Mixed Estimation Function Applied to Neurodegenerative Disorder Gait Signals: low order ARMA modeling

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Résumé − L'approche huberienne est appliquée à des signaux de marche relatifs aux désordres neurodégénératifs. À partir d'une modélisation auto regressive moving average à faible ordre, basée sur l'estimation huberienne, *l'effet mémoire* de la marche humaine est montré. L'approche mathématique est discutée et des résultats expérimentaux basés sur une base de données contenant 16 sujets sains, 15 Parkinson et 19 Huntington sont présentés.

Abstract − Huberian approach is applied to neurodegenerative disorder gait signals. From low order auto regressive moving average modeling based on Huberian estimation, *memory effect* of human walking disorder is shown. Mathematical approach is discussed and experimental results based on a database containing 16 Control subjects (CO), 15 Parkinson's disease (PD), and 19 Huntington's disease (HD) are presented.

1 Introduction

This paper introduces a new use of Huberian estimation function [7] applied to neurodegenerative disorder signals from walking human. Low order Auto Regressive Moving Average (ARMA) modeling of human gait disorder and new indicators to differentiate two neurodegenerative diseases are proposed and discussed. Neurodegenerative disorders have a direct consequence on the human behavior by introducing *natural outliers* (NO) in biomechanic time-signals. These points are crucial in the study of neurodegenerative diseases and provide information of the degree of disorder. Here, PD and HD diseases are studied through the *stride time-signal* (STS) of human gait rhythm, corresponding to the time from initial contact of when one foot to the subsequent contact of the same foot. Human locomotion is regulated by the central nervous system (CNS). PD is a chronic and progressive hypokinetic disorder of the CNS induced by basal ganglia dysfunction. HD is a progressive neurodegenerative disorder with autosomal dominant inheritance. Different approaches exist to analyze gait rhythm time-signals, such as Gaussian approach [11], Huberian framework [6], and cyclostationary analysis [9]. Wu and Krishnan [11] developed a framework through Gaussian statistical analysis applied to PD, amyotrophic lateral sclerosis, and gait maturation in children. The main drawback of studies based on the Gaussian framework is the not well treatment of the NO in the time-signal. Indeed, during the 5-min walking period, every time the subjects reached the end of the hallway, they had to turn around, and finally they continued walking. In these studies based on Gaussian approach, NO are deleted to ensure consistency and convergence of the estimator. Here, all points are treated. ARMA system identification is a well-defined problem in several science and engineering areas such as speech signal processing, adaptive filtering, radar Doppler processing or biomechanics. Based on the fractional signal processing approach, Chaudhary et al [4] proposes a fractional least mean square (LMS) algorithm for parameter estimation of Hammerstein nonlinear ARMA system with exogenous noise. This algorithm has still been used in other studies [2]. Another approach uses a two-stage fractional LMS identification algorithm for parameter estimation of controlled ARMA (CARMA) systems [10]. Among the problems of ARMA identification, the model order estimation is crucial. Most of the time, these estimation procedures are performed by the implicit assumption that the processes are Gaussian. Moreover, these methods are based on the assumption that the signal does not contain outliers or a low density of outliers less than 1%. Here, low order ARMA modeling approach based on Huberian function with low threshold γ to assess parameters and experimental results are performed with real measurements with NO. *Memory effect* of human walking disorder is shown. This paper is organized as follows: Section 2 gives the Huberian mathematical context. Experimental results based on a database containing 16 CO, 15 PD, and 19 HD are shown in Section 3. Conclusions and perspectives are drawn in Section 4.

2 Mathematical framework

2.1 Estimation criterion based on Huberian function

Let $(S, \mathbf{S}, P)$ be a probability space and $\{X_k\}_{k=1}^N$ a sequence of i.i.d. random variables with values in S. Let Θ be a Borel subset in $\mathbb{R}^d$ and Γ a compact subset of $\mathbb{R}$. Let $\rho_\gamma^H : S \times \Theta \times \Gamma \to \mathbb{R}$ be a symmetric function such that $\rho_\gamma^H(\bullet(\theta,\gamma))$ is measurable for each $\theta \in \Theta$ and $\gamma \in \Gamma$. The estimator $\hat{\theta}_N^H$ is defined by a minimum of the form

$$N^{-1}\sum_{k=1}^{N}\rho_\gamma^H\left(X_k(\hat{\theta}_N^H,\hat{\gamma})\right) = \inf_{\theta\in\Theta,\gamma\in\Gamma} N^{-1}\sum_{k=1}^{N}\rho_\gamma^H\left(X_k(\theta,\gamma)\right) \tag{1}$$

with $\rho_\gamma^H(X) = \left\{\frac{X^2}{2} \text{ for } |X| \le \gamma, \gamma|X| - \frac{\gamma^2}{2} \text{ for } |X| > \gamma\right\}$. γ is a threshold to be determined to improve efficiency, convergence, and stability of $\hat{\theta}_N^H$. Let us introduce two index sets in $\theta \in \mathbb{R}^d$ defined by $\nu_2(\theta,\gamma) = \{k : |\varepsilon_k(\theta,\gamma)| \le \gamma\}$ and $\nu_1(\theta,\gamma) = \{k : |\varepsilon_k(\theta,\gamma)| > \gamma\}$ such that $card[\nu_2(\theta,\gamma)] + card[\nu_1(\theta,\gamma)] = N \ \forall\theta \in \mathcal{D}_M,\ \gamma \in \mathcal{D}_\gamma$, where $\varepsilon_k(\theta,\gamma)$ is the prediction error, $\mathcal{D}_M$ and $\mathcal{D}_\gamma$ are compact subsets and M a model structure. Let $M(\theta)$ be a particular model corresponding to the parameter vector value θ. Let us define $\tilde{\theta} = [\theta \ \ \gamma]$. We denote $s_k(\theta,\gamma)$, $k = 1, ..., N$ the sign function such that $s_k(\theta,\gamma) = 1$ for $\varepsilon_k(\theta,\gamma) > \gamma$, $s_k(\theta,\gamma) = -1$ for $\varepsilon_k(\theta,\gamma) < -\gamma$ and $s_k(\theta,\gamma) = 0$ for $|\varepsilon_k(\theta,\gamma)| < \gamma$. Let $\varepsilon_k(\theta,\gamma) = y_k - \hat{y}_{k|k-1}(\theta,\gamma) = y_k - \varphi_k^T(\theta,\gamma)\theta$ be the prediction error where y_k is the process output, $\hat{y}_{k|k-1}(\theta,\gamma)$ the prediction model and $\varphi_k(\theta,\gamma) \in \mathbb{R}^d$ the regressor vector. This criterion contains a L_2 part to treat small prediction errors and a L_1 part to deal with NO. Consider a batch of data from the system $\tilde{Z}^N = [y_1 ... y_N]$. Roughly speaking, we have to determine a mapping from the data $\tilde{Z}^N$ to the set $\mathcal{D}_M \times \mathcal{D}_\gamma$, $\tilde{Z}^N \longrightarrow \hat{\tilde{\theta}}_N^H = \left[\hat{\theta}_N^H \ \ \hat{\gamma}\right] \in \mathcal{D}_M \times \mathcal{D}_\gamma$. The estimation criterion to be minimized is then given by

$$W_N(\theta,\gamma) = \frac{1}{N}\sum_{k\in\nu_2(\theta,\gamma)}\frac{\varepsilon_k^2(\theta,\gamma)}{2} + \frac{\gamma}{N}\sum_{k\in\nu_1(\theta,\gamma)}\left(|\varepsilon_k(\theta,\gamma)| - \frac{\gamma s_k^2(\theta,\gamma)}{2}\right) \tag{2}$$

2.2 ARMA model in Huber's framework

The process output data are denoted as $\delta t_k, k = 1...N$ corresponding to the STS of human gait rhythm from heel toe force sensors underneath the left foot. Now assuming that δt_k is generated according to $\delta t_k = H_0(q)\, e_k$, where $H_0(q)$ is the noise filter and e_k, $k = 1...N$ a random variables sequence with zero mean and variances λ. The ARMA model set is parametrized by a d-dimensional real-valued parameter vector θ, i.e. $\delta t_k = H(q,\theta)\, e_k = \frac{\mathcal{C}(q,\theta)}{\mathcal{A}(q,\theta)} e_k$ with $\mathcal{A}(q,\theta) = 1 + \sum_{i=1}^{n_A} a_i q^{-i}$, $\mathcal{C}(q,\theta) = 1 + \sum_{i=1}^{n_C} c_i q^{-i}$ and $\theta = [a_1 ... a_{n_A} c_1 ... c_{n_C}]^T$. Moreover, q^{-1} is the lag operator such that $q^{-l}\delta t_k = \delta t_{k-l}$, $l \in \mathbb{N}$. We write $\varepsilon_k(\theta,\gamma) = \delta t_k - \hat{\delta t}_k(\theta,\gamma)$ as the prediction error where the prediction model is $\hat{\delta t}_k(\theta,\gamma) = \varphi_k^T(\theta,\gamma)\,\theta$ and the regressor is $\varphi_k(\theta,\gamma) = [-\delta t_{k-1} ... - \delta t_{k-n_A} \ \ \varepsilon_{k-1}(\theta,\gamma) ... \varepsilon_{k-n_C}(\theta,\gamma)]^T$. $\psi_k(\theta,\gamma)$ is the gradient with respect to θ of $\hat{\delta t}_k(\theta,\gamma)$ given by $\psi_k(\theta,\gamma) = \frac{1}{\mathcal{C}(q,\theta)}\varphi_k(\theta,\gamma)$ [8].

2.3 Choice of γ

A new curve ensuring a location of γ in low values leading to a reduction of the bias is shown. This bias is given by

$$\sup_{F_N\in\mathcal{P}_{\Phi_N}(\omega)}\left|\hat{\theta}_N^H - \theta^*\right| = b_N^\omega(\gamma) \le \hat{\kappa}^N f^\omega(\gamma)\,|\mathcal{L}^p| \tag{3}$$

where $\mathcal{L}^p$ is the upper NO and $f^\omega(\gamma)$ a new function named *tuning function*. Moreover κ^N is independent of γ, θ^* is the true parameter, $\mathcal{P}_{\Phi_N}(\omega)$ is the corrupted distribution model and F_N the contaminated Gaussian. From this curve in Fig.1, classical interval $C_\gamma = [1, 1.5]$ appears and a new interval $E_\gamma = [0.001, 0.2]$ is defined. We showed [6] that $f^\omega(\gamma) \approx 0.034\gamma^5 - 0.316\gamma^4 + 1.113\gamma^3 - 1.773\gamma^2 + 1.088\gamma - 0.002$. In absolute value, the slope in E_γ is six times as important as that of the slope in C_γ. Accordingly, the sensitivity to reduce the influence of high NO in E_γ is six times as important. Therefore, this new curve allows to locate a new investigation interval of γ to decrease the effects of NO. Let us define

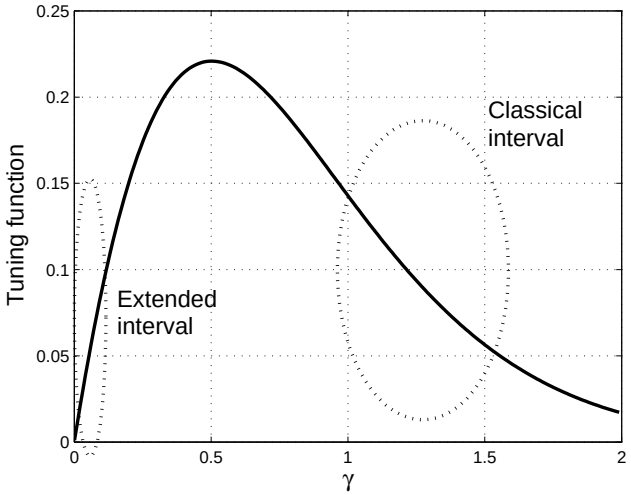

FIG. 1: Tuning function with two main intervals. Classical interval C_γ with $\gamma \in [1, 1.5]$ and extended interval E_γ with $\gamma \in [0.001, 0.2]$.

$\tilde{\Psi}(\theta,\gamma) = [\Psi(\theta,\gamma) \ \ \partial_\gamma W_N(\theta,\gamma)]^T$, where $\tilde{\Psi}(\theta,\gamma) \in \mathbb{R}^{d+1}$ and $\Psi(\theta,\gamma) = \partial_\theta W_N(\theta,\gamma)$ named Ψ-function. We seek an optimal value of γ such that $W_N(\theta,\gamma)$ has a global minimum with probability one (w.p.1) as N tends to infinity with $\overline{W}(\theta,\gamma) = \lim_{N\to\infty} EW_N(\theta,\gamma)$. This involves that

the solution of $\tilde{\Psi}\left(\hat{\theta}_N^H, \hat{\gamma}\right) = 0$ is unique meaning a global minimum of $\hat{\theta}_N^H$ such that $\hat{\gamma} \to \gamma^*$ and $\hat{\theta}_N^H \to \theta^*$ w.p.1 as $N \to \infty$.

3 Experimental results

Experimental results are presented over 16 CO, 15 PD, and 19 HD, left and right feet for different estimation norms and a campaign of estimations is carried out in C_γ with $\gamma^* = 1.5$ and E_γ with $0.001 \leq \gamma^* \leq 0.2$. For each estimator, namely LSE, LSAD (Least Sum of Absolute Deviation), L_∞ and Huberian, comparisons between CO vs PD and HD for left and right feet are given. Table. 1 shows the means of $\gamma*$, $RMSE$, $FIT(\%)$ ($100\left(1 - \frac{y-\hat{y}}{y-<y>}\right)$ where y, $\hat{y}$ and $< y >$ are the process output, the prediction model output and the mean of the process output, respectively), $L_2C(\%)$, $L_1C(\%)$ and the total number of parameters $n = n_A + n_C$. L_2C and L_1C are the L_2 and L_1 contributions respectively given by $L_iC = \frac{card\left[\nu_i(\hat{\theta}_N^H, \gamma^*)\right]}{N}$. These are indicators of the density of NO in the prediction errors. We focus on the main results in Table. 1. L_2, L_1 and L_∞ estimators give bad results with large RMSE, low FIT and large number of parameters between 40 and 70. In C_γ for $\gamma^* = 1.5$, the number of parameters is reduced with $25 \leq n \leq 32$ but not sufficient for a reduced order ARMA modeling. The Huberian approach in E_γ leads to relevant results. In Corbier and Carmona [5] we showed that $d_{\mathrm{M}}^H < d_{\mathrm{M}}^{L_1} < d_{\mathrm{M}}^{L_2}$ where d_{M}^H is the Huberian model order. This result is still verified here for a signal modeling. Main results show that $< \gamma^*_{control} > \approx 2 < \gamma^*_{disease} >$, meaning that there are twice more NO in STS-PD and STS-HD than STS-CO. Indeed, for PD and HD, the estimation requires a low value of γ^* involving a large value of the L_1 contribution close to 70%. For CO, $\gamma^* \approx 0.19$ and $L_1C \approx 58\%$. Figure. 2 and 3 show two ARMA models for left CO ($\gamma^* = 0.05$) and left PD ($\gamma^* = 0.003$) respectively with a FIT close to 83%. In Figure. 3 NO clearly appear in index-times $k = 52$, $k = 113$, $k = 190$ and $k = 247$. We can notice the good behavior of the Huberian reduced order ARMA model during this phase. Equation (4) shows the reduced order ARMA model of left PD for $\gamma^* = 0.003$.

$$\begin{aligned}\delta t_k = 0,712\delta t_{k-1} + 0,022\delta t_{k-2} + 0,018\delta t_{k-3} + 0,181\delta t_{k-4} \\ +0,060\delta t_{k-5} + e_k - 0,236e_{k-1} - 0,065e_{k-2} + 0.141e_{k-3} \\ -0,098e_{k-4} \qquad (4)\end{aligned}$$

The limited number of ARMA parameters contradicts conclusions in [1]. These studies showed a stride intervals of normal human walking which exhibit long-range temporal correlations. They presented a highly simplified walking model by reproducing the long-range correlations observed in stride intervals without complex peripheral dynamics. Based on fractal approach they showed an important point of view related to the long-range *memory effect* of human walking [3]. Our new approach shows a *short-range memory effect* for normal and disease human walking. It remains to investigate this *memory effect* and try to interpret in physiological terms the correlations with the CNS.

4 Conclusion

The main purpose of this paper has been to present a reduced order ARMA estimation method based on a robust approach using Huberian function for the neurodegenerative disorder signal modeling. A new approach has been presented to choose the threshold in Huberian function, allowing a best treatment of the natural outliers contained in the signals. The reduced number of parameters is due to a relevant choice of this threshold in a new interval range. However, it remains to characterize more appreciably the diseases to differentiate the neurodegenerative disorders. future work will focus on mixed L_p estimator [5] to reduce the number of parameters providing new indicators and will investigate the *memory effect* of human walking.

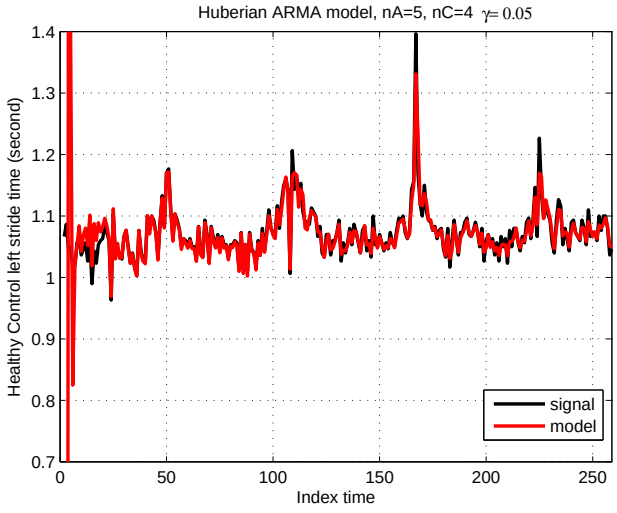

FIG. 2: Left Huberian ARMA model (red line) vs CO real signal (black line) at $\gamma = 0.05$.

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TAB. 1: Means of $\gamma*$, $RMSE$, $FIT(\%)$, $L_2C(\%)$, $L_1C(\%)$ and $n = n_A + n_C$ for different estimation norms. C_γ is the classical interval with $< \gamma* >= 1.5$. E_γ is the extended interval low values of $\gamma*$.

			CO left						**PD left**			
Estimator	$\gamma*$	$RMSE$	FIT	L_2C	L_1C	n	$\gamma*$	$RMSE$	FIT	L_2C	L_1C	n
L_2	–	11.2	10	100	0	70	–	13	9	100	0	70
L_1	–	4.3	42	0	100	41	–	5.2	38	0	100	46
L_∞	–	4.2	25	–	–	45	–	5.3	26	–	–	56
Huber in C_γ	1.5	2.4	42	95	5	25	1.5	3.1	31	96	4	28
Huber in E_γ	0.17	0.09	92	41	59	9	0.09	0.34	78	30	70	9
			CO right						**PD right**			
L_2	–	10.2	9	100	0	70	–	13	9	100	0	70
L_1	–	5.3	44	0	100	39	–	6.2	35	0	100	46
L_∞	–	3.2	26	–	–	46	–	5.5	28	–	–	54
Huber in C_γ	1.5	2.3	44	96	4	27	1.5	3.3	31	96	4	30
Huber in E_γ	0.18	0.08	92	43	57	9	0.09	0.29	78	32	68	9
			CO left						**HD left**			
L_2	–	11.2	10	100	0	70	–	8	17	100	0	70
L_1	–	4.3	42	0	100	41	–	4.1	36	0	100	44
L_∞	–	4.2	25	–	–	45	–	6.3	24	–	–	54
Huber in C_γ	1.5	2.4	42	95	5	25	1.5	3.2	32	96	4	31
Huber in E_γ	0.17	0.09	92	41	59	9	0.08	0.28	78	29	71	9
			CO right						**HD right**			
L_2	–	10.2	9	100	0	70	–	13	9	100	0	70
L_1	–	5.3	44	0	100	39	–	6.2	35	0	100	46
L_∞	–	3.2	26	–	–	46	–	5.1	32	–	–	56
Huber in C_γ	1.5	2.3	44	96	4	27	1.5	3.5	29	95	5	32
Huber in E_γ	0.18	0.08	92	43	57	9	0.07	0.16	87	27	73	9

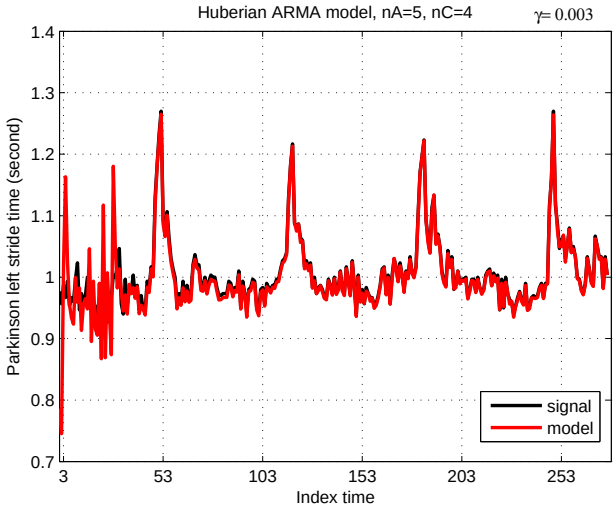

FIG. 3: Left Huberian ARMA model (red line) vs PD real signal (black line) at $\gamma = 0.003$.

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