Eigenvalue-Based Spectrum Sensing With Two Receive Antennas

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Résumé – Le concept de la radio intelligente définit deux types d'utilisateurs: les utilisateurs primaires (UP) qui ont accès aux ressources spectrales d'une façon prioritaire et les utilisateurs secondaires (US) qui exploitent les opportunités de communication laissées vacantes par les UPs. Dans ce papier on s'intéresse au problème de détection des ressources spectrales libres en utilisant les distributions du nombre de conditionnement (NDC) de la matrice de covariance des signaux reçus par l'US. Une nouvelle formule mathématique est proposée pour la distribution du NDC dans le cas d'absence des UPs permettant ainsi de développer un nouveau algorithme de détection. Les résultats des simulations nous permettent de valider la formulation théorique et les hypothèses de bases.

Abstract – Spectrum sensing is the key enabler for dynamic spectrum access as it can allow secondary users to reuse "*free*" spectrum without causing harmful interference to primary users. In this paper, we give the exact distribution of standard condition number (SCN) of dual central uncorrelated Wishart matrices. This allows us to derive accurate closed-form expressions of the detection and false alarm probabilities. Simulation results are presented to validate the accuracy of the derived expressions.

1 Introduction

One of the most promising solutions for the spectrum scarcity and the inefficiency in its usage is cognitive radio (CR), invented by Mitola [1]. Spectrum sensing is a critical component of CR as it is a fundamental requirement for the secondary user (SU) to continuously senses the channel before accessing it to avoid causing interference to the primary user (PU).

Several detection techniques were explored by researchers in the last decade [2]. Among these, Energy detector (ED) is the most popular technique because of the fact that it is simple, non-coherent, and need no prior knowledge about the PU's signal. However, ED requires perfect knowledge of the noise power which reveals the high performance degradation under noise uncertainty conditions [3]. Other techniques, such as matched filter, cyclostationary detection, filter bank, wavelet detection and covariance detection methods were also proposed [2].

Eigenvalue based detector (EBD) has been recently proposed as an efficient way for spectrum sensing in CR [4, 5, 6] for the fact that it does not need any prior knowledge about the noise power or signal to noise ratio. Due to this blindness property, this technique has been shown to overcome the traditional energy detector technique [5]. EBD is based on the eigenvalues of the received signal's covariance matrix and utilize results from random matrix theory. It detect the presence/absence of the PU by exploiting receiver diversity that may consist of cooperation between SUs, multiple antennas, or oversampling. EBD includes the largest eigenvalue (LE) detector proposed in [5], the scaled largest eigenvalue (SLE) detector [6], and the standard condition number (SCN) detector [5, 4, 7, 8, 9, 10].

The standard condition number is defined as the ratio of maximum to minimum eigenvalues. The SCN algorithms rely on asymptotic assumptions that may not be practical. The main limitation of this work, including Marchenko-Pastur (MP) law used in [4], the Tracy-Widom (TW) distribution used in [5], or the usage of Curtiss formula with TW-distribution in [7, 8], is the asymptotic assumptions on the covariance matrix size, i.e., the number of samples and the number of antennas must tends to infinity. In addition to this limitation, the analytical formulas due to TW-distribution could not be implemented online and lookup tables (LUT) should be used instead.

More recently, exact results on the SCN distribution of finite sized Wishart matrices have been found in [11] and applied in CR [9, 10]. The covariance matrix is known as Wishart matrix if the receiver's inputs are assumed Gaussian. The complexity of SCN distribution, derived in [11], quickly increases as the number of samples and/or the number of antennas increase. In addition, it is very important to have an inverse form of the SCN distribution which is difficult using expressions from [11].

In this paper, we consider the dual case (i.e. two antenna CR receiver) and we give a new simple and accurate form for the SCN cumulative distribution function (CDF) and probability density function (PDF) of the SCN of dual Wishart matrices. Accordingly, we provide the exact form of the false-alarm probability and the decision threshold.

The rest of this paper is organized as follows. In section 2, we give the cognitive radio system model. In section 3, the SCN metric is analysed and the detection algorithm is provided. The new analytical forms of the CDF and PDF of the SCN metric as well as the form of the false-alarm probability and decision threshold are also derived. Simulation results are provided in section 4 and the paper conclusion in section 5.

2 System Model

Let us consider a multiple-antenna CR system aiming to detect the presence/absence of a single PU during a sensing period corresponds to N samples and denote by K the number of antennas at the CR receiver. For this detection problem, there are two hypothesizes : $\mathcal{H}_0$ corresponds to the absence of the PU (i.e. free spectrum) ; and $\mathcal{H}_1$ where the PU exists (i.e. spectrum being used). The received vector, at instant n, under both hypothesizes is given by :

$$\mathcal{H}_0 : \ \boldsymbol{y}(n) = \boldsymbol{\eta}(n), \tag{1}$$

$$\mathcal{H}_1 : \ \boldsymbol{y}(n) = \boldsymbol{h}(n)\boldsymbol{s}(n) + \boldsymbol{\eta}(n), \tag{2}$$

where $\boldsymbol{y}(n) = [y_1(n), \cdots, y_K(n)]^T$ is the observed $K \times 1$ complex samples from all antennas at instant n. $\boldsymbol{\eta}(n)$ is a $K \times 1$ complex circular white Gaussian noise. $\boldsymbol{h}(n)$ is a $K \times 1$ complex vector that represents the channels' coefficient between the PU and each antenna at the CR receiver and $\boldsymbol{s}(n)$ stands for the primary signal to be detected.

After collecting N samples from each antenna, the received signal matrix $\boldsymbol{Y}$ is written as (3).

$$\boldsymbol{Y} = \begin{pmatrix} y_1(1) & y_1(2) & \cdots & y_1(N) \\ y_2(1) & y_2(2) & \cdots & y_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ y_K(1) & y_K(2) & \cdots & y_K(N) \end{pmatrix} \tag{3}$$

Without loss of generality, we suppose that $K \leq N$ and we define the received sample covariance matrix as $\boldsymbol{W} = \boldsymbol{Y}\boldsymbol{Y}^\dagger$, where $.^\dagger$ define the Hermitian conjugate.

3 SCN-based Algorithm

Let us denote by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > 0$ the eigenvalues of $\boldsymbol{W}$ then the SCN metric is given by :

$$SCN = \frac{\lambda_1}{\lambda_K}. \tag{4}$$

Denoting by t the decision threshold, then the detection probability (P_d), defined as the probability of correctly detecting the presence of PU, and the false alarm probability (P_{fa}), defined as the probability of detecting the presence of PU while it does not exist, are, respectively, given by (5) and (6).

$$P_d \ = P(SCN \geq t/\mathcal{H}_1) \tag{5}$$

$$P_{fa} = P(SCN \geq t/\mathcal{H}_0) \tag{6}$$

These probabilities depend on the threshold (t) being used. However, if the expression of the P_{fa} and P_d are previously known, then a threshold could be set according to a required error constraints. Then, it is clear that these probabilities depend on the distribution of the SCN metric. If we denote the CDF and PDF of SCN, respectively, by $F_i(.)$ and $f_i(.)$ with index $i \in \{0, 1\}$ indicates the considered hypothesis. Then we can write :

$$P_{fa} = 1 - F_0(t) \tag{7}$$

$$P_d \ = 1 - F_1(t) \tag{8}$$

Accordingly, for a prescribed false-alarm probability P_{fa} [1], the SCN detector algorithm could be described as follows :

1. Compute the Covariance matrix $\boldsymbol{W} = \boldsymbol{Y}\boldsymbol{Y}^\dagger$.
2. Compute the minimum and maximum eigenvalues (λ_1, λ_K) of $\boldsymbol{W}$.
3. Evaluate the SCN value ($SCN = \lambda_1/\lambda_K$).
4. Accept $\mathcal{H}_0$ if and only if $SCN \leq F_0^{-1}(1 - P_{fa})$.

Thus, it is important to have a simple and accurate form of the SCN distribution for the algorithm to work in real time system.

3.1 SCN distribution under $\mathcal{H}_0$

Under $\mathcal{H}_0$, the input of the matrix $\boldsymbol{Y}$ is a complex circular white Gaussian noise with zero mean and unknown variance σ_η^2, then $\boldsymbol{W}$ is well known as a central uncorrelated complex Wishart matrix and is denoted by $\boldsymbol{W} \sim \mathcal{CW}_K(N, \sigma_\eta^2 \mathbf{I}_K)$ where K is the size of the matrix, N is the number of degrees of freedom (DoF), and $\sigma_\eta^2 \mathbf{I}_K$ is the correlation matrix. In this case, the exact generic form of the SCN distribution, $F_0(t)$, could be found in [11] and for $K = 2$ and $K = 3$ in [9, 10] respectively. However, this formula has a complex expression and finding the inverse function F_0^{-1} can be very difficult to do. In this section, we propose new mathematical expressions of the probability density function and the cumulative distribution function of the SCN under null hypothesis $\mathcal{H}_0$. Consequently, expressions of the Pfa and the threshold t_{opt} are provided.

As defined in (4), the standard condition number of dual matrices can be expressed as following :

$$SCN = \frac{\lambda_1}{\lambda_2} = \frac{1 + \sqrt{1 - 4D/T^2}}{1 - \sqrt{1 - 4D/T^2}} \tag{9}$$

If we denote the elements of the matrix $\boldsymbol{W}$ by $w_{i,j}$ with $i, j = 1, 2$, then $T = w_{1,1} + w_{2,2}$ is the trace of the matrix $\boldsymbol{W}$ which is also the sum of the eigenvalues, and $D = w_{1,1} * w_{2,2} - w_{1,2}^2$ is the determinant of the matrix $\boldsymbol{W}$ which is also the product of the eigenvalues. Because our sample size N is greater than 30 [12], the central limit theorem allows us to state that $w_{1,1}$ and $w_{2,2}$ converge in distribution to Gaussian distributions. Moreover, the random variable $(w_{1,1} * w_{2,2})$ has a normal product distribution and $w_{1,2}^2$ obeys to Chi-Square distribution. Using variable substitution method and mathematical manipulation,

1. For a prescribed detection probability, one can consider the same algorithm $\mathcal{H}_1$ hypothesis and SCN distribution under $\mathcal{H}_1$

the random variable $Z = 1 - 4D/T^2$ has a beta distribution with parameters $\alpha = 1$ and $\beta = N/2$. Finally, the probability density of standard condition number (SCN=$\frac{1+Z}{1-Z}$) is given by

$$f_0(t) = \beta t^{\beta-1}(t-1)^{2\alpha-1}\frac{2^{2\beta}}{(1+t)^{2(\alpha+\beta)-1}} \tag{10}$$

where $t \in \mathbb{R}$ and $t \geq 1$. Then, the cumulative distribution function of the random variable SCN is given as

$$F_0(t) = 1 - \frac{2^{2\beta}t^{\beta}}{(1+t)^{2\beta}} \tag{11}$$

Now, given equations (11) and (7), the false-alarm probability can be expressed as :

$$P_{fa} = \frac{2^{2\beta}t^{\beta}}{(1+t)^{2\beta}} \tag{12}$$

The receiver optimization criterion is thus to maximize the detection probability at a fixed false alarm probability (Neyman-Pearson criterion). Therefore, the optimal threshold is given by

$$t_{opt} = 2(P_{fa})^{(-1/\beta)}\left[1+\sqrt{1-(P_{fa})^{(1/\beta)}}\right] - 1 \tag{13}$$

If the standard condition number of the received covariance matrix $\boldsymbol{W}$ is greater than t_{opt}, the channel will be busy ; otherwise channel will be idle.

3.2 SCN distribution under $\mathcal{H}_1$

Under $\mathcal{H}_1$, the exact generic form of $F_1(t)$ is derived in [11], however, any further numerical evaluation requires Nuttall-Q function which could be replaced by Marcum Q-function and a finite weighted sum Bessel functions [13] or by using hypergeometric functions that could be expanded to an infinite sum (See, for example, [14] for $K = 2$ and $N = 2$). Thus, and since both solutions are difficult to manipulate, a third solution is to use the non-central/central approximation that approximates the distribution of the non-central uncorrelated Wishart by the distribution of the central semi-correlated Wishart. The exact general form of the distribution of SCN of central semi-correlated Wishart matrix is provided by [11], and used to approximate $F_1(t)$ for $K = 2$ in [11, 9].

4 simulation

In this section, the theoretical results presented in Section 3.1 are validated via Monte-Carlo simulations. The simulation results are obtained by generating 10^6 random samples of the random matrix $\boldsymbol{Y} \in \mathbb{R}^{K,N}$ according to (3), where $K = 2$. Figure 1 validates the new proposed expression for the cumulative distribution function of the SCN of uncorrelated central dual Wishart matrices with arbitrary degrees N. It is clearly that the analytical curves yield an excellent match with the Matlab simulator output.

Figure 2 shows the performance of the proposed algorithm for a given probability of false alarm, signal-to-noise ratio on

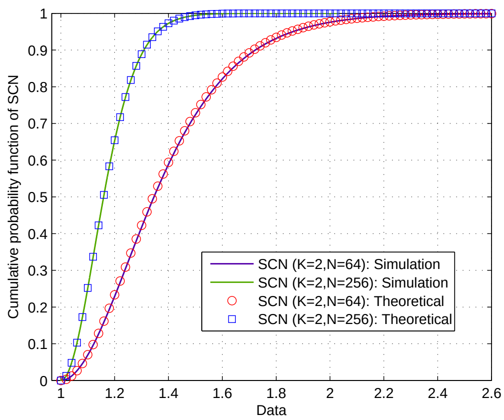

FIGURE 1 – Standard condition number CDF of a 2×2 uncorrelated central dual Wishart matrices

X-axis and probability of detection on Y-axis. We compare the performance of the proposed scheme with that of ideal energy detection algorithm and also the practical energy detector where there are some noise uncertainty. The noise uncertainty makes the energy detector very unreliable where good estimation of the noise power level is not available. Moreover, The proposed algorithm shows significant robustness to the noise uncertainty problem.

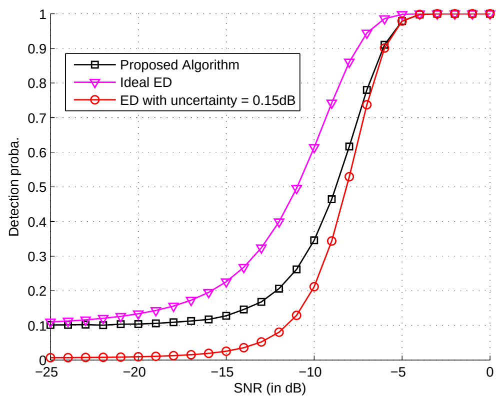

FIGURE 2 – Detection probability as function of the SNR for a fixed false alarm ($P_{fa} = 0.1$), $K = 2$, and $N = 256$, comparison between ideal energy detector, energy detector under noise uncertainty, and proposed algorithm.

5 Conclusion

In this paper, we have presented a new expression of the cumulative distribution function of the standard condition number of uncorrelated central dual Wishart matrices. A new detection algorithm based on this new formula is proposed. We derived accurate expressions of the false alarm probability and the optimal threshold. We have shown that the simulation results validate the accuracy of the derived expressions.

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