Green Power Control for large MIMO systems

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Résumé – Dans ce travail, une nouvelle méthode de contrôle de la puissance distribuée en MIMO (Multiple Input Multiple Output) des canaux d'interférence est proposé. La méthode proposée consiste à traiter le problème d'optimisation en tant que partie de commande de puissance en prenant l'efficacité énergétique comme la métrique de performance. En contraste avec les travaux connexes sur le contrôle de la puissance et de l'efficacité énergétique, cette mesure inclut les effets de l'utilisation des blocs finis pour la transmission, à l'aide des estimations de canal au niveau du récepteur, et compte tenu de la puissance totale consommée par l'émetteur au lieu de simplement la puissance rayonnée. Chaque joueur dans cette jeu, représente un émetteur MIMO qui tente d'améliorer sa propre efficacité énergétique en choisissant le meilleur niveau de puissance d'émission. Dans ce scénario, il est démontré que le jeu en résulte a un équilibre de Nash unique à laquelle l'algorithme de BRD (Best Response Dynamics) convergent. Notre analyse est étayée par les résultats numériques qui indiquent que le contrôle de la puissance distribuée peut être un mécanisme précieux pour améliorer l'efficacité énergétique dans les réseaux sans fil.

Abstract – In this work, a novel method of distributed power control in large MIMO (multiple input multiple output) interference channels is proposed. The proposed method is to treat the optimization problem as a power control game with the energy-efficiency as the performance metric. In contrast with related works on power control and energy-efficiency, this metric includes the effects of using finite blocks for transmitting, using channel estimates at the receiver, and considering the total power consumed by the transmitter instead of just the radiated power. Each player in the resulting game represents a MIMO transmitter that tries to improve its own energy-efficiency by choosing the best transmit power level. In this scenario, it is shown that the resulting game has a unique Nash equilibrium to which the best response dynamics (BRD) algorithm converge. Our analysis is supported by numerical results which indicate that distributed power control can be an invaluable mechanism to improve the energy-efficiency in wireless networks.

1 Introduction

Using large multiple antennas, virtual multiple input multiple output (MIMO) systems, and small cells is envisioned to be one way of contributing to reducing energy consumption and improving data rates [1]. The work reported in this paper concerns interfering MIMO transmitter-receiver pairs in which communication links evolve in a quasi-static manner. The strength of interference is primarily determined by the path differences between the various transmitter-receiver pairs, for example, a base station and a user interfering with a user connected to a neighboring base station. The performance metric considered for measuring energy-efficiency of a MIMO communication corresponds to a trade-off between the net transmission rate (transmission benefit) and the consumed power (transmission cost).

The ultimate goal pursued in this paper is a relatively important problem in signal processing for communications. It consists of tuning the transmit power of the transmitted signal optimally in a distributed manner like in [2]. Several works like [3] use game theory for power control in MIMO system **when channel state information is available both at the transmitter and receiver ends.** The present paper aims at optimizing the transmit power in the sense of energy-efficiency as stated in [4] and [5]. In [4], energy-efficiency is defined as the ratio of the probability that the channel mutual information is greater than a given threshold to the used transmit power for single link systems. In [5], the more general framework of imperfect CSI is considered. This work, extends [5] to a multi-user and multi-cell framework and tackles the problem of distributed power control using results from game theory. The major contribution of this work is to study the power control games for interference channels when **when imperfect channel state information is available only at the receiver end.** This kind of system with no CSI available at the transmitter is more easily deployed as there is no requirement of advanced feedback mechanisms that have to send information on all the channels from the receiver to the transmitter.

2 System model

We consider a MIMO interference channel with K interfering transmitter-receiver pairs. Each of the K transmitter-receiver pairs is equipped with M_k transmit antennas and N_k receive antennas. As the transmitter does not receive any feedback on the channel, uniform power allocation is used among the antennas, but the total transmit power P_k is controlled. Denote by $\underline{P}$, a vector of dimension K, with the transmit power used by the k-th transmitter as it's k-th component. The average SINR (without considering fast-fading effects) of the k-th receiver is a function of $\underline{P}$ and is given by:

$$\gamma_k(\underline{P}) = \frac{P_k g_{k,k}}{\sum_{j \neq k} g_{j,k} P_j + \sigma_k^2} \tag{1}$$

where $g_{i,j}$ is the term modeling channel fading due to path loss, from transmitter i to receiver j, and σ_k^2 translates the noise level at the receiver. We assume that the channel obeys the simple discrete-time block-fading law, where the channel is constant for some time interval, after which it changes to an independent value that it holds for the next interval [6]. Each transmitted block of data is assumed to comprise a training sequence in order for the receiver to be able to estimate the channel; the training sequence length in symbols will be denoted by t_s and the block length in symbols by T_s. Based on the results in [6], an effective SINR $\tilde{\gamma}_k$ can replace the original SINR γ_k to account for the loss due to imperfect channel estimation at the receiver of a user, given by:

$$\tilde{\gamma}_k(\underline{P}) = \frac{\frac{t_s}{M_k}\gamma_k^2(\underline{P})}{1 + \gamma_k(\underline{P}) + \gamma_k(\underline{P})\frac{t_s}{M_k}} \tag{2}$$

For performance evaluation, each transmitter k considers its own energy efficiency η_k, defined as:

$$\eta_k(\underline{P}) = \frac{R_0 \xi \left(1 - \frac{t_s}{T_s}\right) \Pr_{\mathbf{H}}\left[\log\left|\mathbf{I}_M + \frac{1}{M}\tilde{\gamma}_k(\underline{P})\mathbf{H}\mathbf{H}^H\right| \geq \xi\right]}{aP_k + b} \tag{3}$$

where $a > 0$, $b \geq 0$ are parameters, R_0 is a parameter which expresses in Hz (e.g., the system bandwidth); we define ξ as the target spectral efficiency. Pr evaluates the probability over all realizations of the MIMO channel $\mathbf{H}$ which is an independent and identically distributed complex Gaussian matrix of dimension $M_k \times N_k$. The notation $|\mathbf{A}|$ denotes the determinant of the (square) matrix $\mathbf{A}$. The numerator represents the benefit associated with transmitting namely, the net transmission rate of the communication and is measured in bit/s. The rate comprises a term $1 - \frac{t}{T}$ which represents the loss in terms of information rate due to the presence of a training mechanism and a term representing the transmission success probability. Note that a packet is received successfully only if the associated mutual information is above a certain target. The denominator of (3) represents the cost of transmission in terms of power where b is a constant power consumption and P_k the radiated power.

3 Equilibrium analysis of the energy-efficient power control game

Since transmitter k, $k \in \mathcal{K}$, can only control the variable P_k of the $K-$variable function $\eta_k(\underline{P})$ and is assumed to consider the energy-efficiency of his own communication, using a non-cooperative game model is a relevant choice. Additionally, as it is assumed that power control is performed from block to block in an independent manner, using a static or one-shot game model is the most natural choice (see e.g., [2]). A non-cooperative game under strategic form is merely given by an ordered triplet. With the notations of this paper it writes as

$$\mathcal{G}_{\mathrm{X}} = \left(\mathcal{N}, \{\mathcal{P}_i\}_{i \in \mathcal{N}}, \{u_{i,\mathrm{X}}\}_{i \in \mathcal{N}}\right) \tag{4}$$

where the set of players is therefore the set of transmitters, the action space for player i is $\mathcal{P}_i = [0, P_{\max}]$, and $u_{i,\mathrm{X}}$ is the utility function of player i when the arrival rate model is X. As explained in Sec. II, when UDP is assumed, a QoS constraint is imposed on the packet loss. Under this assumption, the utility function is chosen to be the energy-efficiency, i.e. $u_k = \eta_k$.

Proposition 3.1 *The game $\mathcal{G}$ has a unique Nash equilibrium to which the proposed sequential best response dynamics algorithm always converges.*

Proof: The game $\mathcal{G}$ is quasi-concave when for all k, one of the following conditions are satisfied:

(a) $M_k \geq 1$, $N_k = 1$;

(b) $M_k \to +\infty$, $N_k < +\infty$;

(c) $M_k < +\infty$, $N_k \to +\infty$;

(d) $M_k \to +\infty$, $N_k \to +\infty$, $\lim\limits_{M_k \to +\infty, N_k \to +\infty} \dfrac{M_k}{N_k} = \ell < +\infty$;

Using the property of quasi-concavity of $\eta_k(\underline{P})$ proved in [5], we can show that $\mathcal{G}$ has at least one NE using the Reny's existence theorem in [7]. Furthermore, we can show that the player's best-responses are always standard functions ; where the best-response of player k to the (reduced) action profile $\underline{P}_{-k}$ is the single function defined by $\mathrm{BR}_i(\underline{P}_{-k}) = \arg\max\limits_{P_k} u_k(\underline{P})$. A function $F(x)$ is standard, if it satisfies the following properties :

1. $F(x_1) \geq F(x_2)$, if $x_1 \geq x_2$: Monotonic
2. $F(\lambda x) \leq \lambda F(x)$, if $\lambda \geq 1$: Scalable

Clearly as $P_{j \neq k}$ increases, the interference generated increases and so the best response is to play a higher P_k, and so the BR function is monotonic. Additionally as there is a set scaling based on the channel co-efficients $g_{j,k}$, the BR function is also scalable. This shows that the game has BR functions that are standard. Hence using the theorem in [8], $\mathcal{G}$ has a unique NE.

■

In this proposition, we see that for any number of large MIMO or any MISO transmitter-receiver pairs interfering with each other, the existence and convergence to the NE is guaranteed with the BRD algorithm. The implementation of this algorithm is detailed in [9] for a general utility function.

4 Numerical results and conclusion

In this section, we provide one of our numerical results that illustrate the validity of our approach. Specifically, we compare the utility of our completely distributed algorithm to a centralized algorithm for power control and to a scenario where power control is not implemented. In Figure 1, we plot the energy-efficiency v.s the fraction of distance from base station (x) to cell radius (L), i.e $\frac{x}{L}$, resulting in $g_{k,k} = \left(\frac{L}{x}\right)^3$ and $g_{j,k} = \left(\frac{L}{4L-x}\right)^3$ for $j \neq k$, using a cubic path loss exponent and assuming that immediately neighboring cells use different bands. NE corresponds to the points of Nash equilibrium, compared to the Social Fair Optimum (SFO), as in a multi-cellular network, users from different cells have to be served simultaneously. We also plot the case where $P_{max} = 100$ W is used always. For all three cases, we consider transmission at $\xi = 16, 8$ and 4 bps/hertz.

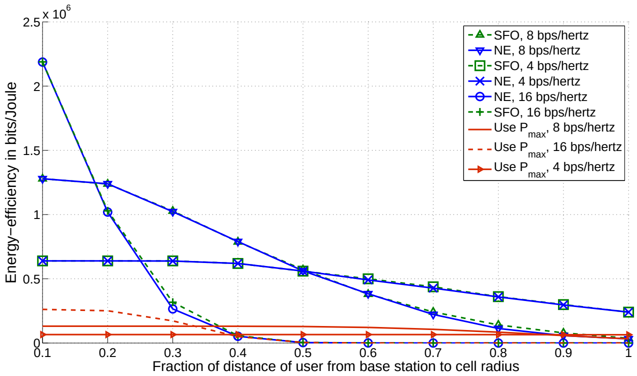

FIG. 1: Energy-efficiency (η_k) v.s distance from base station $\frac{x}{L}$, with $M_k = N_k = 2$, $b = 10$ W, $P_{max} = 100$ W

From Figure 1 it can be seen that the NE performs much more efficiently than using a simple strategy of using all the available power. Also surprisingly, it can be seen that the NE and the SFO are almost equally efficient for all values of $\frac{x}{L}$.

5 Conclusion

Our results indicate that the proposed distributed algorithm can offer an effective alternative to centralized optimizations in situations where the nature of the network makes it difficult to implement a centralized power control mechanism. It is also interesting to note that in terms of energy-efficiency, implementing power control is of great importance. An extension of this work would be to study the power allocation game when imperfect CSI is available at the transmitter of every MIMO systems.

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