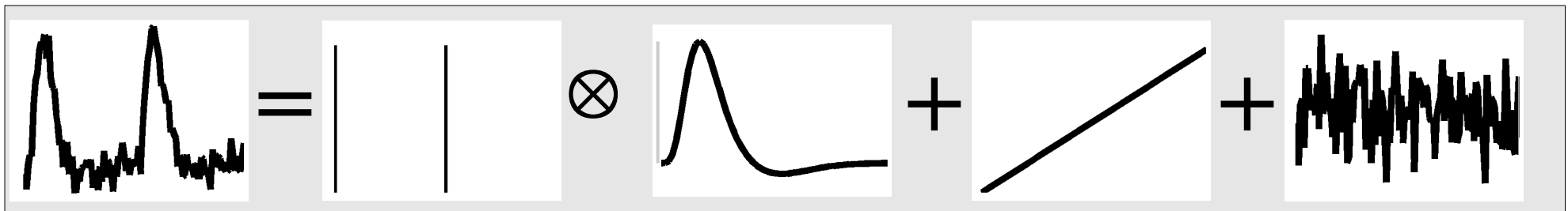


Forward BOLD signal model

Unknown parameters

$$y_j = \sum_{m=1}^M X^m h_j^m + P \ell_j + b_j$$

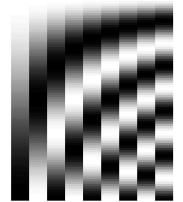
HRF
Drift
Noise statistics voxel V_j



BOLD signal measured in voxel V_j

Arrival time of stimulus m

Orthonormal basis for low frequency drift modelling



Known parameters



Likelihood definition

- **Main hypothesis: noise decorrelated in space**

→ fMRI time series are statistically independent in space:

$$p(\mathbf{y} | \mathbf{h}, \mathbf{l}, \boldsymbol{\theta}_0) = \prod_{j=1}^J p(\mathbf{y}_j | \mathbf{h}_j, \mathbf{l}_j, \theta_{0,j})$$

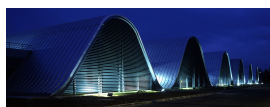
$$\propto \prod_j f_{B_j} \left(\mathbf{y}_j - \sum_{m=1}^M \mathbf{X}^m \mathbf{h}_j^m - \mathbf{P} \mathbf{l}_j \right)$$

Temporal noise model: either **white** or **serially correlated AR(1)**

$$\mathbf{b}_j \sim \mathcal{N}(\mathbf{0}, \epsilon_j^2 \mathbf{I}) \quad \rightarrow \quad \theta_{0,j} = [\epsilon_j^2]$$

$$\mathbf{b}_j \sim \mathcal{N}(\mathbf{0}, \epsilon_j^2 \boldsymbol{\Lambda}_j^{-1}) \quad \rightarrow \quad \theta_{0,j} = [\epsilon_j^2, \rho_j]$$

[Marrelec et al, HBM 2003; Ciuciu et al, IEEE TMI 2003]

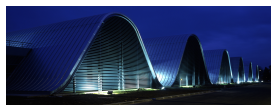


Bayes' rule

$$p(\text{HRF} \mid \text{data}) = \frac{p(\text{data} \mid \text{HRF}) \overset{\text{Prior}}{\boxed{p(\text{HRF})}}}{p(\text{data})}$$

What do we know about the HRF before the data are acquired?

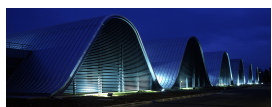
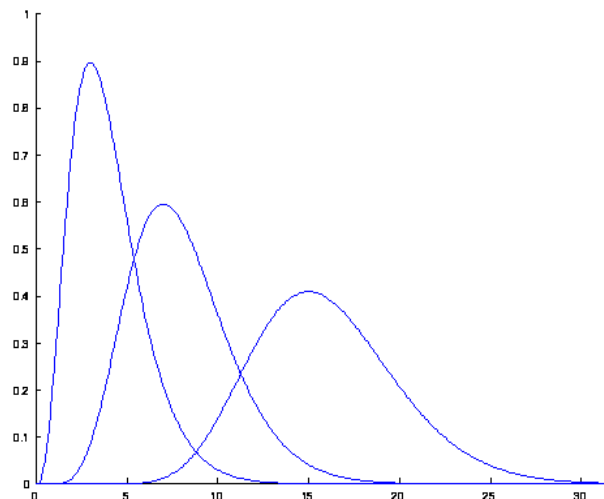
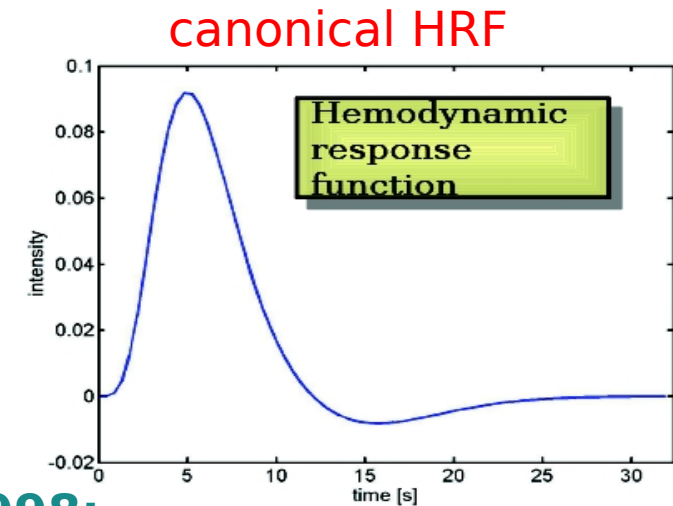
Prior modeling



HRF prior modelling

Parametric approaches

- Canonical HRF: SPM [Friston et al, 1994]
- One function, several parameters
 - Poisson functions: [Friston et al, 1994]
 - Gamma functions: [Boyton et al, 1996]
 - Gaussian functions: [Rajapakse et al, 1998; Kruggel & von Cramon, 1999; Kruggel et al, 2000]

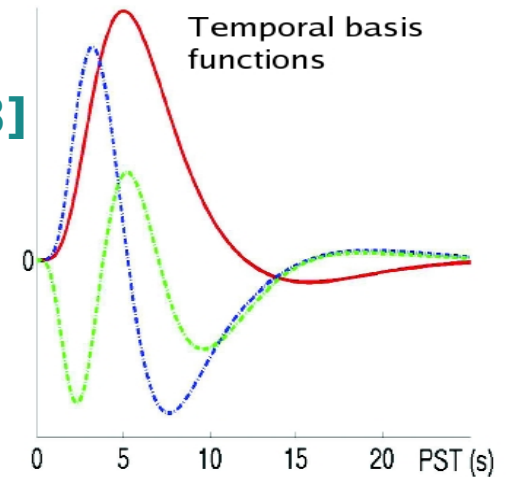


HRF prior modelling

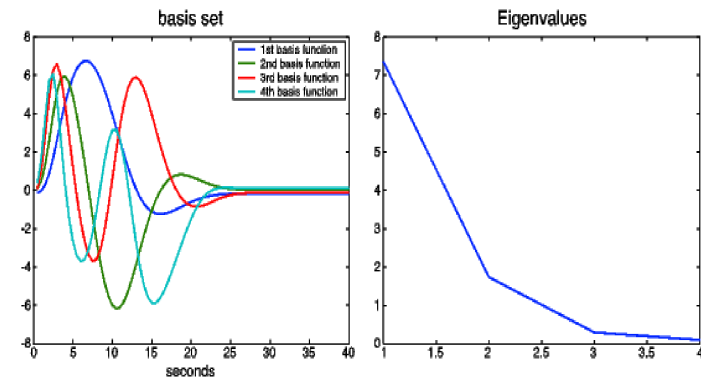
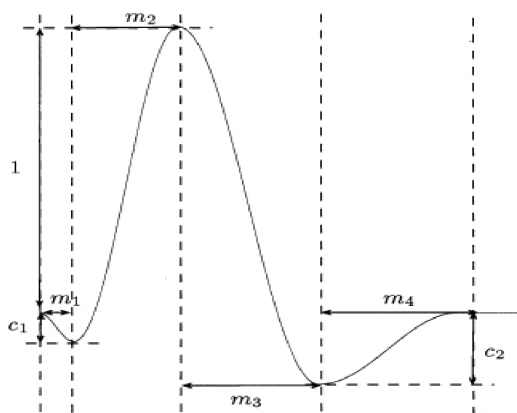
- **Function basis** [Friston et al, NIM 1998]

- Gamma function and its derivative(s)
- polynomial/spline functions:

[Genovese, JASA 2000; Gössl et al, NIM 2001;
Gibbons et al, 2004]



- Half-cosine parameterization:



[Woolrich et al, NeuroImage 2004]



HRF prior modelling

[Marrelec, Ciuciu et al, IPMI'03; Ciuciu et al., 2003]

• Nonparametric approach: smoothing prior

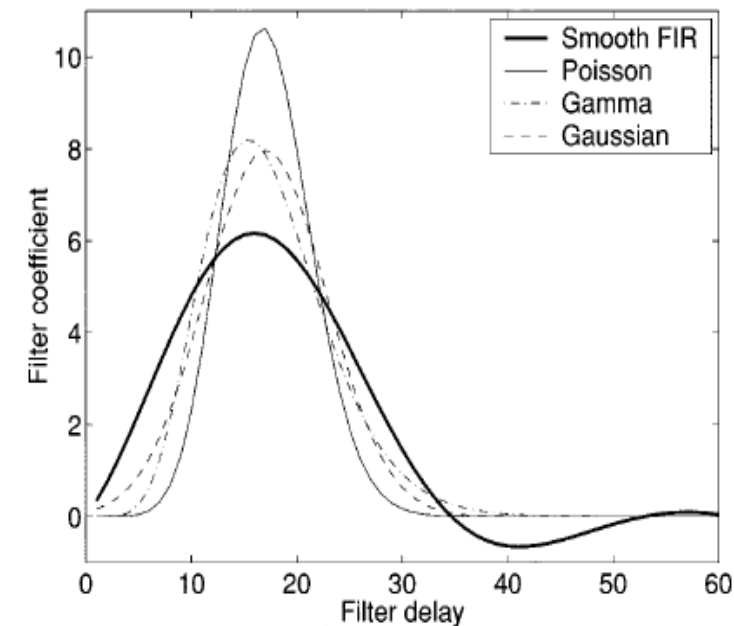
- \mathbf{h}_m is $C^2 \Rightarrow \mathbf{h}_m \sim \mathcal{N}(0, r_m \mathbf{R})$ with $r_m > 0$, and $\mathbf{R}^{-1} = \mathbf{D}_2^t \mathbf{D}_2$
- $\mathbf{h}_m \approx 0$ at the stimulus onset and the baseline return: $h_{0,m} = h_{P\Delta t,m} = 0$
- \mathbf{h}_m and \mathbf{h}_k are independent for $m \neq k$:

$$p(\mathbf{h} ; \mathbf{R}, \theta_{\mathbf{h}}) = \prod_{m=1}^M p(\mathbf{h}_m | \mathbf{R}, r_m)$$

$$\propto \left(\prod_{m=1}^M r_m^{-(P-1)/2} \right) \det(\mathbf{R})^{-M/2} \exp \left(-\frac{\mathbf{h}^t \mathbf{R}_h^{-1} \mathbf{h}}{2} \right)$$

with $\mathbf{R}_h = \text{diag}[\mathbf{r}] \otimes \mathbf{R} = \text{diag}[r_1 \mathbf{R}, r_2 \mathbf{R}, \dots, r_M \mathbf{R}]$, $\theta_{\mathbf{h}} = [r_1, \dots, r_M]$

Smooth FIR vs other standard filters

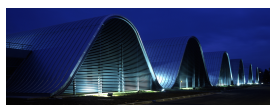


$$\mathbf{D}_2 = \frac{1}{(\Delta t)^2} \begin{pmatrix} -2 & 1 & 0 & & & & \\ \mathbf{1} & -2 & \mathbf{1} & 0 & & & \\ 0 & 1 & -2 & 1 & 0 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 0 & 1 & -2 & 1 & 0 \\ & & & 0 & 1 & -2 & 1 \\ & & & & 0 & 1 & -2 \end{pmatrix}$$

$$(\partial^2 \mathbf{h})_{i,k\Delta t} \approx \frac{h_{i,(k+1)\Delta t} - 2h_{i,k\Delta t} + h_{i,(k-1)\Delta t}}{(\Delta t)^2}$$

$$\|\partial^2 \mathbf{h}_i\|^2 = (\mathbf{D}_{2,i} \mathbf{h}_i)^t (\mathbf{D}_{2,i} \mathbf{h}_i)$$

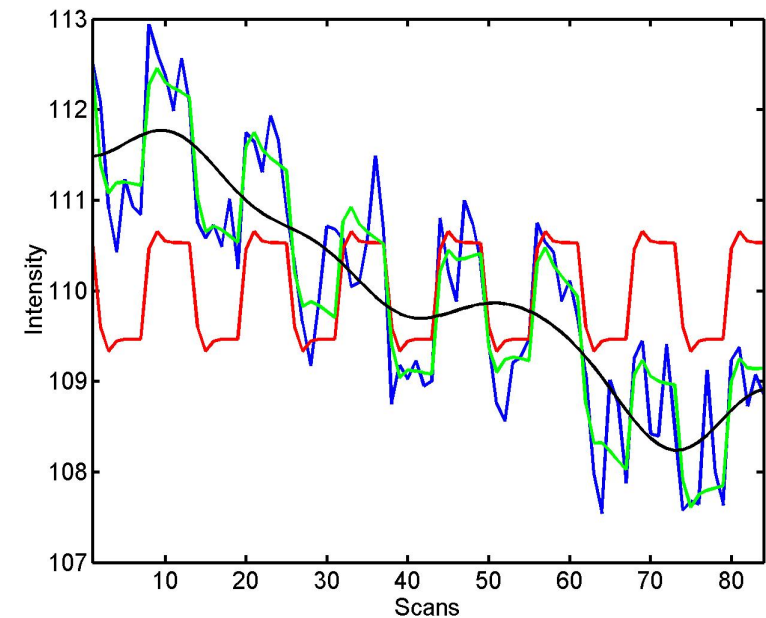
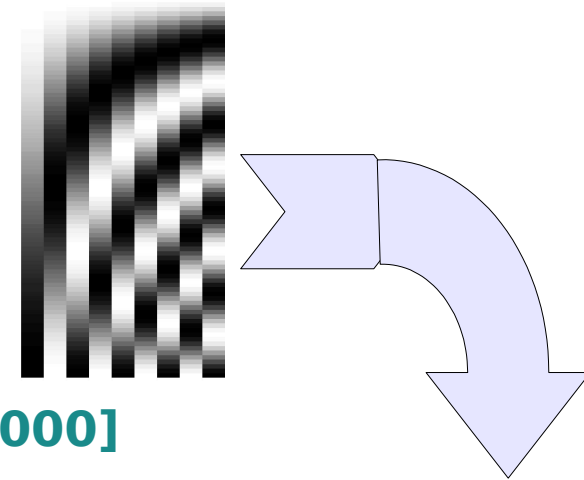
$$= \mathbf{h}_i^t (\mathbf{D}_{2,i}^t \mathbf{D}_{2,i}) \mathbf{h}_i.$$



Drift modelling

Parametric approaches

- Linear subspace spanned by
 - DCT basis function: [\[Friston et al, 2000\]](#)
 - A set of polynomial basis function: [\[Worsley et al, 2000\]](#)
- Wavelet subspace: [\[Meyer, 2003\]](#)



Reduce autocorrelation in the residual noise process

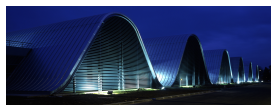


Bayes' rule

$$p(\text{HRF} \mid \text{data}) \propto p(\text{data} \mid \text{HRF}) p(\text{HRF})$$



What do we know about the HRF given the data?
Keystone of learning scheme



Bayesian HRF estimate

- Closed-form MAP estimate

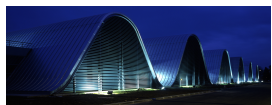
$$p(\mathbf{h}_j | \mathbf{y}_j ; \boldsymbol{\theta}, \ell_j) \sim \mathcal{N}(\hat{\mathbf{h}}_j^{\text{MAP}}, \boldsymbol{\Sigma}_j)$$

$$\boldsymbol{\Sigma}_j^{-1} = \frac{1}{\sigma_{\varepsilon_j}^2} \mathbf{X}^t \mathbf{X} + \mathbf{R}_H^{-1},$$

$$\hat{\mathbf{h}}_j^{\text{MAP}} = \frac{1}{\sigma_{\varepsilon_j}^2} \boldsymbol{\Sigma} \mathbf{X}^t (\mathbf{y}_j - \mathbf{P} \ell_j)$$

- Alternative Marginal MAP estimate:

$$\hat{\mathbf{h}}_j^{\text{MMAP}} = \arg \max_{\mathbf{h}} p(\mathbf{h} | \mathbf{y}_j ; \boldsymbol{\theta}) = \arg \max_{\mathbf{h}} \int p(\mathbf{h}, \ell_j | \mathbf{y}_j ; \boldsymbol{\theta}) d\ell_j.$$



Drift & hyper-parameters

- Nuisance variables and hyper-parameters
 - **Deterministic** parameters: Maximum likelihood estimation

$$(\hat{\theta}^{\text{ML}}, \hat{\ell}_j^{\text{ML}}) = \arg \max_{\theta, \ell_j} [\log p(\mathbf{y}_j; \theta, \ell_j) = \log \int p(\mathbf{y}_j, \mathbf{h}_j; \theta, \ell_j) d\mathbf{h}_j]$$

- EM or ECM algorithm **[Ciuciu et al, IEEE TMI 2003]**

- Drift parameters as **random variables**: marginalization

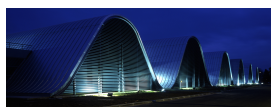
$$\hat{\theta}^{\text{ML}} = \arg \max_{\theta} [\log p(\mathbf{y}_j; \theta) = \log \int p(\mathbf{y}_j, \mathbf{h}_j, \ell_j; \theta) d\mathbf{h}_j d\ell_j]$$

[Marrelec, Ciuciu, IEEE TMI 2004]

- Hyper-parameters as **random variables**: combine marginalization & posterior inference using sampling

$$p(\theta_0, \mathbf{h}_j | \mathbf{y}_j) = \int p(\theta_0, \theta_1, \mathbf{h}_j, \ell_j | \mathbf{y}_j) d\theta_1 d\ell_j$$

$$\begin{cases} \theta_0^{(k)} & \sim p(\theta_0 | \mathbf{h}_j^{(k-1)}, \mathbf{y}_j) \\ \mathbf{h}_j^{(k)} & \sim p(\mathbf{h}_j | \theta_0^{(k)}, \mathbf{y}_j) \end{cases} \quad \text{[Marrelec et al, HBM 2003]}$$



FIR vs. regularized FIR

[Casanova et al, NeuroImage, 2008]

Increased temporal resolution with regularization

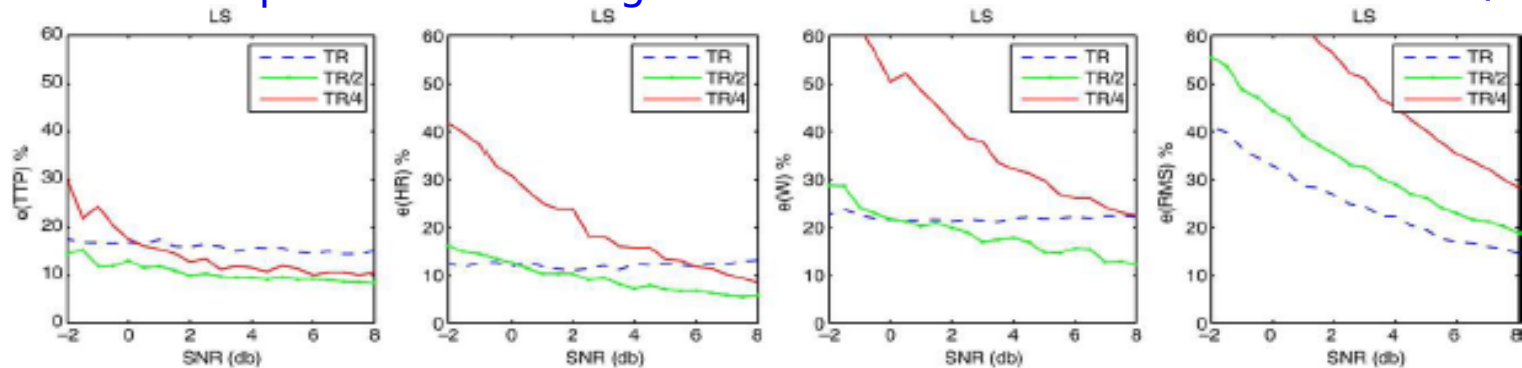
Time-to-peak

Height

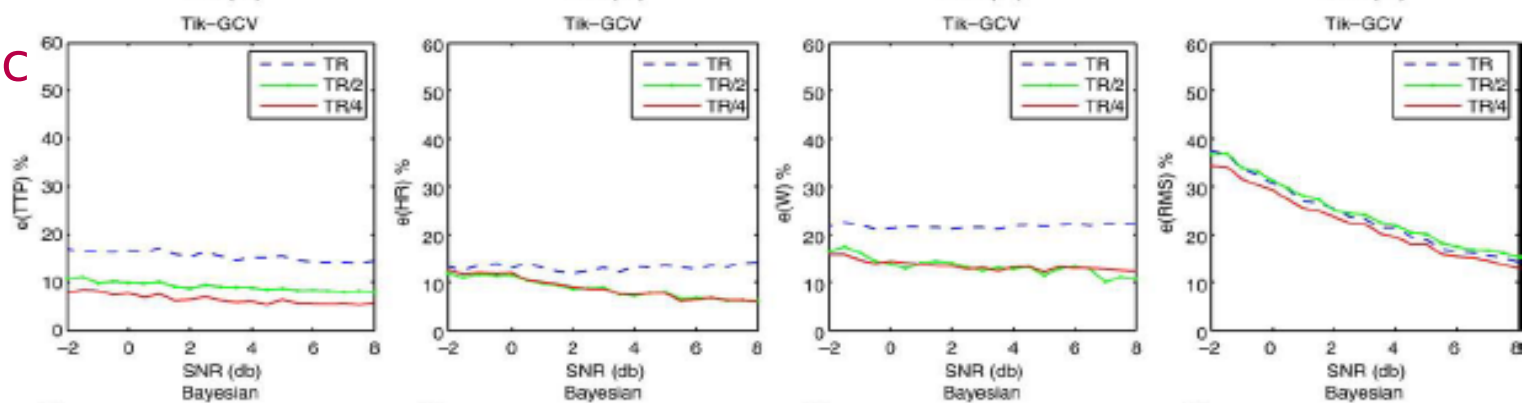
Width

Root Mean Square Error

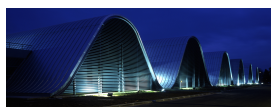
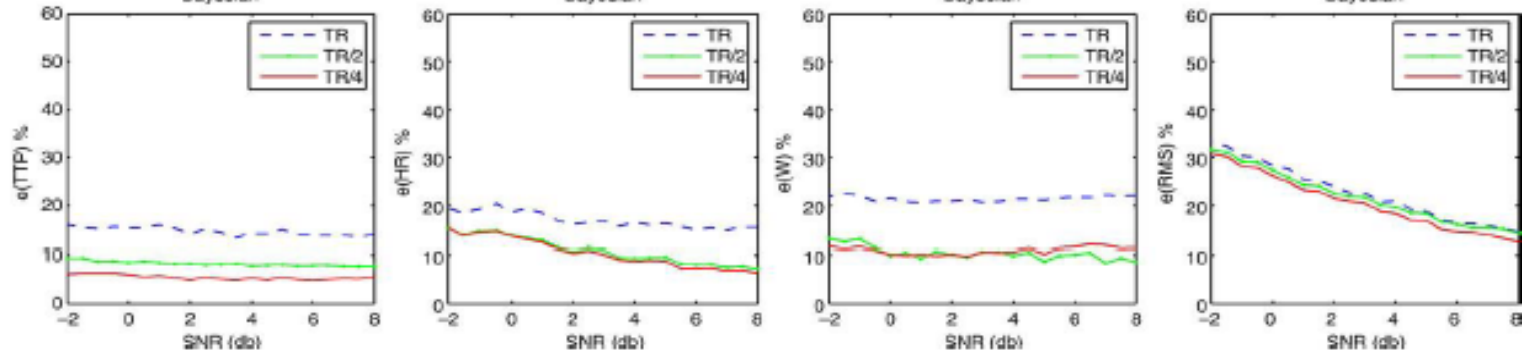
FIR



Deterministic
regularized
FIR



“Bayesian
regularized
FIR”



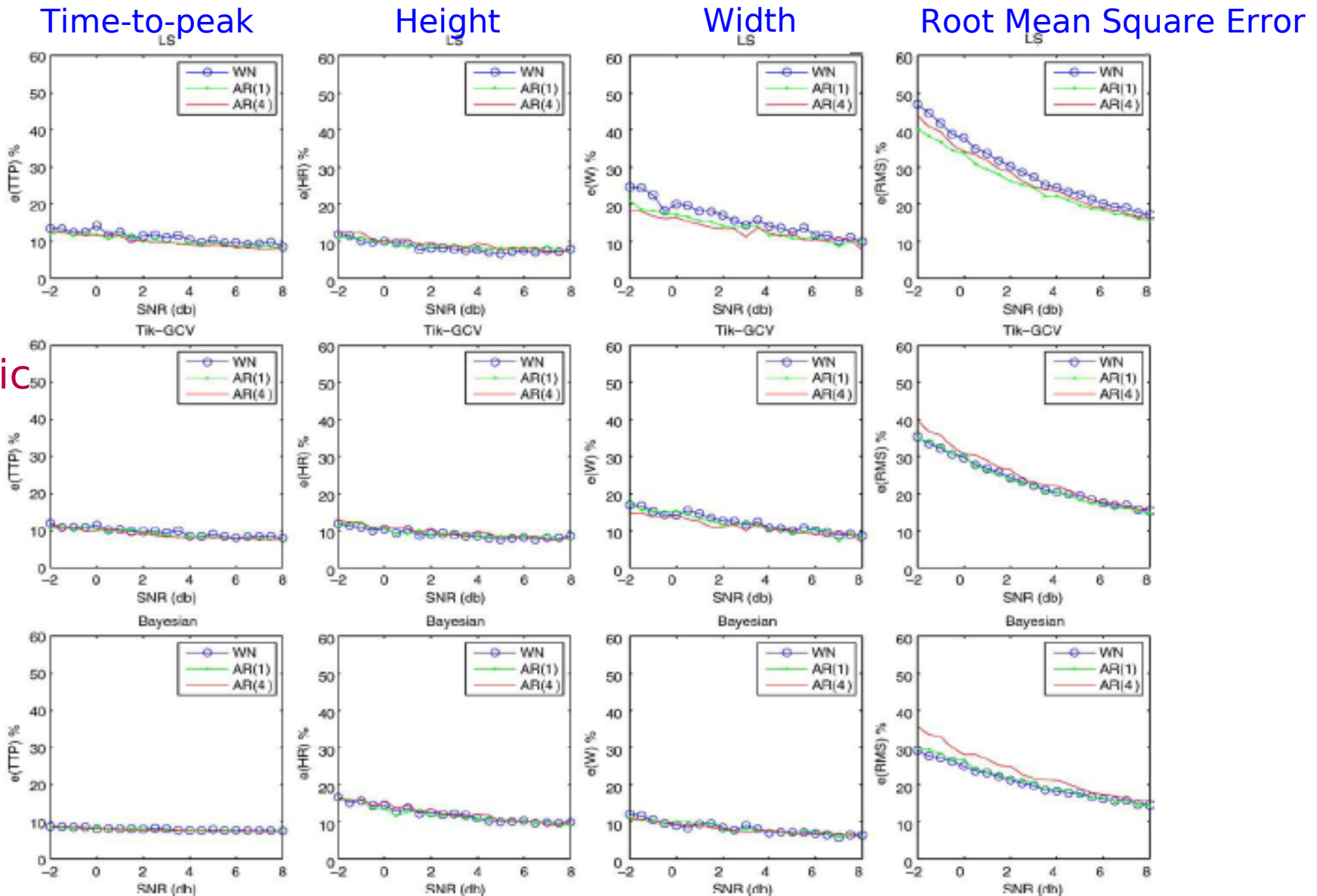
FIR vs. regularized FIR



Exp. ITI pdf
 $\text{ITI}_{\text{mean}} = 3\text{s}$
 FIR

Deterministic
 regularized
 FIR

“Bayesian
 regularized
 FIR”



Little impact of noise autocorrelation for short ISIs



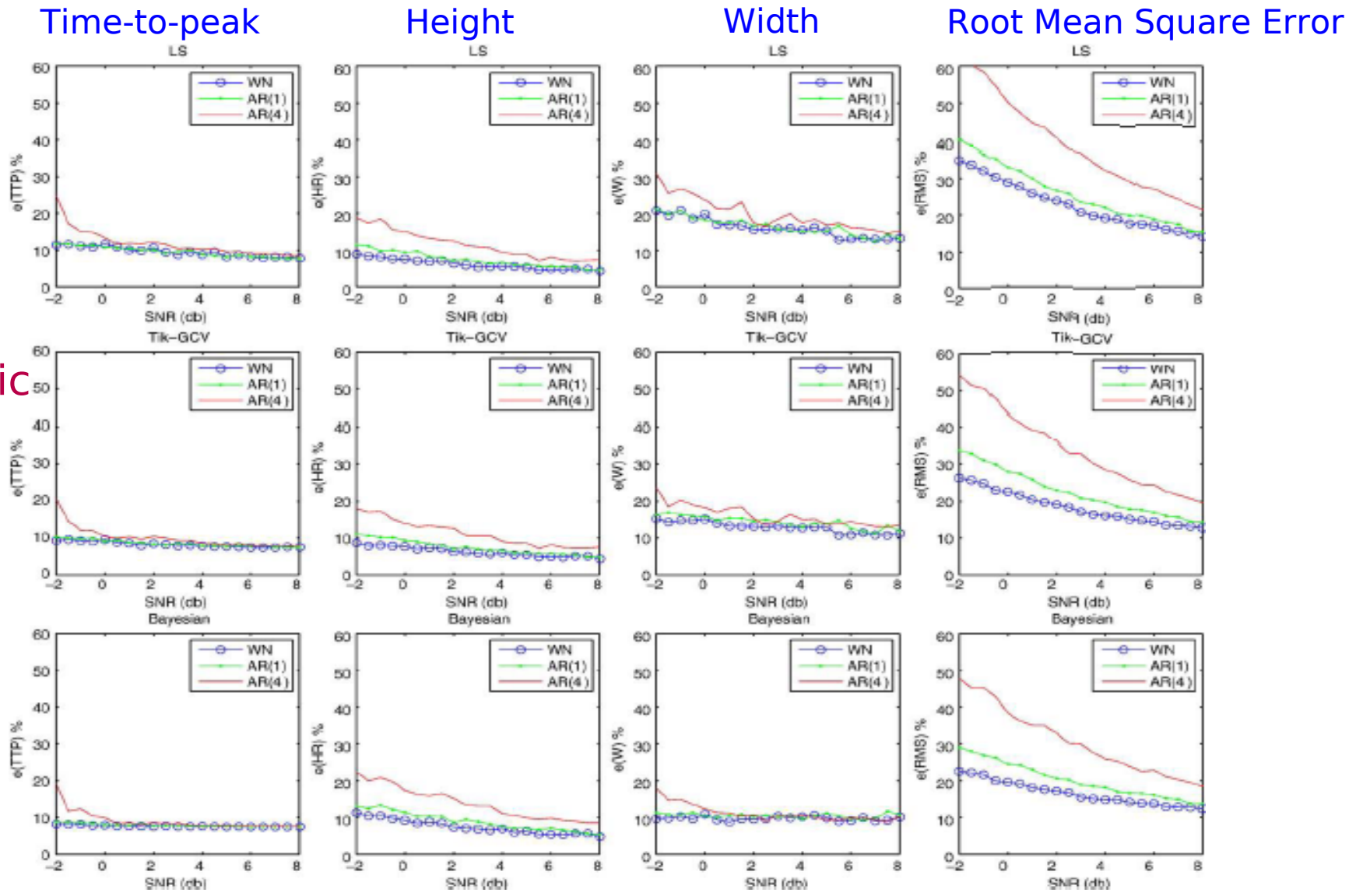
FIR vs. regularized FIR



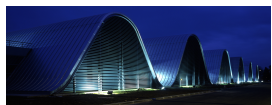
Exp. ISI pdf
 $ITI_{\text{mean}} = 10\text{s}$
 FIR

Deterministic
 regularized
 FIR

“Bayesian
 regularized
 FIR”



Stronger impact of noise autocorrelation for long ITIs
 irrespective of the method



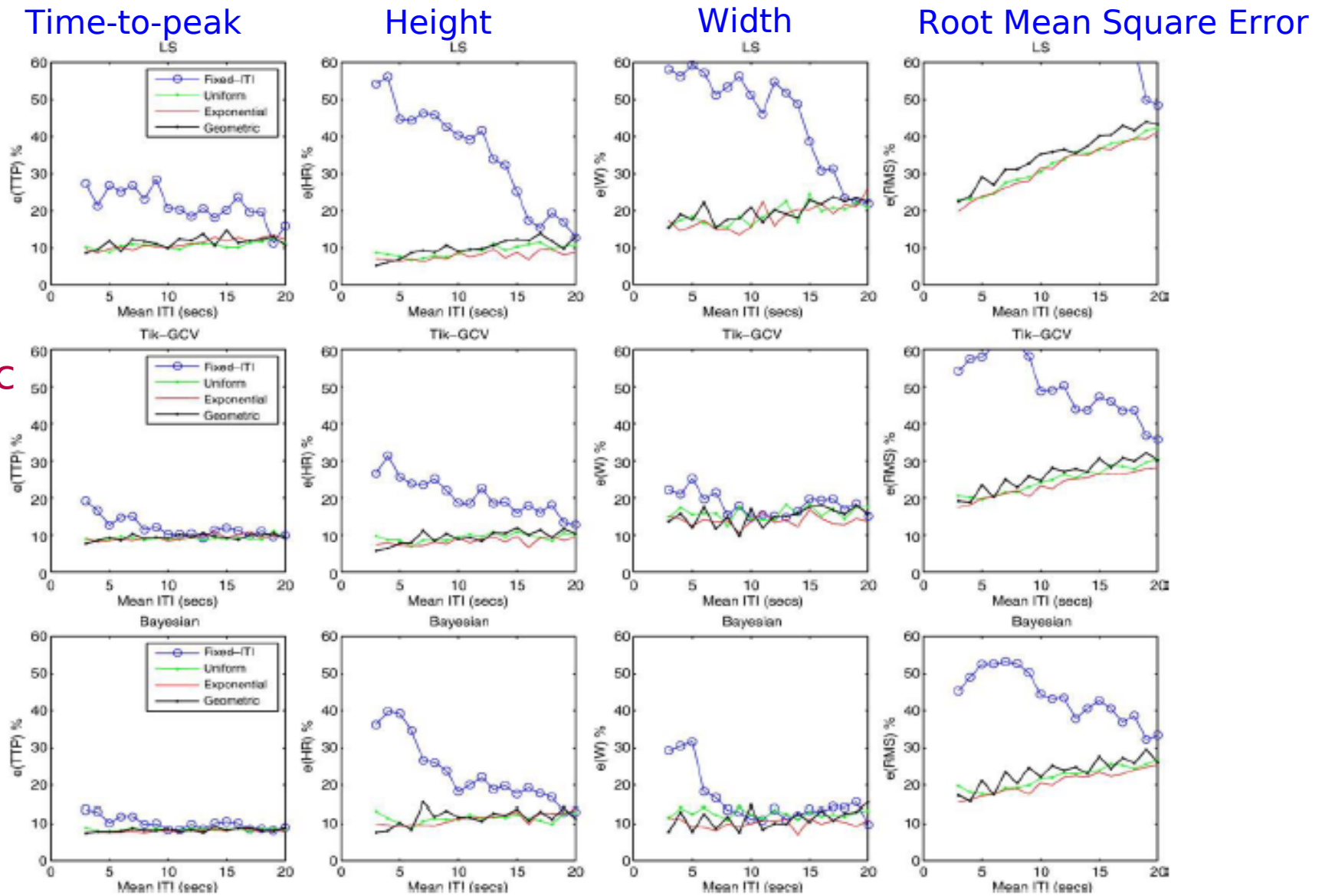
FIR vs. regularized FIR



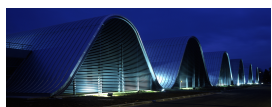
Various ITI densities
FIR

Deterministic regularized FIR

“Bayesian regularized FIR”

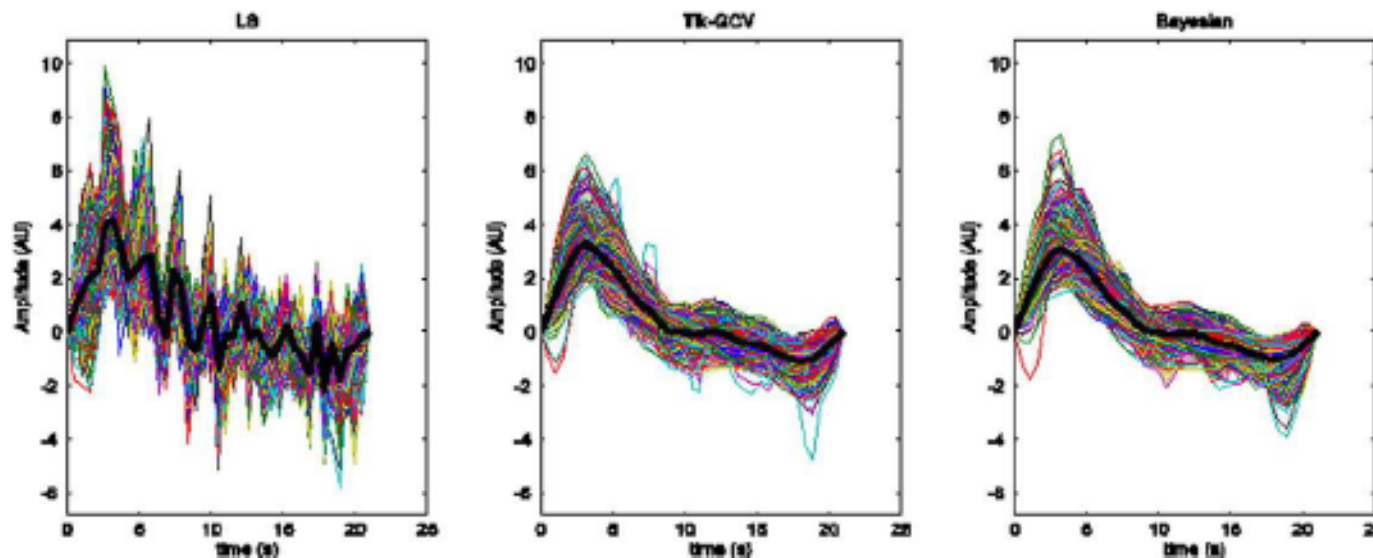
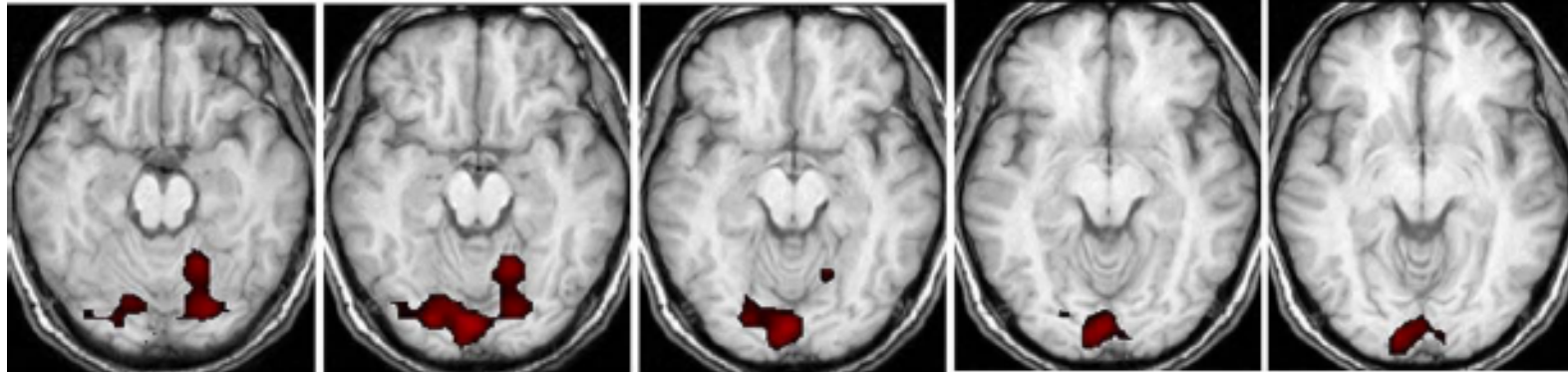


Regularization increases estimation efficiency at fixed ITI and performs even better using random designs



FIR/regularized FIR: real data

[Casanova et al, NeuroImage, 2008]



Average HRF estimate

Standard FIR estimates: unstable for TR/4 temporal grid
 Regularized FIR models: similar & meaningful results



Results on real fMRI datasets

Speech perception experiment: (silence, phonological, acoustic, control)

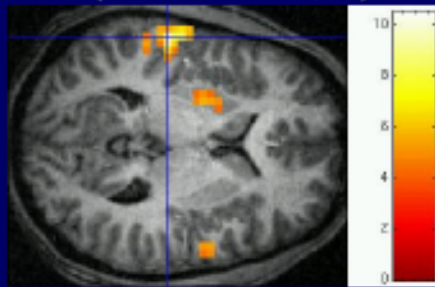
sound - silence

(phonological - control) & phonological - control
(acoustic - control)

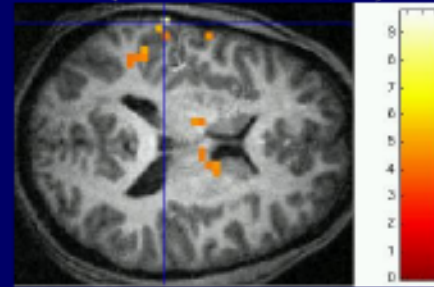
$V_1 = (-60, -24, 4)$ mm

$V_2 = (-68, -28, 8)$ mm

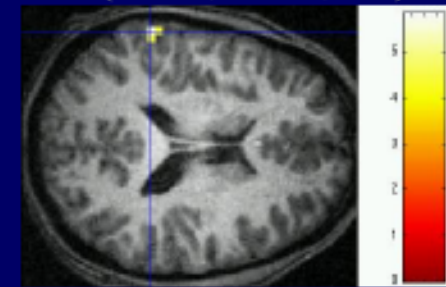
$V_3 = (-64, -40, 16)$ mm



(a) Heschl gyrus

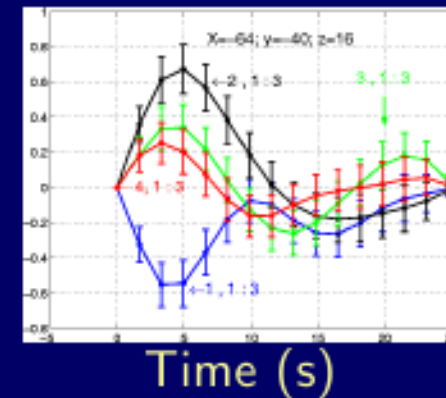
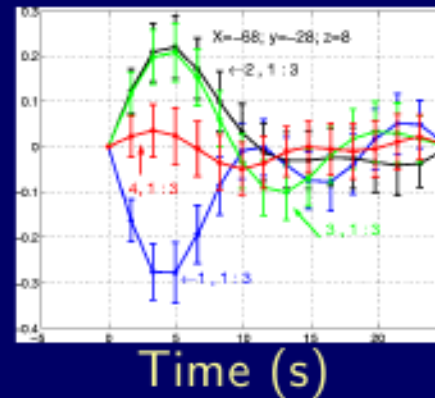
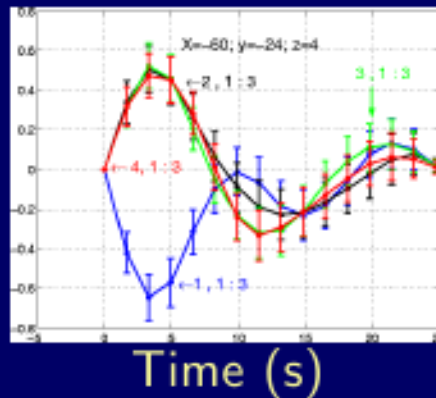


(b) Planum temporale

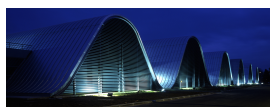


(c) Planum temporale

HRF amplitude



HRF estimation performed on Sessions 1-3



Improved detection

Better sensitivity

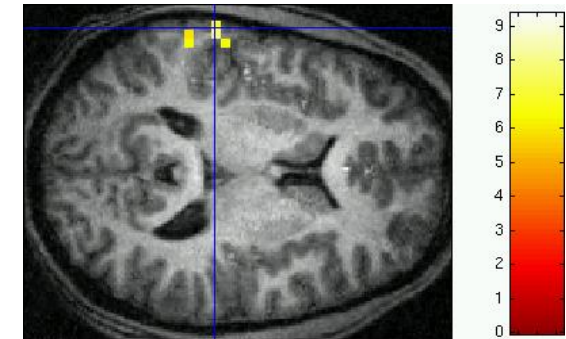
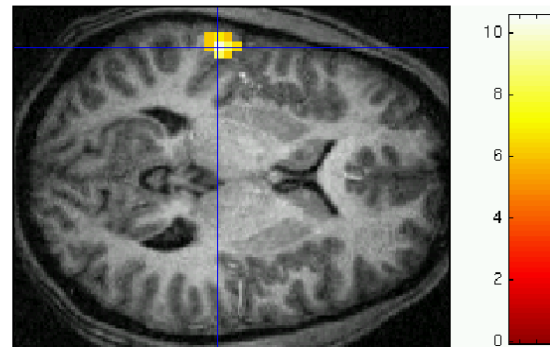
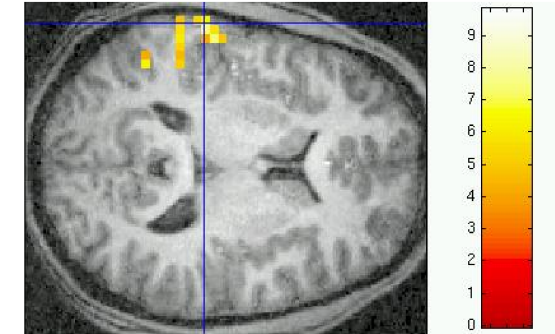
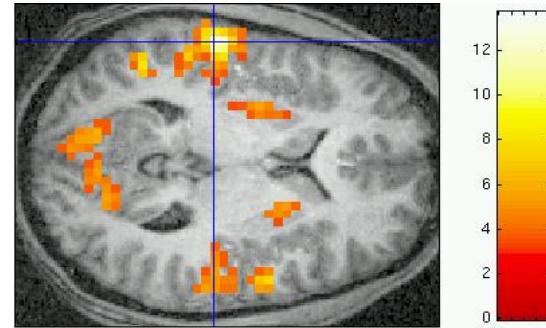
GLM built using
HRF estimate

$$f_m(t_n) \triangleq (x^{(m)} \star \hat{h}_m)(t_n)$$

GLM built upon
canonical HRF

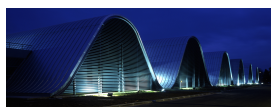
$$f_m(t_n) \triangleq (x^{(m)} \star h_c)(t_n)$$

[Ciuciu et al, ISBI, 2002]



$$V_1 = (-60, -24, 4) \text{ mm}$$

$$V_2 = (-68, -28, 8) \text{ mm}$$

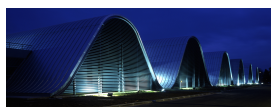


Between-trial variability

- Modelling the trial by trial variability

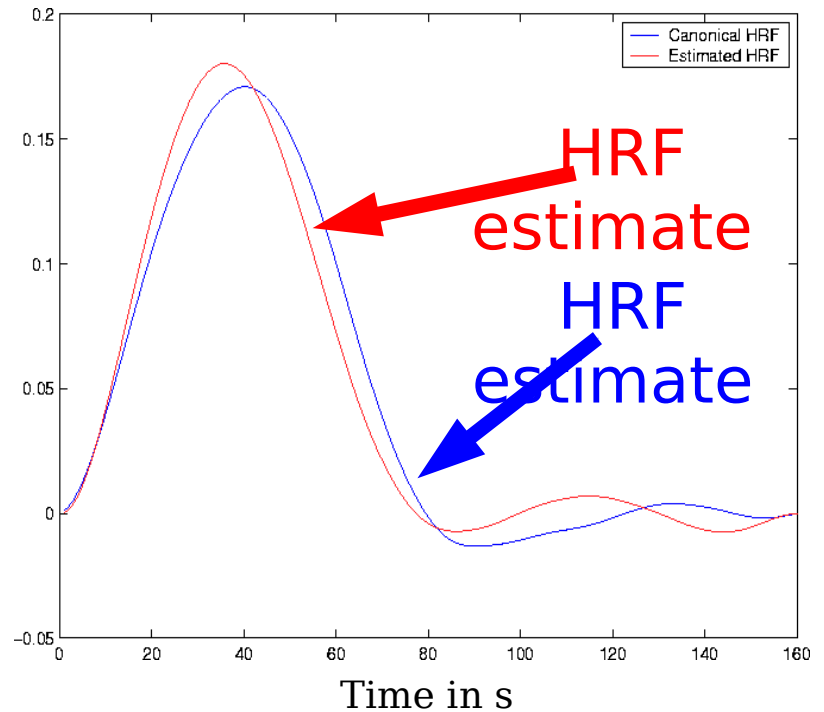
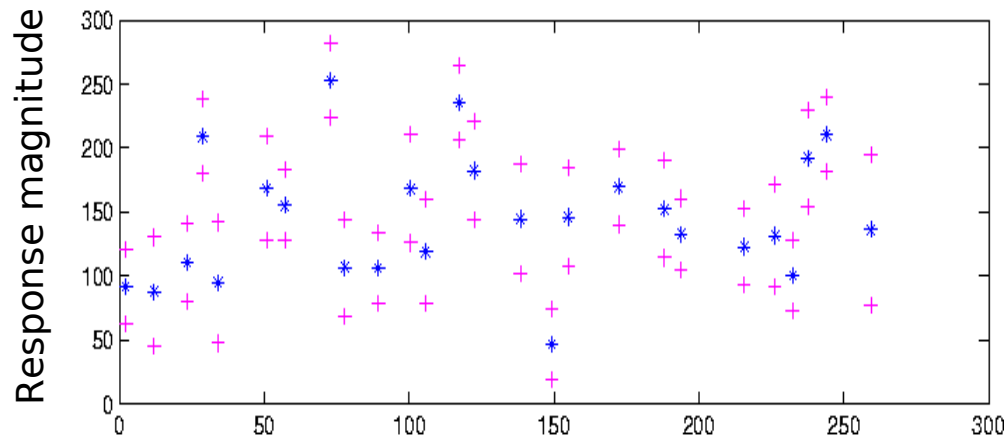
$$\mathbf{y}_j = \sum_{m=1}^M \sum_{k=1}^{K_m} \alpha_k^m \mathbf{X}_k^m \mathbf{h}_j + \mathbf{P} \ell_j + \mathbf{b}_j$$

- Independence between trial magnitudes $\alpha = (\alpha_k^m)$
[Donnet, Ciuciu et al, ISBI 2004]
- Dependence on the past:
 - Habituation modelling: repetition-suppression phenomenon
[Ciuciu et al, ICASSP 2009]

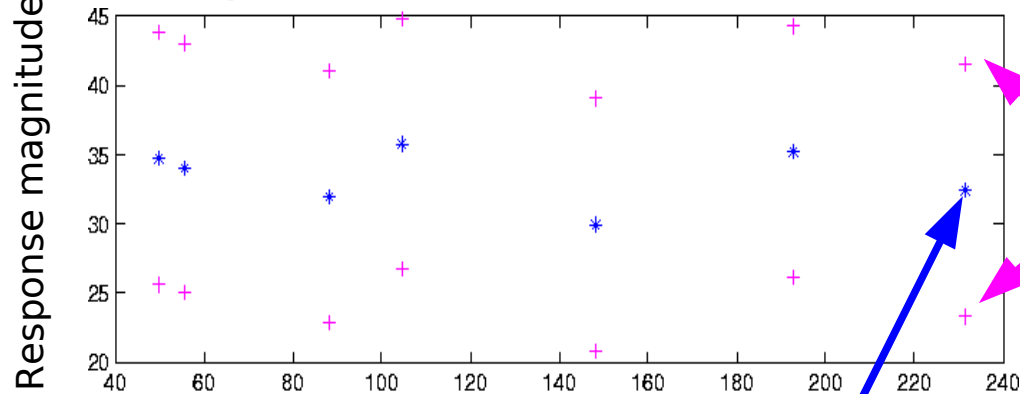


Extract temporal information

Responses to right button click



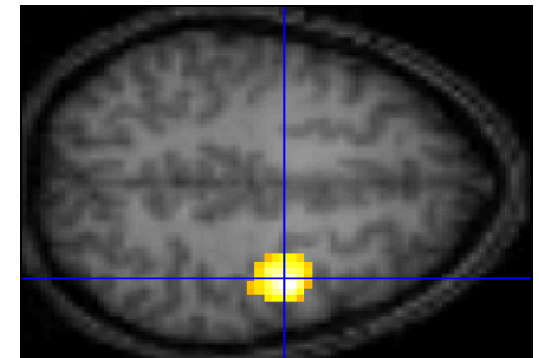
Responses to visual stimulus



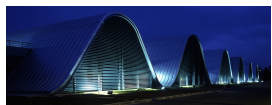
Standard deviation

Trial magnitude

Data from left motor cortex



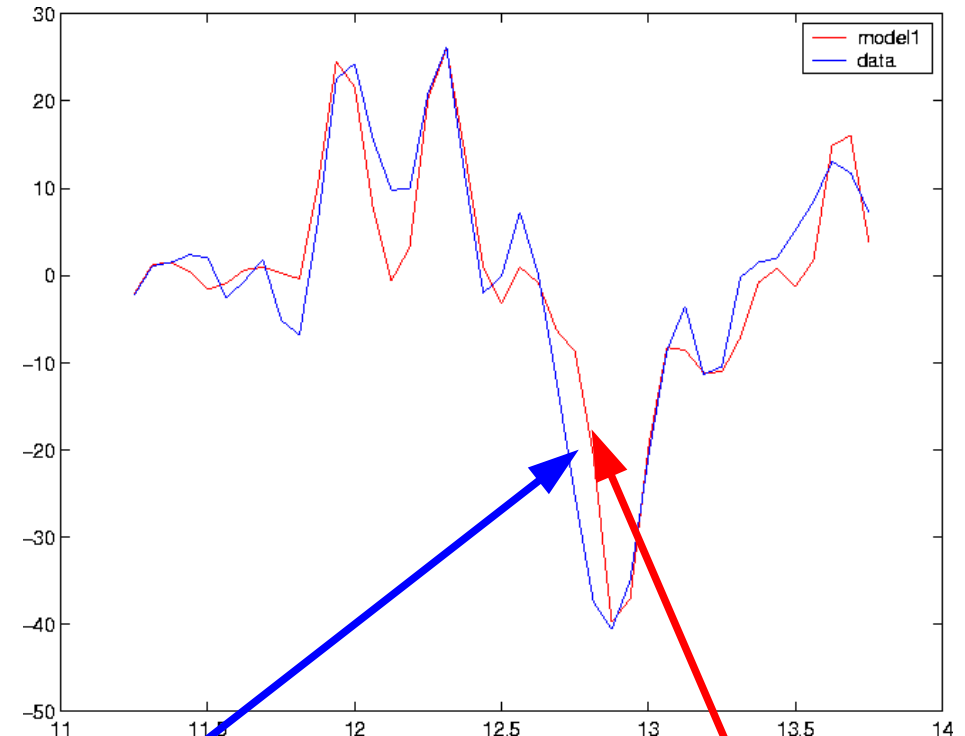
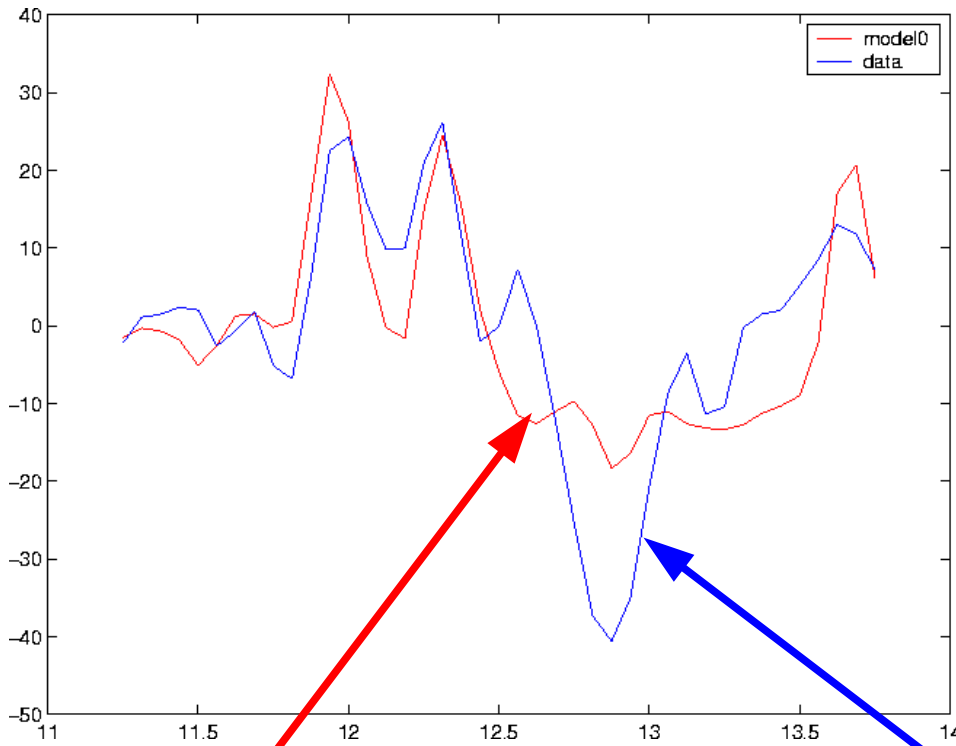
[Donnet et al., NeuroImage 2006]



Extract temporal information

$\mathcal{P} = \{\text{visual, Right click}\}$

[Donnet et al., ISBI 2004]



Predicted time series with a convolution model

Predicted time series by a non-stationary model

same fMRI time course



Conclusions

- Precise estimation of the evoked BOLD response
 - Efficient & random design
 - Reasonable Signal-to-Noise ratio
 - Regularization necessary
- FIR modelling
 - Sufficient for $ITI > 2s$.
 - Otherwise: inadequate to capturing non-linear effects
 - Able to account for trial-by-trial variability
- Voxelwise HRF estimation approaches
 - Computationally costly
 - Only a scanner induced spatial resolution
 - ... Less robust than regionwise counterparts

