Achievable rate regions for cooperative cognitive radio networks with complex channels and circular normal additive noises

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Résumé – Dans cet article, nous dérivons les régions de débit atteignables pour un système de radio cognitive coopératif composé d'une liaison licenciée et d'une liaison opportuniste, qui est aidée par un relais full-duplex utilisant CF ou DF, lorsque tous les canaux sont complexes et les bruits sont supposés suivre une distribution normale circulaire.

Abstract – In this paper, we derive the achievable rate regions for a cooperative cognitive radio network composed of a licensed link and an opportunistic link, which is helped by a full-duplex relay node performing either CF or DF, when all channels are modeled as complex and the noises are assumed to follow a circular normal distribution.

Next generations of wireless network are facing major challenges, such as increasing the network capacity and throughput, improving the energy-efficiency, satisfying ultra-low latency, while serving an unprecedented large number of users with limited radio resources. Various techniques, ranging from cognitive radio, cooperative communications, full-duplexing to millimeter waves have hence been proposed to address these goals [1].

Both full-duplexing technology and opportunistic spectrum access increase the spectral efficiency by either multiplexing both the transmission and the reception on the same frequency band [2] or by allowing an opportunistic use of under-utilized licensed bands under Quality of Service (QoS) constrains protecting the licensed communication [2, 3].

Cooperative communications aim at increasing the network capacity and throughput by exploiting both the increasing number of connected devices (potential relay nodes) and the nature of the wireless medium, which makes all communicated messages available at any node within range. Various relaying schemes have been proposed in information theory for the relay channel, such as Decode-and-Forward (DF), where the relay decodes the sent message before re-transmission; and Compress-and-Forward (CF), where the relay only quantizes its received signal [4]. None of the above relaying schemes is optimal in all cases with respect to the system parameters; however, they perform well over various extensions of the relay channel [5], such as the two-way relay channel [6], the multiway relay channel [7, 8], the diamond relay channel [9], and the interference relay channel [10, 11].

In this paper, we consider a cooperative cognitive radio network, where the opportunistic transmission is assisted by a fullduplex operating relay and assume, as in our previous study [12], that the opportunistic direct link is present and that no interfering link, neither from the licensed network to the secondary one nor from the secondary network to the licensed one, can be neglected. Contrary to our previous study, we assume here that all channels are complex and that all additive white Gaussian noises follow a circular complex normal distribution, which changes the achievable rate regions.

Our main contribution is precisely the derivation of the new achievable rate regions under DF and CF relaying, in the complex channels' case and when the noises follow a circular complex normal distribution. Hence, our obtained results extend our previous study [12] as well as [13] (regarding the achievable rate under CF with correlated noises), in which the channels were assumed real and the noises normal ones.

As opposed to the interference relay channels in [10, 11], in our problem, the relay only helps the secondary transmission and considers as additive noise the primary signal. For DF, the major difference is that the primary message is not decoded at the relay and only the secondary message is re-encoded in our setting compared to [10]. For CF, the compression is adapted solely to the secondary receiver as opposed to two different receivers [11] (via single or bi-level compression). Hence, our achievable rates in the Gaussian case cannot be derived directly from the expressions provided in [10, 11]. In spite of

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the similarity of our model with the interference relay channel, the resulting resource optimization problems for the secondary network are very different because of the additional cognitive radio constraint protecting the primary link.

1 System model





The cooperative cognitive radio network under study, composed of a primary transmission between U_P and D_P ; and a secondary transmission between U_S and D_S helped by a fullduplex operating relay R is depicted in Figure 1. The relay, which either performs CF or DF, is assumed to be able to cancel out any self-interference.

The relay and the two destinations receive the signals

$$Y_R = h_{SR}X_S + h_{PR}X_P + Z_R$$

$$Y_i = h_{Ri}X_R + h_{ii}X_i + h_{ji}X_j + Z_i,$$

where $i \in \{S, P\}, j = \{S, P\} \setminus i$. X_R, X_S and X_P , of average power P_R , P_S and P_P , denote respectively the message transmitted by the relay, the opportunistic user and the primary user. Z_R and Z_i represent the additive noises at the relay and at the destination D_i and are such that $Z_R \sim C\mathcal{N}(0, N_R)$, $Z_i \sim C\mathcal{N}(0, N_i)$. We also assume that all channel gains are complex, static during the transmission, and known at the transmitter and receiver ends, as per usual in information theoretic works.

Throughout the paper, we assume that the primary message is not decoded at the relay and secondary destination and can hence be treated as additional noise. We thus consider the following equivalent noises $\tilde{Z}_R = Z_R + h_{PR}X_P \sim C\mathcal{N}(0,\tilde{N}_R)$ with $\tilde{N}_R = N_R + |h_{PR}|^2 P_P$ and $\tilde{Z}_S = Z_S + h_{PS}X_P \sim C\mathcal{N}(0,\tilde{N}_S)$ with $\tilde{N}_S = N_S + |h_{PS}|^2 P_P$ at the relay and secondary destination respectively. Further, note that the two later noises are correlated such that $\mathbb{E}[\tilde{Z}_R\tilde{Z}_S^*] = h_{PR}h_{PS}^*P_P$, where $\mathbb{E}[.]$ denotes the mathematical expectation. Likewise, the message from the secondary user and the relay are considered additional noise at the primary destination.

Notations. We denote by $C(x) = \log_2(1 + x)$ the capacity of the point-to-point channel. Let $\mathcal{R}\{\cdot\}$, $(\cdot)^*$, and $|\cdot|$ denote the real part, the complex conjugate, and the absolute value of a complex number, respectively. At last, $\mathbb{H}(\cdot)$ represents the differential entropy, and $\mathbb{I}(\cdot; \cdot)$ is the mutual information.

2 Preliminaries

We start by presenting two theoretical and preliminary results relating the differential conditional entropies to the estimator minimizing the mean squared error, which is proved to be linear when the considered random variables are Gaussian ones. These results can be found in [14, 15], but are reported below for the sake of completeness.

Lemma 1. Let $\hat{X}(Y)$ be the estimator of a random variable (r.v.) X when we have side information Y. The estimator minimizing the estimation error is given by $\hat{X}(Y) = \mathbb{E}[X|Y]$ and the mean squared error of the estimation is such that

$$\mathbb{E}[|X - \mathbb{E}[X|Y]|^2] \ge \frac{1}{\pi e} 2^{\mathbb{H}(X|Y)},\tag{1}$$

where the equality is met if X and Y are Gaussian. Moreover, if X and Y are jointly Gaussian with zero mean, then the optimal estimator is linear and is given by

$$\mathbb{E}[X|Y] = \frac{\mathbb{E}[XY^*]}{\mathbb{E}[|Y|^2]}Y$$

and the mean squared estimation error is given by

$$\mathbb{E}[|X - \mathbb{E}[X|Y]|^2] = \mathbb{E}[|X|^2] \left(1 - \frac{|\mathbb{E}[XY^*]|^2}{\mathbb{E}[|X|^2]\mathbb{E}[|Y|^2]}\right).$$

Proof. The lower bound in (1) follows from the Corollary of [14, Theorem 8.6.6]. The optimal estimator is obtained by minimizing the mean squared error given as $\mathbb{E}[|X - \hat{X}(Y)|^2|Y]$ and is given as $\hat{X}(Y) = \mathbb{E}[X|Y]$.

Now, assume that X and Y are such that $X \sim C\mathcal{N}(0, \sigma_X^2)$, $Y \sim C\mathcal{N}(0, \sigma_Y^2)$ with the correlation coefficient defined as $\rho = \frac{|\mathbb{E}[XY^*]|}{\sigma_X \sigma_Y}$. It can be shown that there exist two independent random variables $Z_1, Z_2 \sim C\mathcal{N}(0, 1)$, such that

$$X = \sigma_X \sqrt{1 - \rho^2} Z_1 + \frac{\mathbb{E}[XY^*]}{\sigma_Y} Z_2, \quad Y = \sigma_Y Z_2.$$

Hence, the optimal estimator is linear such that $\mathbb{E}[X|Y] = \frac{\mathbb{E}[XY^*]}{\mathbb{E}[|Y|^2]}Y$. Then, the mean squared estimation error expression follows easily.

Below, we extend Lemma 1 in the case where we have side information composed of two r.v. $Y = (Y_1, Y_2)$.

Corollary 1. Let $\hat{X}(Y_1, Y_2)$ be the estimator of X when we have two r.v. Y_1 and Y_2 as side information. The estimator minimizing the estimation error is given by $\hat{X}(Y_1, Y_2) = \mathbb{E}[X|Y_1, Y_2]$ and the mean squared estimation error is such that

$$\mathbb{E}[|X - \mathbb{E}[X|Y_1, Y_2]|^2] \ge \frac{1}{\pi e} 2^{\mathbb{H}(X|Y_1, Y_2)}.$$

Moreover, if X, Y_1, Y_2 are jointly Gaussian with zero mean, then $\mathbb{E}[X|Y_1, Y_2] = a_{opt}Y_1 + b_{opt}Y_2$ with a_{opt} and b_{opt} given below.

i) If Y_1 and Y_2 are independent r.v., then

$$a_{\text{opt}} = \frac{\mathbb{E}[XY_1^*]}{\mathbb{E}[|Y_1|^2]} \quad b_{\text{opt}} = \frac{\mathbb{E}[XY_2^*]}{\mathbb{E}[|Y_2|^2]}$$

ii) Otherwise, we have

$$\begin{split} a_{\rm opt} &= \frac{\mathbb{E}[XY_2^*]}{\mathbb{E}[Y_1Y_2^*]} - b_{\rm opt} \frac{\mathbb{E}[|Y_2|^2]}{\mathbb{E}[Y_1Y_2^*]} \\ b_{\rm opt} &= \frac{-\mathbb{E}[XY_1^*] \mathbb{E}[Y_1Y_2^*] + \mathbb{E}[XY_2^*] \mathbb{E}[|Y_1|^2]}{\mathbb{E}[|Y_1|^2] \mathbb{E}[|Y_2|^2] - |\mathbb{E}[Y_1Y_2^*]|^2} \end{split}$$

In both cases, the mean squared estimation error equals

$$\mathbb{E}[|X - \mathbb{E}[X|Y_1, Y_2]|^2] = \mathbb{E}[|X|^2] - a_{\text{opt}}\mathbb{E}[Y_1X^*] - b_{\text{opt}}\mathbb{E}[Y_2X^*].$$

3 Decode-and-Forward

In this section, we provide an achievable rate region obtained when the relay performs DF. For completeness, we start by the discrete channels' case, followed by the complex additive white Gaussian case.

Proposition 1. In the discrete memoryless channels' case, the achievable rate region over our cooperative cognitive radio network for Decode-and-Forward relaying is given by

$$R_P = \max_{p(x_P)} \mathbb{I}(X_P; Y_P)$$
$$R_S = \max_{p(x_S, x_R)} \min\{\mathbb{I}(X_S; Y_R | X_R), \mathbb{I}(X_S, X_R; Y_S)\}.$$

Proof. The primary rate corresponds to the capacity of the point-to-point channel between the primary user U_P and its destination D_P . The secondary rate is obtained as for DF relaying over the relay channel and is given in [15, Theorem 16.2]. \Box

Theorem 1. The following rate region is achievable over our cognitive radio network with additive Gaussian noises $Z_i \sim C\mathcal{N}(0, N_i), i \in \{S, P, R\}$ when the relay performs Decodeand-Forward:

$$\begin{aligned} R_P &\leq C \left(\frac{|h_{PP}|^2 P_P}{|h_{RP}|^2 P_R + |h_{SP}|^2 P_S + 2\mathcal{R} \{h_{RP}^* h_{SP} \gamma\} \sqrt{P_S P_R} + N_P} \right) \\ R_S &\leq C \left(\min \left\{ f_R(\gamma), f_S(\gamma) \right\} \right), \text{ with } |\gamma| \in [0, 1] \text{ and} \end{aligned}$$

$$f_R(\gamma) = \frac{|h_{SR}|^2 P_S(1 - |\gamma|^2)}{N_R + |h_{PR}|^2 P_P},$$

$$f_S(\gamma) = \frac{|h_{SS}|^2 P_S + |h_{RS}|^2 P_R + 2\mathcal{R}\{h_{SS}h_{RS}^*\gamma\}\sqrt{P_S P_R}}{N_S + |h_{PS}|^2 P_P}$$

Proof. The optimal distributions of X_R and X_S , analogously to the real case, are $X_R \sim \mathcal{CN}(0, P_R)$, $X_S = X'_S + \gamma \sqrt{\frac{P_S}{P_R}} X_R$ with $|\gamma| \in [0, 1]$ and $X'_S \sim \mathcal{CN}(0, (1 - |\gamma|^2)P_S)$ orthogonal to X_R . Thus, $\mathbb{E}[X_S X_R^*] = \gamma \sqrt{P_S P_R}$ and with $|\gamma| \leq 1$ the Cauchy-Schwarz inequality $|\mathbb{E}[X_S X_R^*]|^2 \leq P_S P_R$ is satisfied. Furthermore, it can be noted that this choice of coding is equivalent to the use of coding by superposition. Similarly, the optimal choice for the primary distribution is $X_P \sim \mathcal{CN}(0, P_P)$.

To complete the proof, we need to derive the expression of the three mutual information terms in Proposition 1. Computation of $\mathbb{I}(X_S; Y_R | X_R)$. $\mathbb{I}(X_S; Y_R | X_R) = \mathbb{H}(Y_R | X_R) - \mathbb{H}(Z_R + h_{PR} X_P)$ $\stackrel{(a)}{=} C\left(\frac{|h_{SR}|^2 (1 - |\gamma|^2) P_S}{N_R + |h_{PR}|^2 P_P}\right)$

where (a) follows from Lemma 1.

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$$\begin{split} & \textbf{Computation of } \mathbb{I}(X_S, X_R; Y_S).\\ & \mathbb{I}(X_S, X_R; Y_S) = \mathbb{H}(Y_S) - \mathbb{H}(\widetilde{Z}_S) \\ & = C \left(\frac{|h_{SS}|^2 P_S + |h_{RS}|^2 P_R + 2\mathcal{R} \{h_{SS} h_{RS}^* \gamma\} \sqrt{P_S P_R}}{N_S + |h_{PS}|^2 P_P} \right) \end{split}$$

Computation of
$$\mathbb{I}(X_P; Y_P)$$
.

$$\mathbb{I}(X_P; Y_P) = \mathbb{H}(Y_P) - \mathbb{H}(Z_P + h_{SP}X_S + h_{RP}X_R)$$

$$= C\left(\frac{|h_{PP}|^2 P_P}{|h_{RP}|^2 P_R + |h_{SP}|^2 P_S + 2\mathcal{R}\{h_{RP}^* h_{SP}\gamma\}\sqrt{P_S P_R} + N_P}\right)$$

4 Compress-and-Forward

In this section, we provide an achievable rate region obtained when the relay performs CF. Similarly to the previous section, we start by the discrete channels' case, followed by the complex additive white Gaussian case.

Proposition 2. In the discrete memoryless channels' case, the achievable rate region over our cognitive radio network for Compress-and-Forward relaying is given by

$$R_P = \max_{p(x_P)} \mathbb{I}(X_P; Y_P)$$
$$R_S = \max \mathbb{I}(X_S; \hat{Y}_R, Y_S | X_R)$$

where the maximum for R_S is taken over all distributions such that $\mathbb{I}(X_R; Y_S) \geq \mathbb{I}(Y_R; \hat{Y}_R | X_R, Y_S)$.

Proof. The primary achievable rate is the same as in Proposition 1. The secondary rate is obtained as for CF relaying over the relay channel and is given in [15, Theorem 16.4, Remark 16.3].

Theorem 2. The following rate region is achievable over our cognitive radio network with additive Gaussian noises $Z_i \sim C\mathcal{N}(0, N_i)$, $i \in \{S, P, R\}$ when the relay performs Compress-and-Forward:

$$R_P \leq C \left(\frac{|h_{PP}|^2 P_P}{|h_{RP}|^2 P_R + |h_{SP}|^2 P_S + N_P} \right)$$

$$R_S = C \left(\frac{|h_{SS}|^2 P_S D + P_S \mathbb{E}[|h_{SS} \widetilde{Z}_R - h_{SR} \widetilde{Z}_S|^2]}{\widetilde{N}_S (\widetilde{N}_R + D) - |\mathbb{E}[\widetilde{Z}_R \widetilde{Z}_S^*]|^2} \right)$$
in $D = \frac{P_S \mathbb{E}[|h_{SS} \widetilde{Z}_R - h_{SR} \widetilde{Z}_S|^2] + \widetilde{N}_R \widetilde{N}_S - |\mathbb{E}[\widetilde{Z}_R \widetilde{Z}_S^*]|^2}{|V|^{1/2}}.$ Further

with $D = \frac{\Gamma_{SE}[|h_{SS}Z_R - h_{SR}Z_S|] + N_R N_S - |E[Z_R Z_S]|]}{|h_{RS}|^2 P_R}$. Furthermore, we have

$$\mathbb{E}[|h_{SS}Z_R - h_{SR}Z_S|^2] \\= |h_{SS}|^2 \widetilde{N}_R + |h_{SR}|^2 \widetilde{N}_S - 2P_P \mathcal{R}\{h_{SS}h_{SR}^*h_{PR}h_{PS}^*\} \\|\mathbb{E}[\widetilde{Z}_R \widetilde{Z}_S^*]|^2 = |h_{PR}|^2 |h_{PS}|^2 P_P^2.$$

Proof. Choose $X_S \sim C\mathcal{N}(0, P_S)$, $X_R \sim C\mathcal{N}(0, P_R)$, $\hat{Y}_R = Y_R + Z, Z \sim C\mathcal{N}(0, D)$, where the distortion Z is assumed to be independent of all other messages. In our case, the additive noises at the secondary relay and destination are correlated due to the presence of wireless links between the primary user and the secondary network, and, hence

$$\mathbb{E}[\tilde{Z}_R \tilde{Z}_S^*] = \mathbb{E}[(Z_R + h_{PR} X_P)(Z_S + h_{PS} X_P)^*]$$
$$= h_{PR} h_{PS}^* P_P \neq 0.$$

Computation of the secondary rate $\mathbb{I}(X_S; \widehat{Y}_R, Y_S | X_R)$. First, by applying the chain rule, we obtain:

$$\mathbb{I}(X_S; \widehat{Y}_R, Y_S | X_R) = \mathbb{I}(X_S; \widehat{Y}_S | X_R) + \mathbb{I}(X_S; Y_S | X_R, \widehat{Y}_R)$$
$$= C\left(\frac{|h_{SR}|^2 P_S}{\widetilde{N}_R + D}\right) + \mathbb{H}(Y_S | X_R, \widehat{Y}_R) - \mathbb{H}(Y_S | X_S, X_R, \widehat{Y}_R)$$

Second, we compute $\mathbb{H}(Y_S|X_R, \widehat{Y}_R)$ as follows.

$$\begin{split} \mathbb{H}(Y_S|X_R, Y_R) &= \log_2(\pi e \mathbb{E}[|Y_S - \mathbb{E}[Y_S|X_R, Y_R]|^2]) \\ \stackrel{(b)}{=} \log_2\left(\pi e \left(|h_{SS}|^2 P_S + \widetilde{N}_S - \frac{|h_{SS}h_{SR}^* P_S + \mathbb{E}[\widetilde{Z}_R^* \widetilde{Z}_S]|^2}{|h_{SR}|^2 P_S + \widetilde{N}_R + D} \right) \right) \end{aligned}$$

where (b) follows from Corollary 1 with X_R and \hat{Y}_R independent.

Third, we need to derive $\mathbb{H}(Y_S|X_S, X_R, \widehat{Y}_R)$.

$$\mathbb{H}(Y_S|X_S, X_R, \widehat{Y}_R) = \mathbb{H}(\widetilde{Z}_S|\widetilde{Z}_R + Z)$$

$$\stackrel{(c)}{=} \log_2\left(\pi e\left(\widetilde{N}_S - \frac{|\mathbb{E}[\widetilde{Z}_S \widetilde{Z}_R^*]|^2}{\widetilde{N}_R + D}\right)\right)$$
here (a) for the source 1.

where (c) follows from Lemma 1.

Combining all the above, we obtain the secondary rate:

$$\mathbb{I}(X_S; Y_R, Y_S | X_R) = C \left(\frac{|h_{SS}|^2 P_S D + P_S \mathbb{E}[|h_{SS} \widetilde{Z}_R - h_{SR} \widetilde{Z}_S|^2]}{\widetilde{N}_S(\widetilde{N}_R + D) - |\mathbb{E}[\widetilde{Z}_R \widetilde{Z}_S^*]|^2} \right)$$

Computation of the distortion *D*.

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First, we derive the expression of $\mathbb{I}(Y_R; \widehat{Y}_R | X_R, Y_S)$.

$$\begin{split} \mathbb{I}(Y_R; \widehat{Y}_R | X_R, Y_S) &= \mathbb{H}(\widehat{Y}_R | X_R, Y_S) - \mathbb{H}(\widehat{Y}_R | X_R, Y_S, Y_R) \\ \stackrel{(d)}{=} C \left(\frac{|h_{SR}|^2 P_S + \widetilde{N}_R - \frac{|h_{SS}^* h_{SR} P_S + \mathbb{E}[\widetilde{Z}_R \widetilde{Z}_S^*]|^2}{|h_{SS}|^2 P_S + \widetilde{N}_S}}{D} \right) \end{split}$$

where (d) follows from Corollary 1 with X_R and Y_S not independent. Second, we compute $\mathbb{I}(X_R; Y_S)$ below.

$$(X_R; Y_S) = \mathbb{H}(Y_S) - \mathbb{H}(Y_S | X_R)$$
$$= C\left(\frac{|h_{RS}|^2 P_R}{|h_{SS}|^2 P_S + \tilde{N}_S}\right)$$

Finally, $\mathbb{I}(X_R; Y_S) \ge \mathbb{I}(Y_R; \hat{Y}_R | X_R, Y_S)$ reduces to

$$D \ge \frac{P_S \mathbb{E}[|h_{SS}Z_R - h_{SR}Z_S|^2] + N_R N_S - |\mathbb{E}[Z_R Z_S^*]|^2}{|h_{RS}|^2 P_R}.$$

5 Conclusion

In this paper, we derive achievable rate regions for a cooperative cognitive radio network, where the relay performs CF or DF in a full-duplex manner. Due to the presence of the primary transmission, the equivalent additive noises of the secondary network are correlated. We further consider that all channels are complex and that the additive noises follow complex circular normal distributions, hence, our obtained rate regions extend the ones in our previous study [12], as well as the achievable rate of CF under correlated noises of [13].

These new achievable rate regions will be exploited in our future work to derive the optimal power allocation of the secondary network, i.e. both relay and opportunistic user, that maximizes the secondary rate while satisfying some primary QoS constraints.

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