SIMO communication with impulsive and dependent interference - the Copula receiver.

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Résumé – Dans ce papier, nous proposons une méthode pour modéliser la dépendance entre des bruits impulsifs. Nous utilisons la notion de copule ce qui nous permet de représenter les dépendances d'*upper* et de *lower tail*, ce qui n'est pas le cas des coefficients de corrélation classique (qui de plus, ne sont pas adaptés aux lois α-stables, souvent utilisées pour modéliser des bruits impulsifs). Afin d' illustrer l'approche par les copules, nous considérons une configuration simple avec une seule antenne de transmission et deux antennes de réception. Nous pouvons alors construire un récepteur adapté. Nous déterminons anlaytiquement le rapport de vraisemblance qui se décompose en deux parties : une dépendant uniquement des marginales et une dépendant de la copule. Nous pouvons ensuite illustrer l'impact de la structure de dépendance sur les régions de décision.

Abstract – In this paper, we propose solutions for modelling dependence in impulsive noises. We use the copula framework that allows to represent the upper and lower tail dependencies that can not be captured by classical correlation (which, besides, is not adapted to α-stable distributions often considered in modelling impulsive noise). To illustrate the copula approach we consider a simple configuration with a single transmit antenna and two receive antennas and an adapted receiver architecture. We can derive the likelihood ratio that exhibits two components: one from the marginals and one from the copulas. We can then illustrate the impact of the dependence structure on the decision regions.

1 Introduction

Impulsive interference is encountered in many situations, e.g. in power line communications, with ultra-wide band technology, or in dense networks.

In this paper, we consider a simple detection problem in a block fading scenario. Each data symbol is transmitted over wireless channels and $K = 2$ versions of each symbol are received. We only consider the case $K = 2$ for clear analytical expressions and simple illustrations. Extension to higher dimensions however still raises computation and model selection questions. This transmission structure can be motivated by many different practical wireless communication systems, like a rake receiver [1], a single-input-multiple-output system, a cooperative communication system involving multiple relays or in impulse radio Ultra Wide Band systems where repetitions of the transmitted symbol occur [2].

For a single transmitted symbol, the received signal $\mathbf{Y} \in \mathbb{R}^K$ is : $\mathbf{Y} = s_n\mathbf{h}_n + \mathbf{I}_k + \mathbb{N}_n$, where s_n is the unknown transmitted symbol at time n, $\mathbf{h}_n \in \mathbb{R}^K$ is the block fading channel coefficients, $\mathbf{I}_k \in \mathbb{R}^K$ is the impulsive interference and $N_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$ is the thermal noise.

In this paper we make the assumption that the channel state is perfectly known and that interference is dominating. Besides we assume independence between different time instant n so that we will drop this index for simplicity of writing. The studied case can then be summarized by

$$\mathbf{Y} = \mathbf{S} + \mathbf{I} \tag{1}$$

where $\mathbf{S}$ is a vector containing the repeated sample $\mathbf{s}$ and $\mathbf{I}$ the interference vector.

Many papers have considered the case where $\mathbf{I}$ is composed of independent and identically distributed samples. Depending on the impulsive interference distribution assumption, it is more or less complicated to derive the optimal receiver and sometimes suboptimal approaches are considered [3], especially when $\mathbf{I}$ is considered as an α-stable random vector.

In this paper we consider an α-stable interference distribution but we do not consider any longer that the components of $\mathbf{I}$ are independent. We take the example of a SIMO situation : if a strong interference is received on one antenna, the probability of receiving a strong interference sample on another antenna is not negligible. This upper tail dependence can not be captured by traditional correlation function that, anyway, can not be used for α-stable random vectors. We propose to use the copula framework to model the dependence structure. It allows to separately model the marginal distributions and the dependence structure. To our knowledge, such a framework has not been considered in previous papers, except [4]. Copulas have been used in communication but rather to model the dependence of several sources and make some source separation [6] or the correlated shadowing to improve MIMO performance [7].

2 Copulæ

Copulæ are a very useful way to model structures of dependence between random variables [5]. The fundamental result justifying this usefulness is the Sklar's Theorem : it ensures that under the condition that the cumulative distributions of the random variables are continuous, there exists a unique copula C such that $\forall(x,\ldots,x_d)$, we have

$$H(x_1,\ldots,x_d) = C\left(F_1(x_1),\ldots,F_d(x_d)\right). \qquad (2)$$

where H is the joint distribution of the random vector $(X_1,\ldots,X_d)$. Hence, a copula is a function $C : [0,1]^d \mapsto [0,1]$ which couples the marginals F_i between themselves. The name *copula* comes from this last remark. In Fig. 1 we represent the interference samples when $\mathbf{I}$ has independent components. The representation is done directly on the sample or after a transformation through the repartition function of the marginals $(F_i(.))$ to have the representation of the copula.

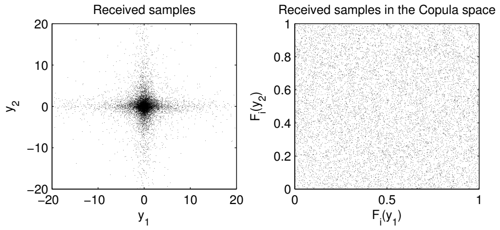

FIGURE 1 – In the left plot samples are independent and they form a cross. This can be explained saying that large values are rare and the occurrence of two large values on the same vector is very unlikely. In the right plot (X and Y axis are $F_i(y_1)$ and $F_i(y_2)$), the points are uniformly distributed which signifies the independent structure.

2.1 Archimedean copulæ

In the following, we consider a particular class of bivariate Archimedean copulæ. The interest of this class is, first of all, the easiness with which they can be constructed. The multivariate Archimedean copulæ have the following form : for all $(u_1,\cdots,u_d)\in[0,1]^d$,

$$C(u_1,\cdots,u_d) = \phi^{-1}\left(\phi(u_1)+\cdots+\phi(u_d)\right). \qquad (3)$$

The function ϕ is called the generator of the copula and is a continuous and convex function such that $\phi(1)=0$. It appears that all Archimedean copula is symmetric in its variables.

We will focus on two families of Archimedean copulæ, both indexed by a single parameter. The Clayton and the Gumbel families of copulæ model asymmetric dependence in tails.

Definition 2.1. *For all $\theta > 0$,The Clayton copula of parameter θ is defined on $[0,1]^d$ by*

$$C(u_1,\cdots,u_d) = \left(u_1^{-1/\theta}+\cdots+u_d^{-1\theta}-(d-1)\right)^{-\theta}.$$

In particular, it is obtained when ϕ^{-1} is the Laplace transform of a Gamma distribution.

In Fig. 2 we have a similar representation as in Fig. 1 but introducing the dependence structure of the Clayton copula. The

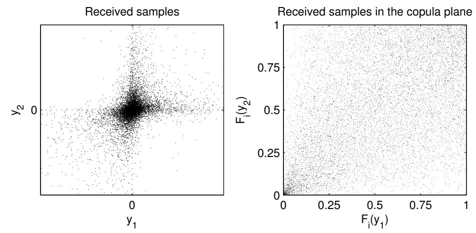

FIGURE 2 – Interference samples for Cauchy marginals and Clayton copula.

cross in the left plot tends to disappear and points, especially in the bottom left quadrant, are differently positioned. This results from the non zero asymmetric tail dependence introduced by the Clayton copula.

Definition 2.2. *For all $\theta \geq 1$,The Gumbel copula of parameter θ is defined on $[0,1]^d$ by*

$$C(u_1,\cdots,u_d) = \exp\left(-(\sum_{i=1}^{d}(-\log u_i))^{1/\theta}\right).$$

In particular, it is obtained when ϕ^{-1} is the Laplace transform of a α-stable distribution.

In Fig. 3 we represent the dependence structure of the Gumbel copula on the received samples and after the transform through the marginals.

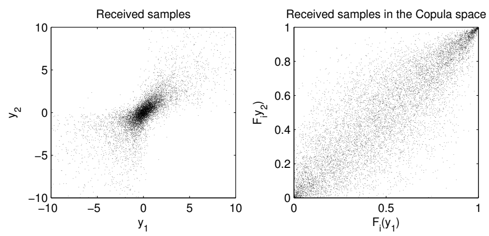

FIGURE 3 – Received samples for Cauchy marginals and Gumbel copula.

3 Log Likelihoood Ratio for dependent variables

In the two-dimensional case and with a binary input, our system model in (1) can be written :

$$\begin{cases} y_1 = s + i_1 \\ y_2 = s + i_2. \end{cases}, \tag{4}$$

where $s \in \{-1, 1\}$. Two repetitions y_1 and y_2 of this bit are obtained and $\mathbf{I} = (i_1, i_2)$ is a bivariate interference vector. The two coordinates i_1 and i_2 are not independent. The LLR for each $\mathbf{Y} \in \mathbb{R}^2$ is given by the ratio

$$\Lambda(y_1, y_2) = \log \frac{\mathbb{P}(y_1 = s + i_1,\, y_2 = s + i_2 \mid s = 1)}{\mathbb{P}(y_1 = s + i_1,\, y_2 = s + i_2 \mid s = -1)}. \tag{5}$$

Let f be the joint density of the couple (i_1, i_2), (5) becomes

$$\Lambda(y_1, y_2) = \log \frac{f(y_1 - 1, y_2 - 1)}{f(y_1 + 1, y_2 + 1)}. \tag{6}$$

3.1 Independent interferences

Fig. 4 illustrates the two decision regions in the case of two independent Cauchy distributions with $x_0 = 0$ and $\delta = 1$. The X and Y axis are the values of the components of the received vector $\mathbf{Y}$. We consider two possible transmitted symbols, $\{-1, 1\}$, meaning that the transmitted vector is either $(1, 1)$ or $(-1, -1)$, denoted by two circles on the figure. The white region corresponds to the decision 1, meaning that $\Lambda \geq 0$ and the black one to -1, i.e., $\Lambda < 0$

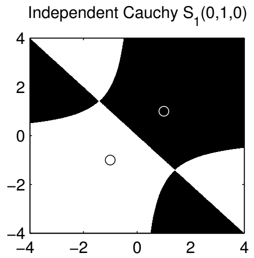

FIGURE 4 – Decision regions for an independent Cauchy interference.

A Gaussian noise would correspond to a linear boundary, corresponding to an Euclidean distance : the whole region above the diagonal going from $(-4, 4)$ to $(4, -4)$ would be black, the other white. Impulsiveness significantly modifies those boundaries and the optimal decision does not necessarily depends on the closest (in euclidean distance) possibly transmitted symbol. On the contrary, a large sample is probably due to a large noise and the corresponding value can not be trusted (when the Gaussian receiver would put much trust on it). It results in the shifted black and whites area where the smallest value makes the decision. However, it necessitates non linear complex operations to implement an optimal receiver.

3.2 Dependent interferences

If we now consider that i_1 and i_2 are dependent and that we can express this dependence through an Archimedean copula, the form of the LLR will change, indeed :

$$\begin{aligned} \Lambda(x, y) &= \log \frac{f_i(x-1) f_i(y-1) c(F_i(x-1), F_i(y-1))}{f_i(x+1) f_i(y+1) c(F_i(x+1), F_i(y+1))} \\ &= \Lambda_{\perp}(x, y) + \Lambda_c(x, y), \end{aligned} \tag{7}$$

where c is the density of the copula and is defined by

$$c(u, v) = \frac{\partial^2 C}{\partial u \partial v}(u, v); \tag{8}$$

f_i and F_i respectively the probability density function and the cumulative distribution of the interference. We can notice that $\Lambda_{\perp}$ represents the independent part of the LLR. The second term

$$\Lambda_c(x, y) = \log \frac{c(F_i(x-1), F_i(y-1))}{c(F_i(x+1), F_i(y+1))} \tag{9}$$

is the part of the LLR depending on the copula and represents the dependence structure. It can however be tricky to derive because it also depends on the marginals.

In the case of the Clayton copula, the consequence on the decision region is shown in Fig. 5, left plot. We clearly see that the lower tail dependence significantly modifies the decision regions. The area corresponding to -1 is significantly larger due to the higher probability of having two samples influenced by a negative value.

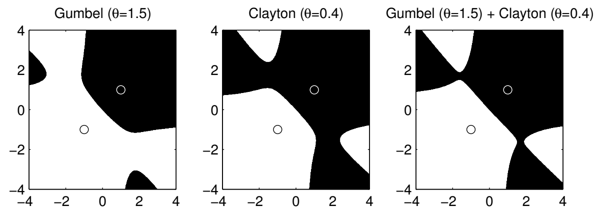

FIGURE 5 – Decision region for Cauchy marginals and Clayton (left), Gumbel (middle) or a mix of Gumbel and Clayton (right) copula.

In the case of the Gumbel copula, the consequence on the decision region is shown in Fig. 5, middle plot. We again clearly see that the upper tail dependence significantly modifies the decision regions, in that case in the reverse way as the Gumbel copula because we have a different tail dependence (upper tail dependence instead of lower tail)..

We finally plot in Fig. 5, right plot the decision regions for a mix of Clayton and Gumbel copula. This allow to keep a symmetric interference, allowing upper and lower tail dependence, which is more relevant to the considered scenario. The decision regions have more similarities with the independent case. However there are still differences at the boundaries. The effect on higher dimensions remain to be studied.

4 Applications on a SIMO model

4.1 Receiver design

The optimal receiver in terms of minimizing the Bit Error Rate (BER) is the Maximum Likelihood (ML) detector. It is

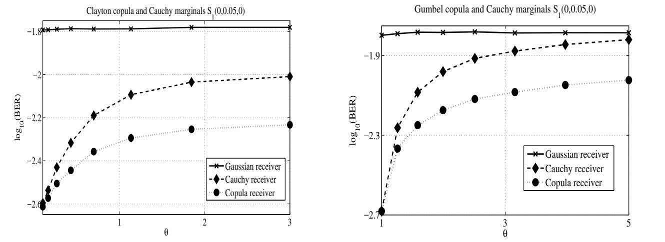

FIGURE 6 – BER for Cauchy noise and Gumbel (left) and Clayton (middle) copulae as a function of the copula parameter.

given by the solution to the following optimization problem :

$$\hat{s} = \arg\max_{s\in\Omega} \mathbb{P}(\mathbf{Y}|s) \tag{10}$$

where Ω is the set of possible transmitted bits.

In the binary case, $\Omega = \{-1, 1\}$ and the problem is reduced to obtaining the sign of the LLR defined in (7).

$$\begin{aligned}\widehat{s} &= sign\left(\Lambda(x, y)\right)\\ &= sign\left(\log\Lambda_{\perp}(x, y) + \Lambda_c(x, y)\right), \end{aligned} \tag{11}$$

Fig. 6 left (resp. middle) compares the performance of the linear Gaussian receiver, a Cauchy receiver assuming independent Cauchy interference and a copula receiver that knows both the marginal and the dependence structures. In that case the dependence is captured by a Clayton (resp. Gumbel) copula.

Obviously when the parameter gets close to zero (resp. one), the dependence is low and, if the Cauchy receiver outperforms the Gaussian one, there is no need to introduce the dependence structure. However, when the dependence increases (θ gets larger), the performance of the Cauchy receiver quickly degrades when the copula receiver is able to maintain a better performance level.

Finally, we apply our Copula receiver to a SIMO case with two receive antennas. A Poisson field of interfer is simulated but the channel coefficient have a correlated phase (phase shift of the LOS contribution uniformly distributed on $[-\pi/4, \pi/4]$). For the given configuration, we vary the parameters of the copula keeping the symmetry. The gain in performance is limited

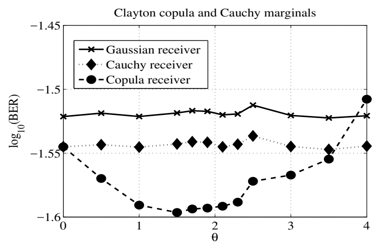

FIGURE 7 – BER for SIMO model for mixture of Clayton and Gumbel copulæ.

but this is easily explained by the small dimension considered (only 2). It is clear that taken the dependence structure into account allow a further gain compared to the independent receiver. This last one gives better performance than the Gaussian receiver.

5 Conclusion

We proposed in this paper a way to model dependency in impulsive interference. Usual tools (based on correlation) do not allow to well capture the dependence structure of such an impulsive interference, especially when the α-stable model is used.

In the case of Cauchy marginals and copulæ from the Archimedean family and with a binary input, we are able to derive analytical expressions of the decision rule based on the likelihood ratio. The results on the decision regions show that dependent interference has a significant impact on the optimal decision that we should make. Consequently, we compared receivers that takes this dependency into account to receivers that do not. We show that the latter can rapidly degrade if a dependence structure is present when the former manage to maintain good performances. We illustrate the possible benefit on a SIMO example.

The densification of networks and their heterogeneity make interference an important issue in wireless communication. The dependence structure is certainly a crucial point for an efficient implementation of such networks. Will copula play a role ?

6 Acknowledgement

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