Compressed sensing MR image reconstruction using data-driven tight frame

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Résumé - L'acquisition comprimée (compressed sensing—CS) est une technique permettant de réduire le temps d'acquisition en imagerie par résonance magnétique (IRM) en sous-échantillonnant l'espace k. La parcimonie ou la compressibilité est la prémisse fondamentale de l'acquisition comprimée. Des transformations prédéfinies telles que la transformée en ondelettes discrète, les framelets et la shearlet ont été largement utilisées pour obtenir des représentations parcimonieuses. Nous proposons une nouvelle méthode de reconstruction des images IRM basée sur CS à l'aide de la technique de data-driven tight frame (data-driven TF). La méthode consiste à diviser une image IRM en patches, à apprendre une série de filtres en utilisant la technique de data-driven TF à partir de ces patches, et à reconstruire l'image finale à l'aide de la technique de Fast Iterative Shrinkage Thresholding Algorithm (FISTA) avec un schéma itératif. Les résultats montrent que la méthode proposée convient à tous les schémas d'échantillonnage de l'espace k et améliore significativement la qualité de l'image par rapport aux méthodes de reconstruction des images IRM existantes.

Abstract - Compressed sensing (CS) is a promising technique to reduce the acquisition time of magnetic resonance imaging (MRI) by using under-sampled k-space data. Sparsity or compressibility is the fundamental premise of the compressed sensing. The predefined transforms, such as the discrete wavelet transform (DWT), the framelets and the shearlet, have been widely used to provide sparse representations. This paper proposes a new CS reconstruction method for MR images using a data-driven tight frame (data-driven TF). The method consists of dividing the MR image into overlapping patches, learning a set of filters forming the data-driven TF from these patches, and reconstructing the final image using the Fast Iterative Shrinkage Thresholding Algorithm (FISTA) with an iterative scheme. The results show that the proposed reconstruction method is suitable for different sampling schemes, including variable density and radial k-space sampling, and improves significantly the image quality compared with existing CS-MR image reconstruction techniques.

1 Introduction

Magnetic resonance imaging (MRI) is a non-invasive medical imaging technique widely used in clinics. However, its imaging speed is limited by the amount of acquired data. Various approaches have been proposed to reduce the acquisition time of MRI as much as possible without degrading image quality [1, 2].

Recently, Compressed Sensing (CS) [3, 4] appeared as a new approach to reconstructing signals with high quality from significantly under-sampled data, and showed its great potential for accelerating MRI acquisitions [5, 6]. MR image reconstruction based on CS assumes that the image has a sparse representation in certain domain (pixel or transform domain). The total variation (TV) regularization and discrete wavelet transform (DWT) are widely used as sparsifying transforms for CS-MR image reconstruction [5, 7]. However, the TV regularization can cause blocking effect although the edges are preserved, and the traditional DWT fails to capture singularities in higher dimensional, such as edges and contours [8]. To overcome the disadvantages, multi-scale geometric analysis methods have been introduced into CS-MR image reconstruction, such as Contourlet [9], Framelets [10] and Shearlet [8].

Unlike the predefined transform, dictionary learning approaches learn an dictionary from the patches of a particular image or undersampled data itself for sparse image representation and have been shown to offer better reconstruction performance when applied to CS-MR image reconstruction [11]. Taking into account the similarity between image patches, nonlocal processing was introduced for image restoration, such as nonlocal total variation (NLTV) [12]. Instead of using patch as the basic unit of sparse representation, a patch-based nonlocal operator was introduced for CS-MR image reconstruction [13]. It exploits the nonlocal self-similarity of images, enabling this approach to achieve lower reconstruction error compared with the conventional CS-MRI reconstruction methods.

In this study, we present a new CS reconstruction method for MR images by introducing the data-driven tight frame (data-driven TF) proposed in [14]. The method consists of dividing the MR image into overlapping patches, learning a set of filters forming the data-driven TF from these patches, and reconstructing the final image using the Fast Iterative Shrinkage Thresholding Algorithm (FISTA) with an iterative scheme.

The rest of the paper is organized as follows. The proposed CS-MRI reconstruction method is detailed in Section 2. The experiments are presented in Section 3, followed by some conclusions in Section 4.

2 Theory and Algorithm

Suppose x is an MR image and F_u is a partial Fourier transform. The CS-MR image reconstruction problem is formulated as:

$$x = \arg\min_x \|F_u x - b\|_2^2 + \lambda \cdot R(x) \tag{1}$$

where b is the under-sampled measurements of the image x in k-space, $R(x)$ is a regularizing functional that represents additional constraints according to some prior knowledge in order to find the optimal solution, and $\lambda > 0$ refers to a balancing parameter.

Instead of predefined transforms, we use the data-driven TF, which has been successfully used in [14] for image denoising problems, as sparsifying transforms for $R(x)$. The data-driven TF aims to adaptively learn a set of filters from input data itself to sparsely represent data. Given an image x, a data-driven TF can be learned by solving the following minimization problem:

$$\min_{c,\Phi} \|c - \Phi x\|_2^2 + \alpha \cdot \|c\|_0, \quad \text{subject to } \Phi^T \Phi = I \tag{2}$$

where Φ is the analysis operator and the rows of Φ form a tight frame, c is the coefficient vector that sparsely approximates the coefficients Φx, and $\alpha > 0$ is a regularization parameter. The algorithm for solving equation (2) is presented in Appendix A. More details about the data-driven TF can be found in [14].

By taking into account the above adaptive discrete tight frame, we reformulate the CS-MR image reconstruction method as:

$$x = \arg\min_x \left\{ \frac{1}{2}\|F_u x - b\|_2^2 + \lambda \cdot \|\Phi x\|_1 \right\} \tag{3}$$

Let $f(x) = \frac{1}{2}\|F_u x - b\|_2^2$, which is a convex and smooth function with the Lipschitz constant L, and $g(x) = \lambda \cdot \|\Phi x\|_1$ denoting a regularizing functional. Equation (3) can be solved using the Fast Iterative Shrinkage Thresholding Algorithm (FISTA) introduced in [15]. Specifically, the problem can be solved by a proximal mapping operation:

$$prox_\rho(\phi)(x) = \arg\min \left\{ \phi(u) + \frac{1}{2\rho}\|u - x\|_2^2 \right\} \tag{4}$$

where ρ is the inverse of the Lipschitz constant L of ∇f and $\nabla f = \left(\frac{1}{2}\|F_u x - b\|_2^2 \right)' = F_u^T (F_u x - b)$ with F_u^T indicating the inverse partial Fourier transform.

The proposed algorithm is outlined as follows.

INPUT:

K : the maximum number of iterations;

n : the filter size of data-driven TF;

λ : the regularization parameter;

tol : the tolerance parameter.

INIT: $\rho = 1/L, t^1 = 1, x^0 = r^1 = 0, k = 0;$

OUTPUT:

x : the reconstructed image.

REPEAT:

$k = k + 1;$

Generate the transform operator Φ^k from x^k according to Equation (2):

$x_g = r^k - \rho \nabla f(r^k);$

$x^k = prox_\rho\left(2\lambda \|\Phi^k x\|_1\right)(x_g);$

$t^{k+1} = \dfrac{1 + \sqrt{1 + 4(t^k)^2}}{2};$

$r^{k+1} = x^k + \dfrac{t^k - 1}{t^{k+1}}(x^k - x^{k-1});$

UNTIL $k > K$ **OR** $\dfrac{\|x^{k-1} - x^k\|_2}{\|x^k\|_2} < tol$.

3 Experimental Results

To evaluate the performance of the proposed method, MR Coronal brain images of size 256 x 256 (Note: data from Ref. [7]) and two different sampling schemes (i.e. random variable density and radial sampling) are used (Figure 1). In Figure 1(b)-(f) are shown the k-space sampling masks, where the sample ratio in k-space is set to be approximately 15% (i.e. keeping 15% of the complete k-space data).

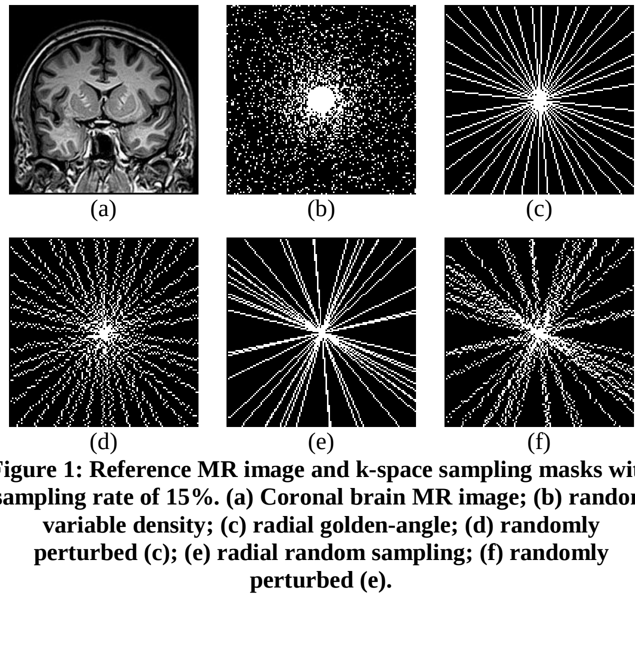

Figure 1: Reference MR image and k-space sampling masks with sampling rate of 15%. (a) Coronal brain MR image; (b) random variable density; (c) radial golden-angle; (d) randomly perturbed (c); (e) radial random sampling; (f) randomly perturbed (e).

To quantitatively compare different methods, three quantitative indexes, including the peak-signal-to-noise ratio (PSNR), relative l2 norm error (RLNE) and mean structural similarity (MSSIM) [16] are calculated.

The proposed method is also compared with three state of the art reconstruction methods used in CS-MRI: the FCSA [7], Shearlet+TGV [8] and PANOCS [13] methods.

The observation measurement b is corrupted by complex Gaussian white noise with an input SNR (ISNR). The associated ISNR is defined as $\mathrm{ISNR} = 20\log_{10}\left(\sigma_x/\sigma_n\right)$, where σ_x and σ_n denote the standard deviation of the original image and the noise, respectively. The ISNR is set to 30 dB, the filter size $n = 4$ and the regularization parameter λ is set as $0.095 \times \sigma_n$.

3.1 Visual Comparisons

The MR images reconstructed using different methods are shown in Figure 2. Visually, the proposed method yielded better image quality than other three methods.

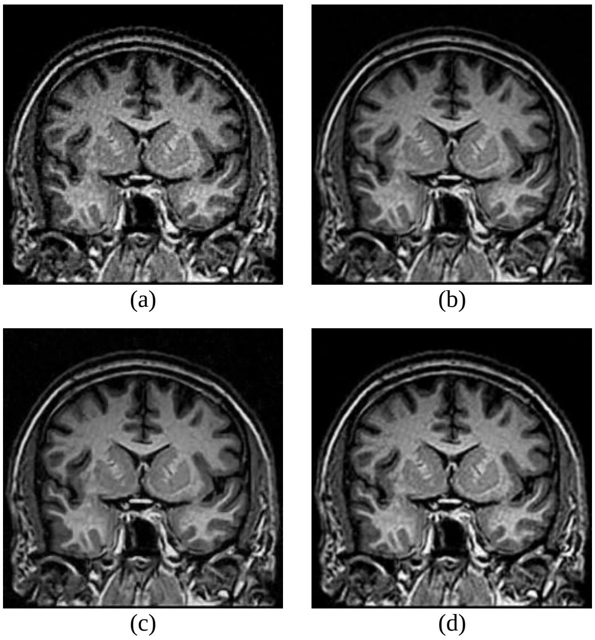

Figure 2: Results of reconstruction on Coronal brain MR images using sampling scheme in Figure 1 (b) with 15% sampling. Image reconstructed using (a) FCSA [7]; (b) Shearlet+TGV [8]; (c) PANOCS [13] and (d) Proposed.

The quantitative assessment between the four methods is given in Tab 1. It can be seen that the proposed method exhibits better reconstruction quality than other three methods, by yielding higher PSNRs, smaller RLNEs, and greater MSSIMs.

Tab 1 : Comparison of different reconstruction methods in terms of PSNR, RLNE and MSSIM using the sampling scheme in Figure 1(b) with 15% sampling.

Method	PSNR	RLNE	MSSIM
FCSA	26.07	0.14	0.84
Shearlet+TGV	27.21	0.13	0.87
PANOCS	26.16	0.14	0.82
Proposed	**28.01**	**0.11**	**0.89**

3.2 Effects of Sampling Rates

Figure 3 gives the performance comparison between the four methods using different sampling schemes with sampling rates of 10%~50%. It can be seen that the performance of our method is almost always better than other three methods, which implies that with the same image reconstruction quality, the proposed method requires only fewer samples and therefore allows reducing acquisition time significantly.

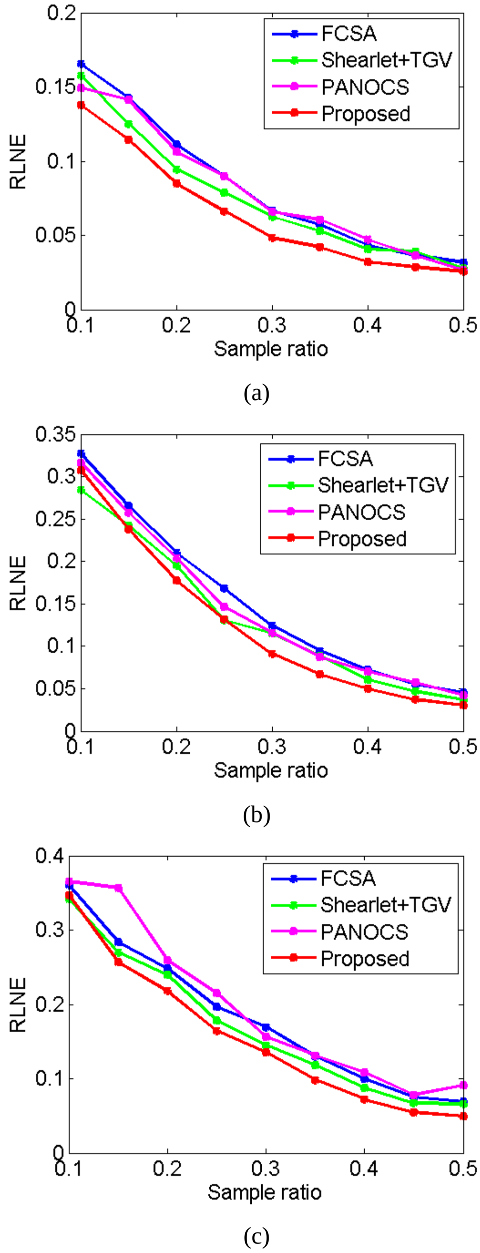

Figure 3: Performance comparisons (RLNE vs. different sampling rates) using different sampling schemes: (a) random variable density; (b) radial golden-angle; (c) radial random sampling.

4 Conclusion

We have proposed a new method for CS-MR image reconstruction using a data-driven tight frame (data-driven TF). The latter provides a better sparse approximation of MR images, which has allowed us to achieve better reconstruction performance. The involved optimization problem is efficiently solved by a first-order fast method. The results demonstrated that the proposed method can be applied to different sampling schemes and improves the reconstruction quality compared to state of the art CS-MRI reconstruction methods.

Appendix A

The algorithm for solving equation (2) is found below.

INPUT: An image x .

INIT: Initialize tight frame filter $\{\phi_i^0\}_{i=1}^{n^2}$.

OUTPUT: The tight frame Φ defined by filters $\{\phi_i^K\}_{i=1}^{n^2}$.

REPEAT

$k = k+1;$

(1) Fix framelet filters $\{\phi_i^k\}_{i=1}^{n^2}$, update the framelet coefficient c^{k+1}:

$$c^{k+1} = \arg\min_{c} \alpha \|c\|_0 + \frac{1}{2}\|d - \Phi^k x\|_2^2 \quad \text{(A-1)}$$

(2) Fix the framelet coefficient c^{k+1} , update the framelet filter $\{\phi_i^{k+1}\}_{i=1}^{n^2}$

$$\{\phi_i^{k+1}\}_{i=1}^{n^2} = \arg\min_{\{\phi_i\}_i^{n^2}} \|c^{k+1} - \Phi x\|_2^2 \quad \text{s.t. } \Phi^T\Phi = I \quad \text{(A-2)}$$

UNTIL $k > K$.

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